Stanford Exploration Project, Report 115, May 22, 2004, pages 367–382

Analytical flattening with adjustable regularization

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ABSTRACT

We add an adjustable regularization parameter to the analytical flattening method which integrates dips in the Fourier domain. The regularization penalizes roughness in depth in the integration result which, in turn, insures that the flattening result is monotonic and continuous. This preserves the data which is necessary for multiple flattening passes or for undoing the flattening result. Because we preform the integration in the Fourier domain, this method is still highly efficient. 2D field gathers and a stacked section are provided as examples. This can easily be extended to 3D, allowing adjustable regularization along shot gathers.

INTRODUCTION

When flattening seismic data, it is important that the relative position of adjacent data points be preserved. In other words, it is important that the process be continuous and monotonic. If the flattening process is not monotonic, then points can be swapped creating artifacts in the data. This is important if the flattened data is to be unflattened or if multiple iterations are to be performed as these artifacts can worsen with each iteration. Simply smoothing the dip in depth prior to integrating should achieve a monotonic result, however, it is more desirable to penalize roughness in depth in a least squares sense while the dip is being integrated. In short, we want to apply an adjustable model styling goal to insure smoothness of the dip integration result.

In Lomask and Claerbout (2002); Lomask (2003) dips are integrated (summed) by a Fourier method with regularization. However, this regularization was not adjustable. Although it did insure that the integrated dip result was smooth, in many cases the depth regularization goal over-whelmed the dip integration goal, causing the result to be too smooth.

In this paper, we present a modification to the analytical flattening method that uses an adjustable weighting parameter for regularization in depth. This method is applied to several 2D field gathers provided by WesternGeco and compared to 2D field gathers that are flattened without regularization. A 2D section created by applying NMO prior to stacking is compared to the same 2D section created by flattening prior to stacking. We show that applying the analytical flattening method with regularization can preserve the integrity of the data even after multiple passes of flattening.

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METHODOLOGY

In Lomask and Claerbout (2002), we found that we could integrate local dip information ($\mathbf{p}_x$) into total time shifts ($\mathbf{t}_x = t(x)$) quickly in the Fourier domain with:

$$\mathbf{t}_x \quad \approx \quad \mathrm{FFT_{1D}}^{-1}\left[\frac{\mathrm{FFT_{1D}}\left[\nabla' \mathbf{p}_\mathrm{x}\right]}{-Z_x^{-1}+2-Z_x}\right], \tag{1}$$

where $Z_x = e^{iw\Delta x}$.

We also found that if we initialized the dips in the y direction ($\mathbf{p}_y$) to zero, then this equation would apply some kind of regularization in the y direction:

$$\mathbf{t} \quad \approx \quad \mathrm{FFT_{2D}}^{-1}\left[\frac{\mathrm{FFT_{2D}}\left[\nabla' \mathbf{p}\right]}{-Z_x^{-1}-Z_y^{-1}+4-Z_x-Z_y}\right], \tag{2}$$

where $\mathbf{t} = t(x,y)$, $\mathbf{p} = (\mathbf{p}_x, \mathbf{p}_y)$, $Z_x = e^{iw\Delta x}$ and $Z_y = e^{iw\Delta y}$. This would cause the integration to be smooth in the y direction. However, we were not able to control how smooth it would be.

Here we will add an adjustable regularization parameter (ϵ) to equation (2). We begin with the fitting goal:

$$\nabla \mathbf{t} \quad = \quad \mathbf{p}. \tag{3}$$

We can minimize the difference between the estimated slope and the theoretical slope with:

$$0 \quad \approx \quad \nabla \mathbf{t} \quad - \quad \mathbf{p}. \tag{4}$$

Next, we write the quadratic form to be minimized as:

$$Q(\mathbf{t}) = (\nabla \mathbf{t} - \mathbf{p})'(\nabla \mathbf{t} - \mathbf{p}). \tag{5}$$

Because the gradient is ($\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$), we can write:

$$Q(\mathbf{t}) = \begin{bmatrix} \frac{\partial \mathbf{t}}{\partial \mathbf{x}} - \mathbf{p}_x \\ \\ \frac{\partial \mathbf{t}}{\partial \mathbf{y}} - \mathbf{p}_y \end{bmatrix}' \begin{bmatrix} \frac{\partial \mathbf{t}}{\partial \mathbf{x}} - \mathbf{p}_x \\ \\ \frac{\partial \mathbf{t}}{\partial \mathbf{y}} - \mathbf{p}_y \end{bmatrix}. \tag{6}$$

This can be rewritten as:

$$Q(\mathbf{t}) = \left(\frac{\partial \mathbf{t}}{\partial \mathbf{x}} - \mathbf{p}_x\right)^2 + \left(\frac{\partial \mathbf{t}}{\partial \mathbf{y}} - \mathbf{p}_y\right)^2. \tag{7}$$

The second term in equation (7) is the regularization term and only needs a scalar parameter ϵ to adjust its weight relative to the first term. Now we have:

$$Q(\mathbf{t}) = \left(\frac{\partial \mathbf{t}}{\partial \mathbf{x}} - \mathbf{p}_x\right)^2 + \epsilon^2\left(\frac{\partial \mathbf{t}}{\partial \mathbf{y}} - \mathbf{p}_y\right)^2. \tag{8}$$

Working backwards we see that it is now necessary to define a gradient operator that has an epsilon weight applied to one direction as:

$$\nabla_\epsilon = \left(\frac{\partial}{\partial x}, \epsilon \frac{\partial}{\partial y} \right). \tag{9}$$

It is also necessary to apply the scalar to the dip in the y direction as:

$$\mathbf{p}_\epsilon = (\mathbf{p}_x, \epsilon \mathbf{p}_y). \tag{10}$$

Lastly, the y components of the z-transform in the denominator of equation (2) also need to be scaled. The final analytical solution with an adjustable regularization parameter is:

$$\mathbf{t} \quad \approx \quad \mathrm{FFT_{2D}}^{-1} \left[\frac{\mathrm{FFT_{2D}} \left[\nabla_\epsilon' \mathbf{p}_\epsilon \right]}{-Z_x^{-1} - \epsilon Z_y^{-1} + 2 + 2\epsilon - Z_x - \epsilon Z_y} \right], \tag{11}$$

where $Z_x = e^{iw\Delta x}$ and $Z_y = e^{iw\Delta y}$.

EXAMPLES

We use the adjustable regularized flattening method on field CMP gathers from the Gulf of Mexico Mississippi Canyon provided by WesternGeco. Methods for using flattening for velocity analysis are currently being developed ((Guitton et al., 2004; Wolf et al., 2004)).

CMP gathers

Figure 1a shows a raw CMP gather. Figure 1b shows the same data after constant velocity (2000 $\mathtt{ms}^{-1}$) NMO has been applied. This reduces the maximum dip and therefore makes the dip estimation more robust. Figure 1c shows the result of one iteration of flattening with no regularization applied ($\epsilon = 0$). This means that each slice of dip is integrated independently. The estimated dip can be seen in Figure 2a. Fluctuations in dip can cause horizontal striations in the time shifts as in Figure 2b. These sudden changes in time shift can cause points to be swapped in depth or time. This can create the artifacts seen in Figure 1c. These artifacts cause errors in the dip estimation which cause the problem to grow with subsequent iterations. The estimated dip for the third iteration can be seen in Figure 1c. These dips are very small compared to the dips in 1a because the figures are clipped differently. However, notice the severity of the striations in the time-shifts after the third iteration shown in Figure 2d.

Figures 3 and 4 show the same data as Figures 1 and 2 except using regularization ($\epsilon = 2$). Notice that the integrated dip shown in Figure 4b is much smoother than the result shown in Figure 2b.

After 3 iterations of flattening, Figure 3d, the gather is flat and the data is still preserved. If necessary, this data can be easily unflattened because the data integrity is intact.

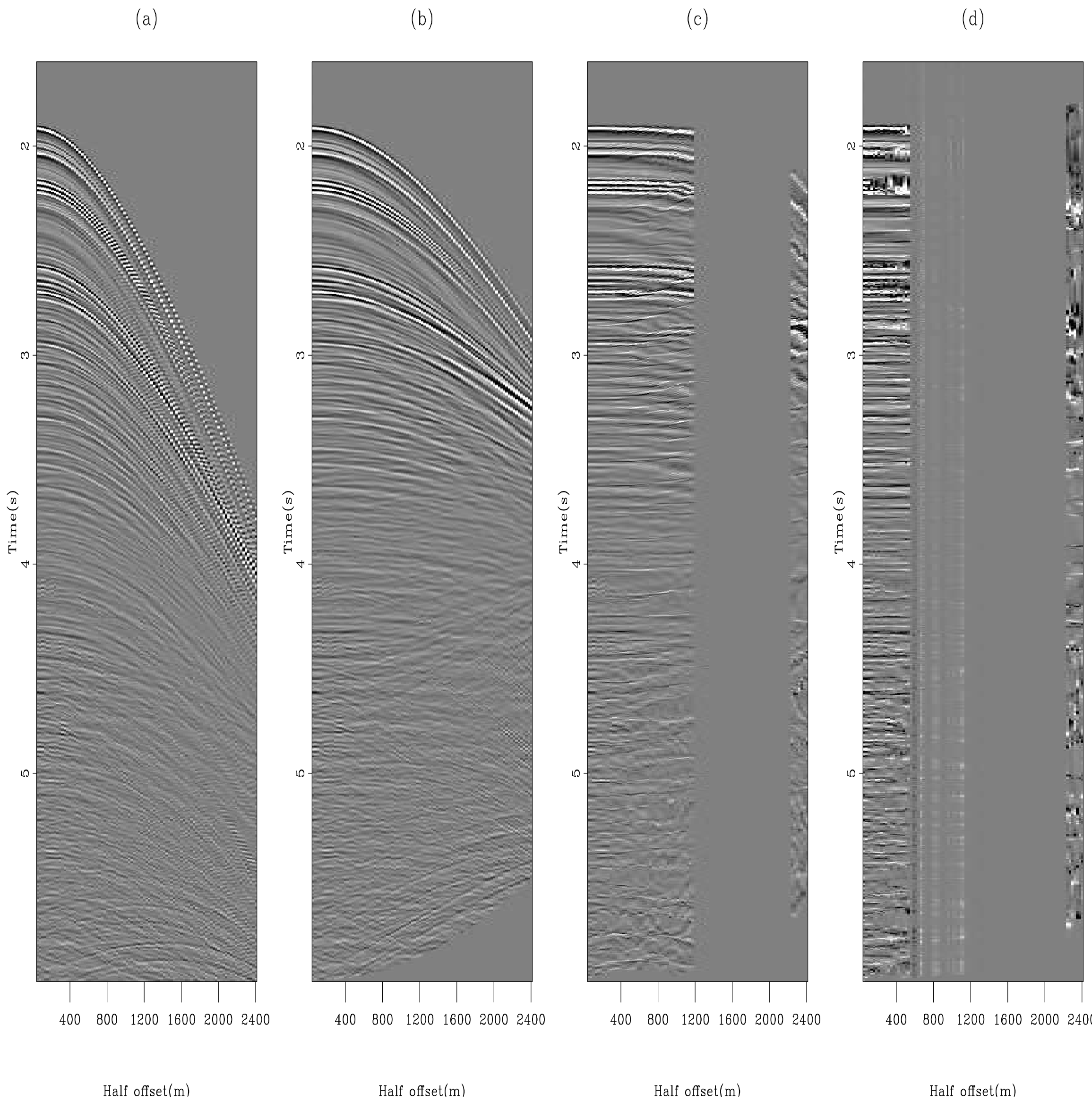

Figure 1: (a) Raw CMP gather. (b) CMP gather with constant velocity applied, 2000 $\mathtt{ms}^{-1}$. (c) Flattened version of (b) with no regularization, one iteration. (d) Flattened version of (b) with no regularization, three iterations. Notice the artifacts in (c) and (d) caused by data points being shifted without maintaining continuity and monotonicity. jesse2-cmp0 [ER,M]

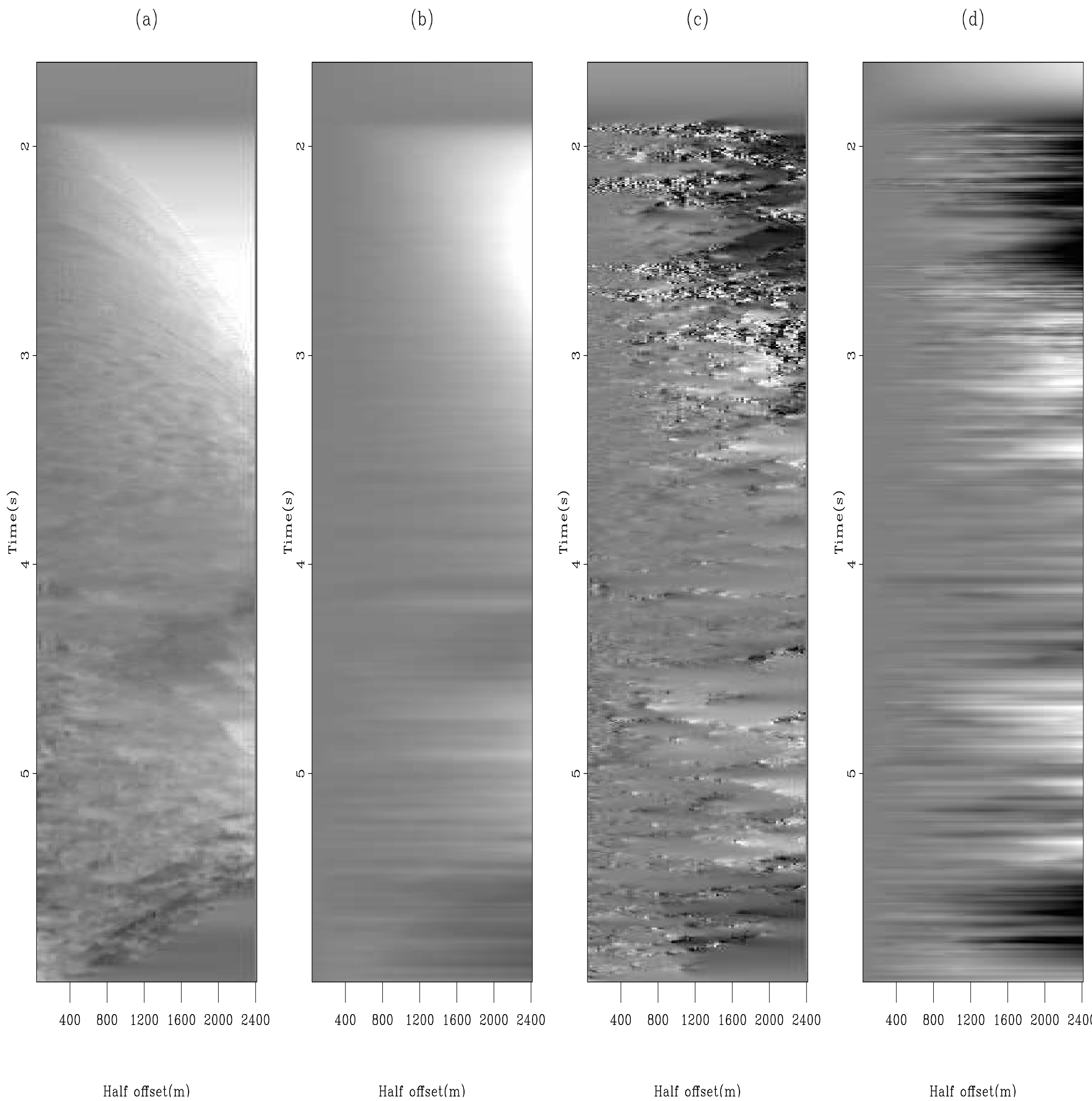

Figure 2: (a) Estimated dip from (b) in Figure 1. (b) Integration result of the dip in (a). (c) Estimated dip for third iteration. (d) Integration result of the dip in (c). jesse2-intcmp0 [ER,M]

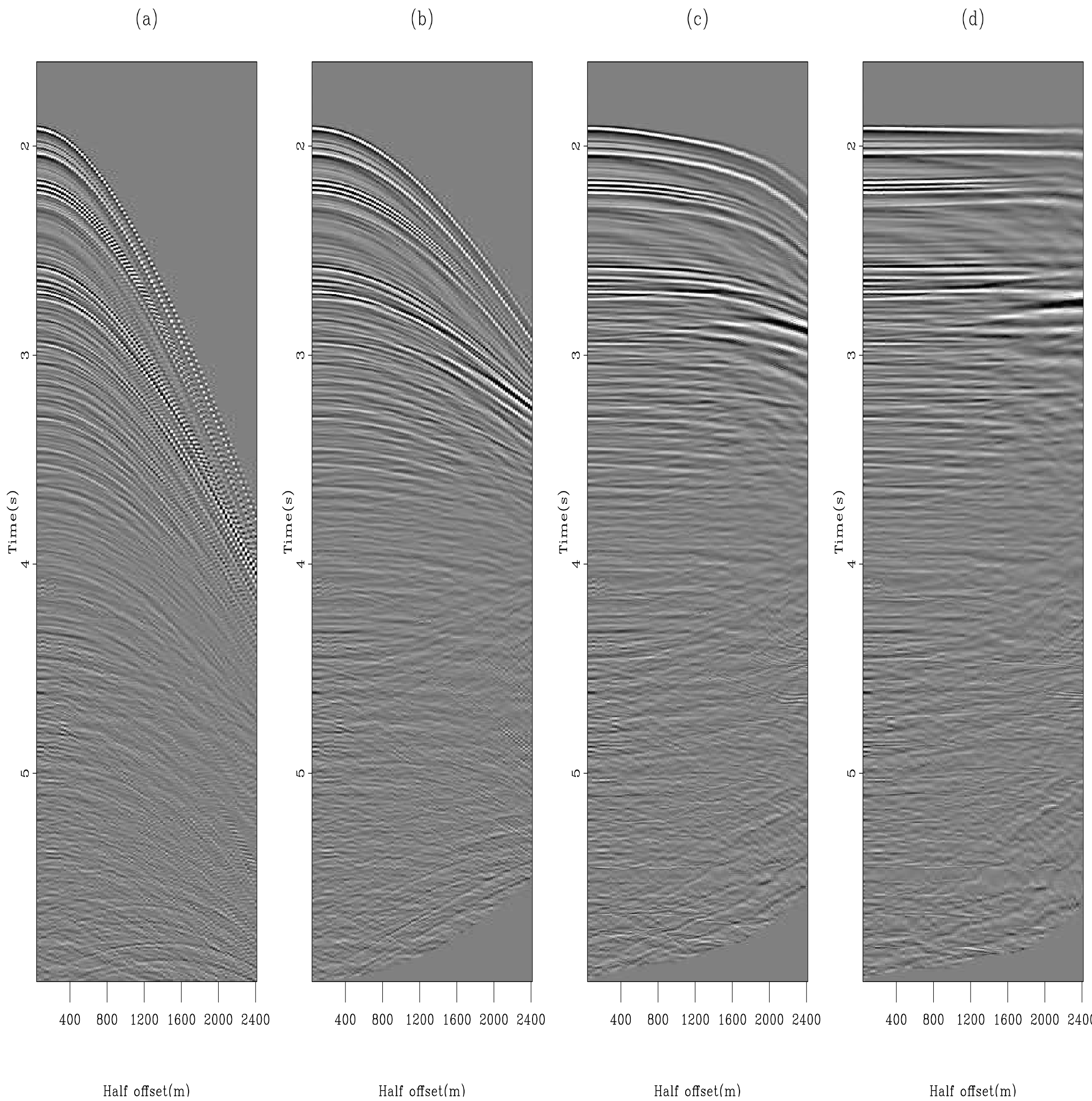

Figure 3: (a) Raw CMP gather. (b) CMP gather with constant velocity applied. (c) Flattened version of (b) with regularization (ϵ=2), one iteration. (d) Flattened version of (b) with regularization (ϵ=2), three iterations. Notice the significant improvement of the flattening result compared to Figure 1. jesse2-cmp1 [ER,M]

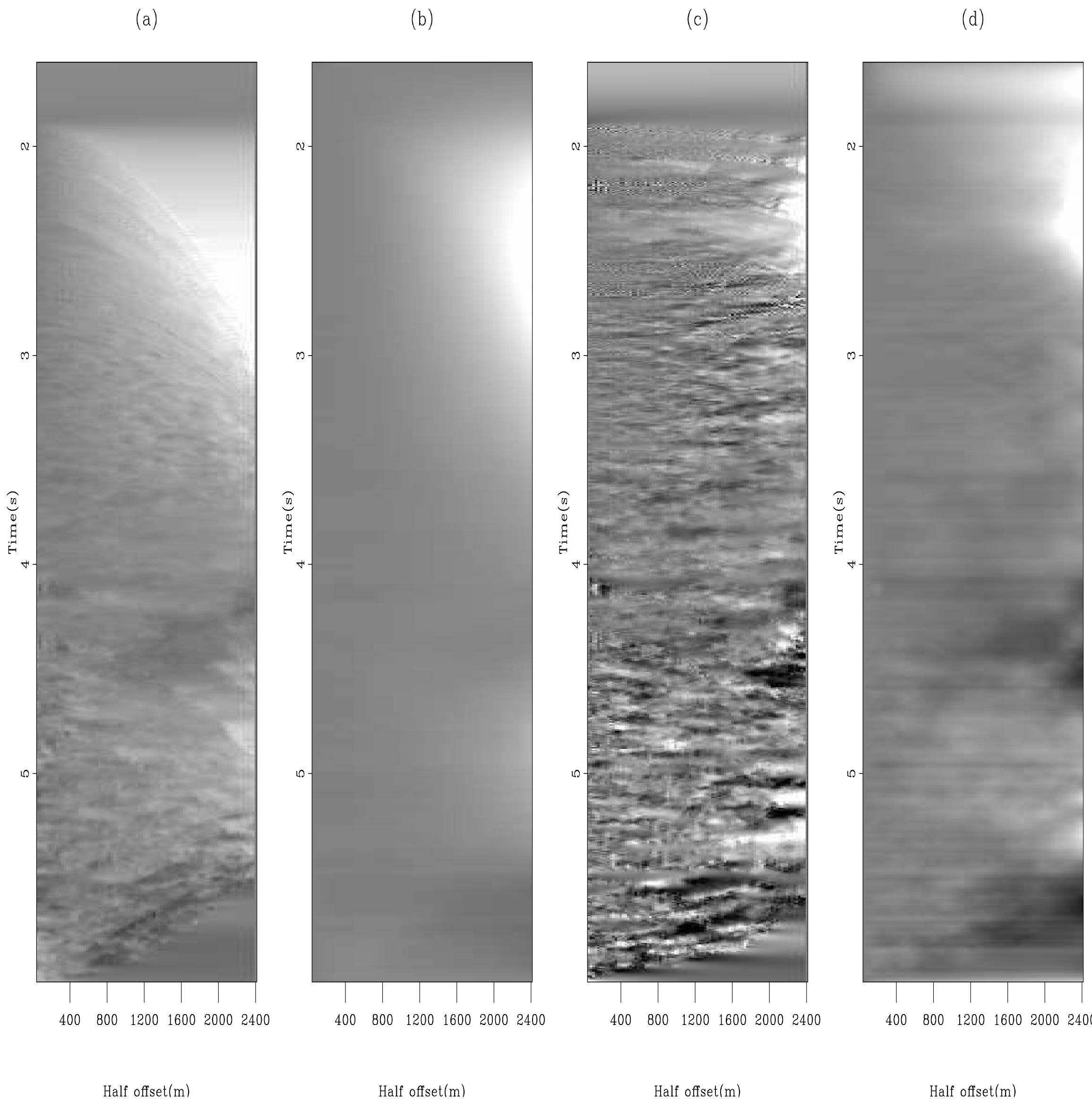

Figure 4: (a) Estimated dip from (b) in Figure 3. (b) Integration result of the dip in (a). (c) Estimated dip for third iteration. (d) Integration result of the dip in (c). jesse2-intcmp1 [ER,M]

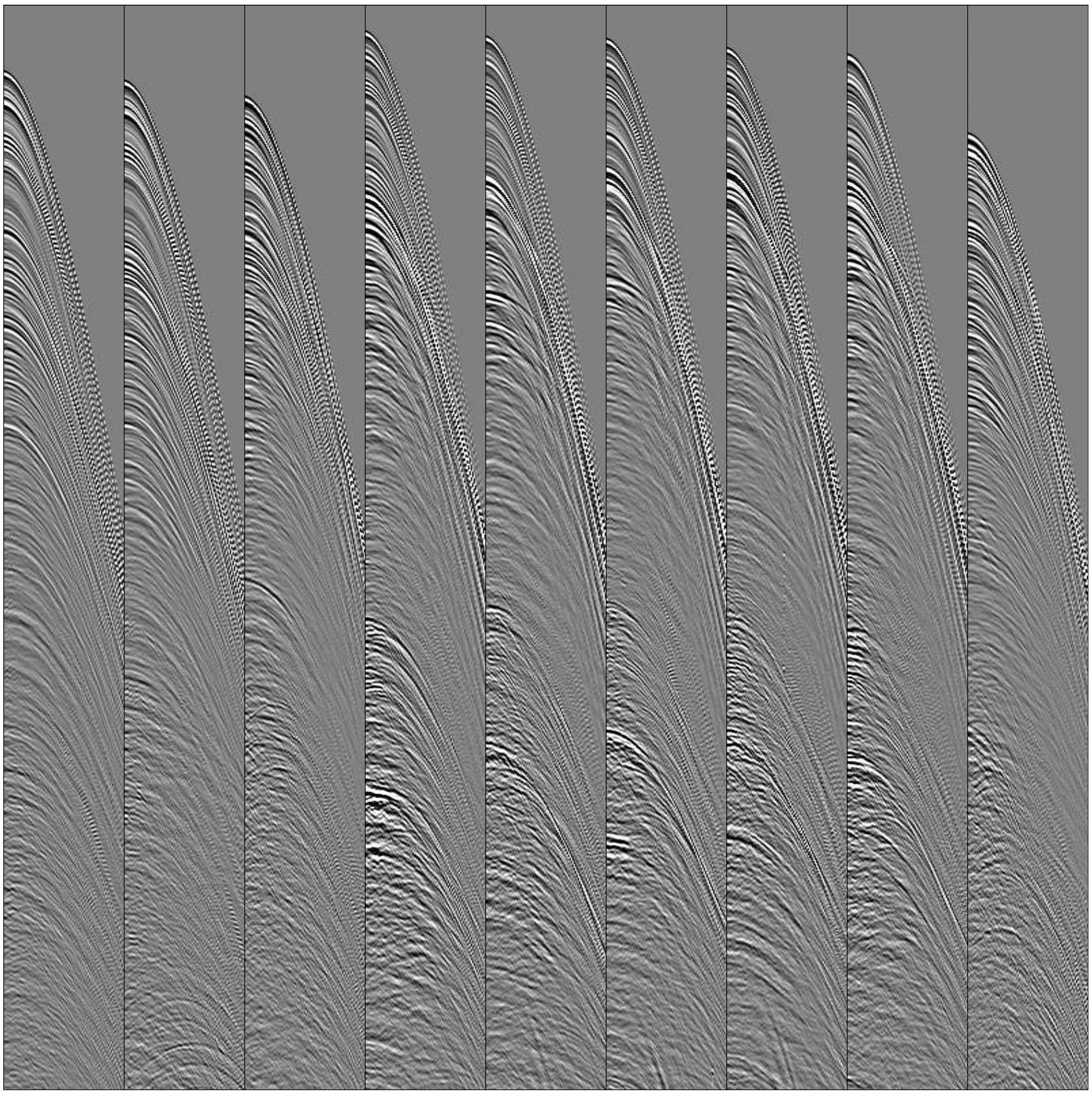

Figure 5: 9 sample gathers from the same data used to create a 2D stack section. Each gather is 2600 meters apart. jesse2-compcmpo [ER]

2D stacked section

Nine sample CMP gathers are shown in Figure 5. Radon de-multiple pre-processing has been applied but significant multiple energy can still be seen in the bottom half of the gathers. For comparison to the flattening method, conventional processing is first applied. Using the simple velocity field shown in Figure 6, NMO is applied to the gathers and the results are shown in Figure 7. This velocity is obviously not exact but many primaries are almost flat. The data is stretched badly and could benefit from a stretch mute.

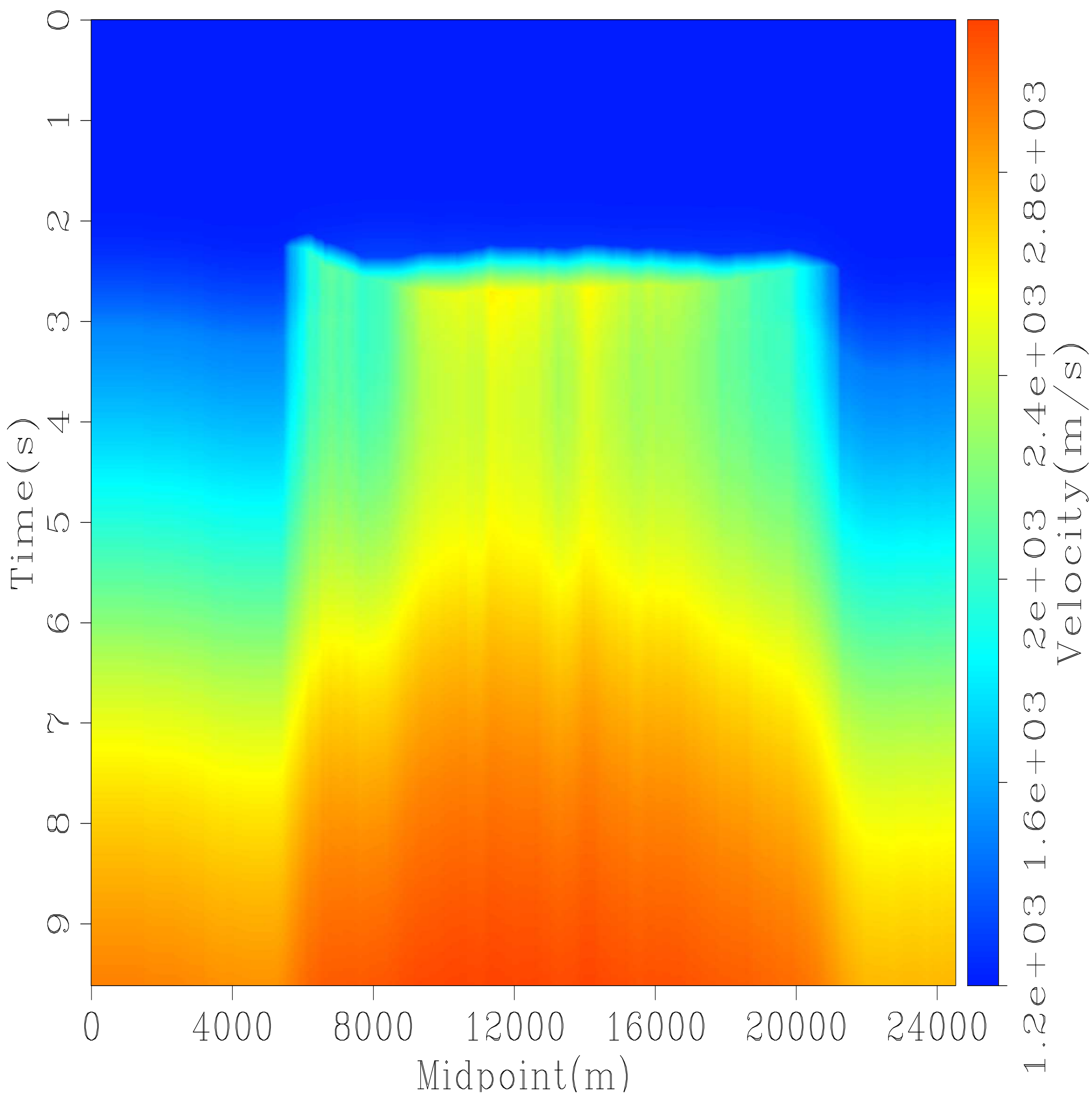

Figure 6: RMS velocity model. jesse2-v_rms [ER]

To reduce dip estimation errors, we first apply NMO with a constant velocity of 2000 ms^{-1}. Applying NMO with this simple velocity does not require knowing any velocity information

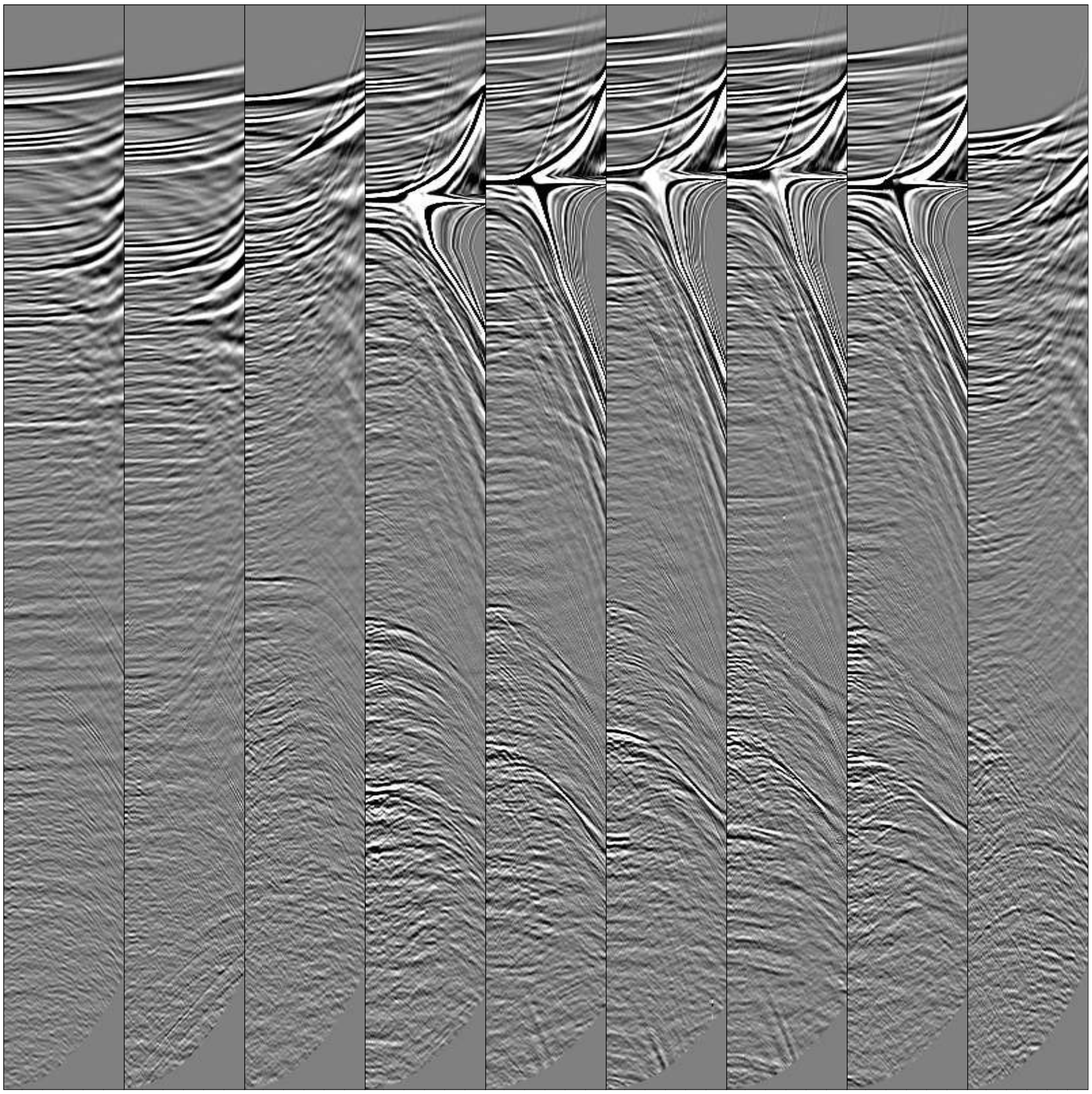

Figure 7: Same gathers as Figure 5 with NMO applied using the velocity function in Figure 6. This velocity is obviously incorrect in several areas but it does flatten many of the primaries and it does not flatten the multiples. jesse2-compnmo [ER]

at all. Next, the gathers are flattened. The nine flattened gathers are shown in Figure 10. These gathers don't have the same stretching errors as in Figure 7 but events with the strongest amplitude are flattened regardless of whether they are primaries, multiples, or headwaves. Evidence of this can be seen in the lower half where the flattening method has flattened the undesirable multiple energy. At the top of the gathers, the flattening method flattens the gathers significantly better than the NMO in Figure 7.

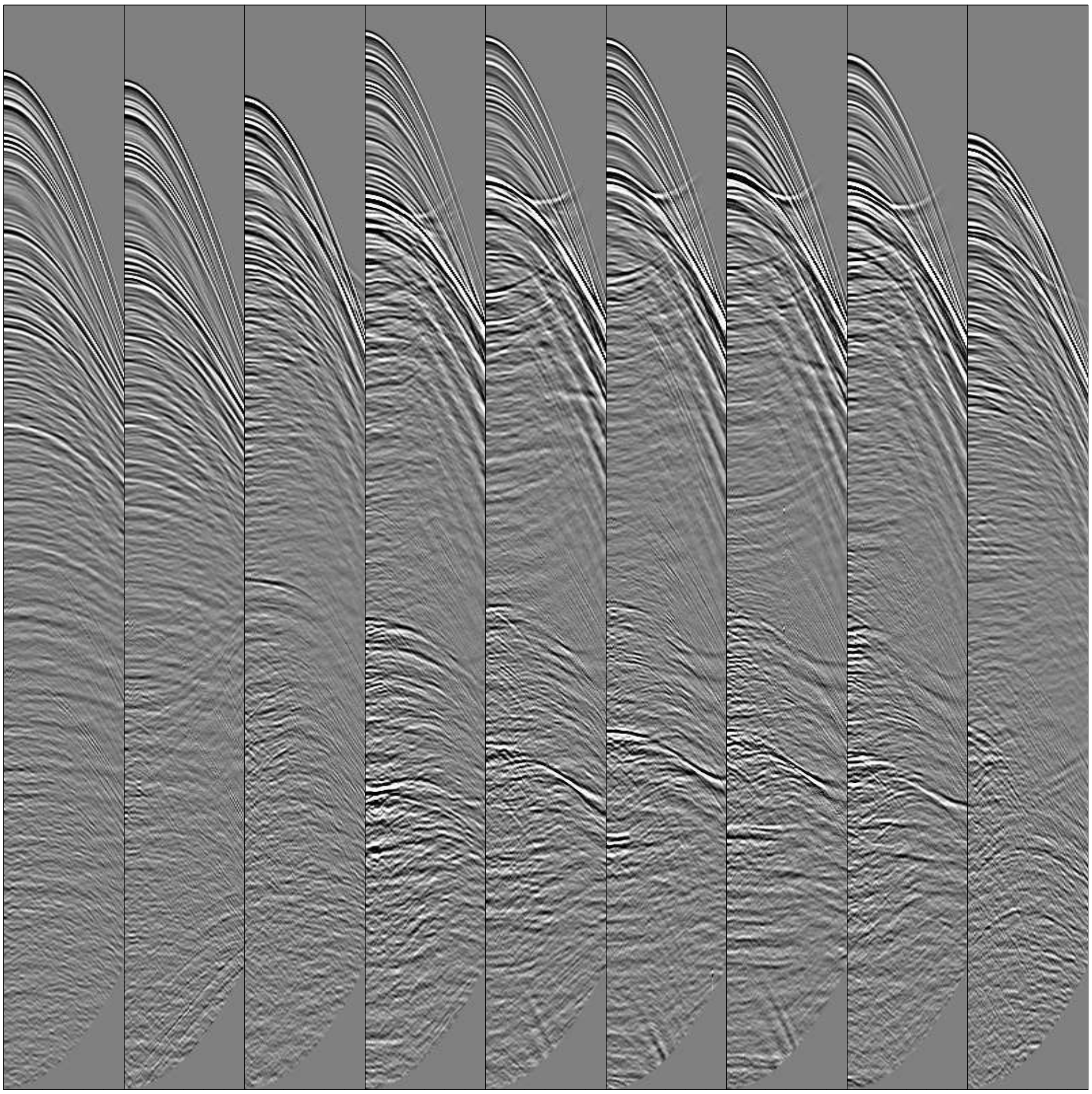

Figure 8: Same gathers as Figure 5 with NMO applied using a constant velocity function, 2000 ms^{-1}. jesse2-compcmp [ER]

The resulting stacked sections with conventional processing and flattening applied can be seen in Figures 9 and 11, respectively. The flattening method does a better job imaging the diffractions seen in the ellipse at the top of the section. However, in the conventional processing image, the primaries are imaged better in the lower left ellipse. In the ellipse on the lower right, the flattening does a much better job imaging the bright reflections.

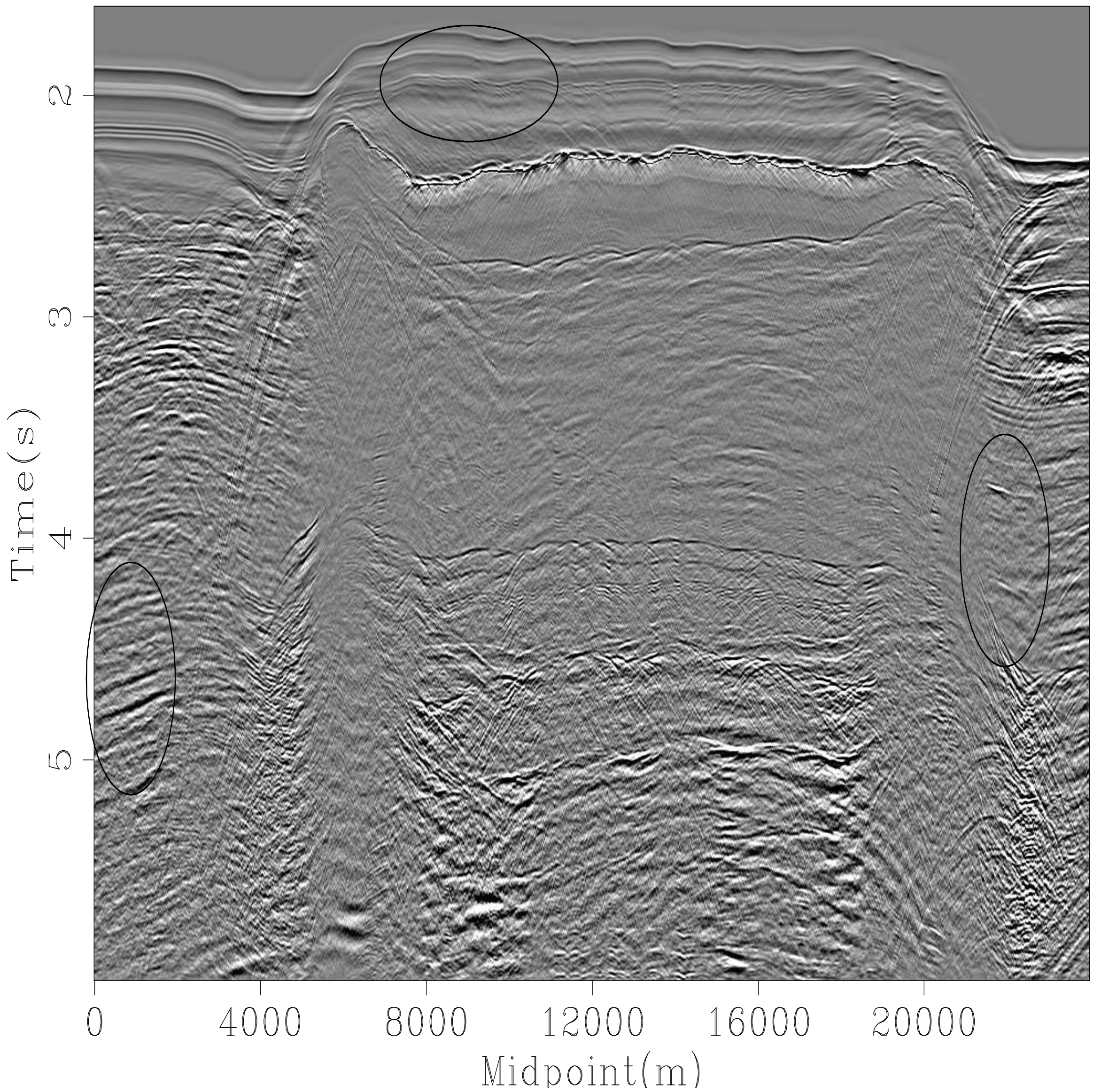

Figure 9: Stacked 2D section using the velocity in Figure 6. jesse2-stacknmo [ER]

In the stacked section after flattening was applied in Figure 11, there is significant jitter between CMP's. This is because each CMP was flattened independently. We can easily correct this by estimating dip from gather to gather and solve the analytical flattening method in 3D. An adjustable parameter that controls the weight of the flattening between gathers can be added as well.

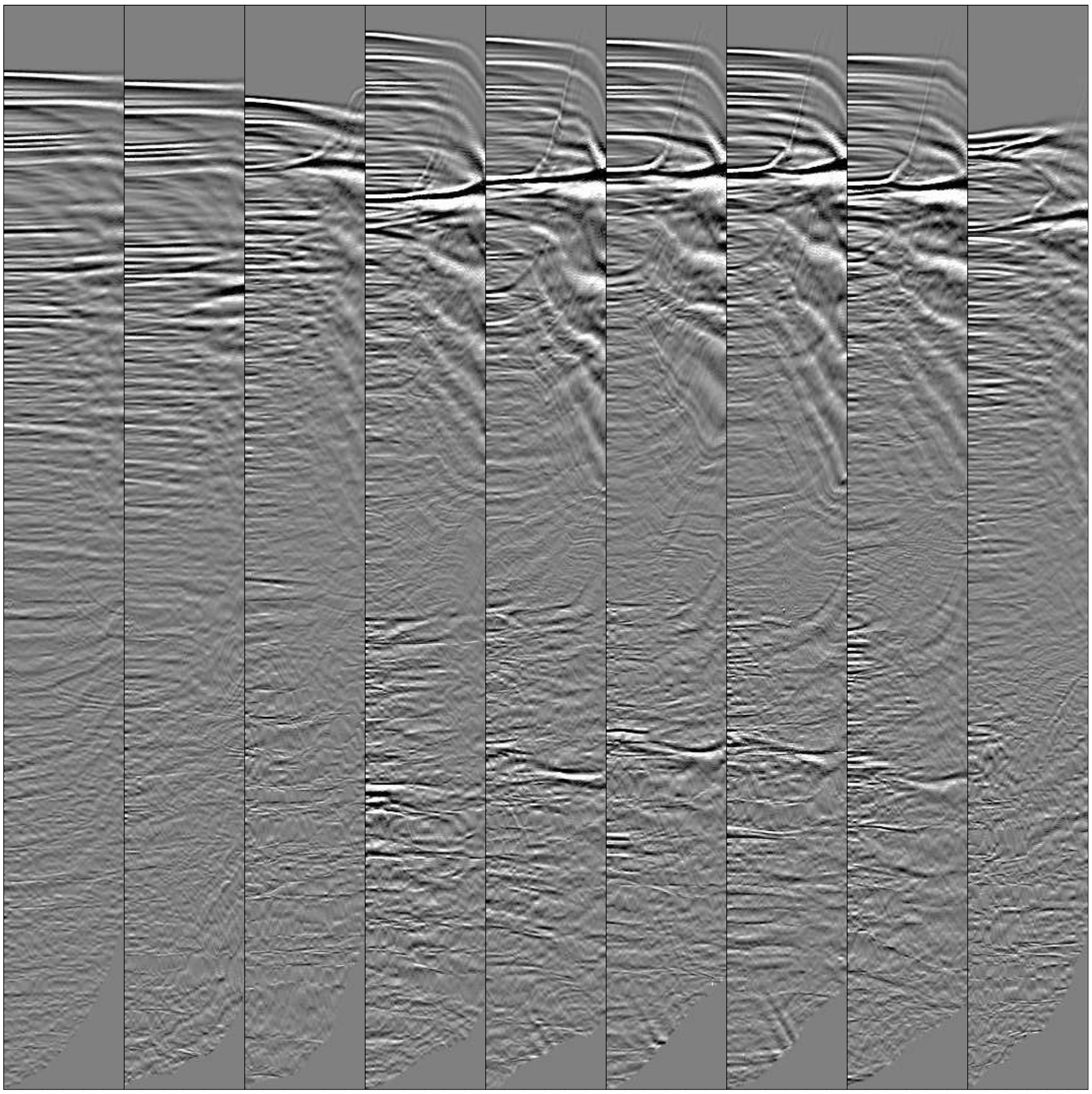

Figure 10: Flattened version of the gathers in Figure 8. jesse2-compflat [ER]

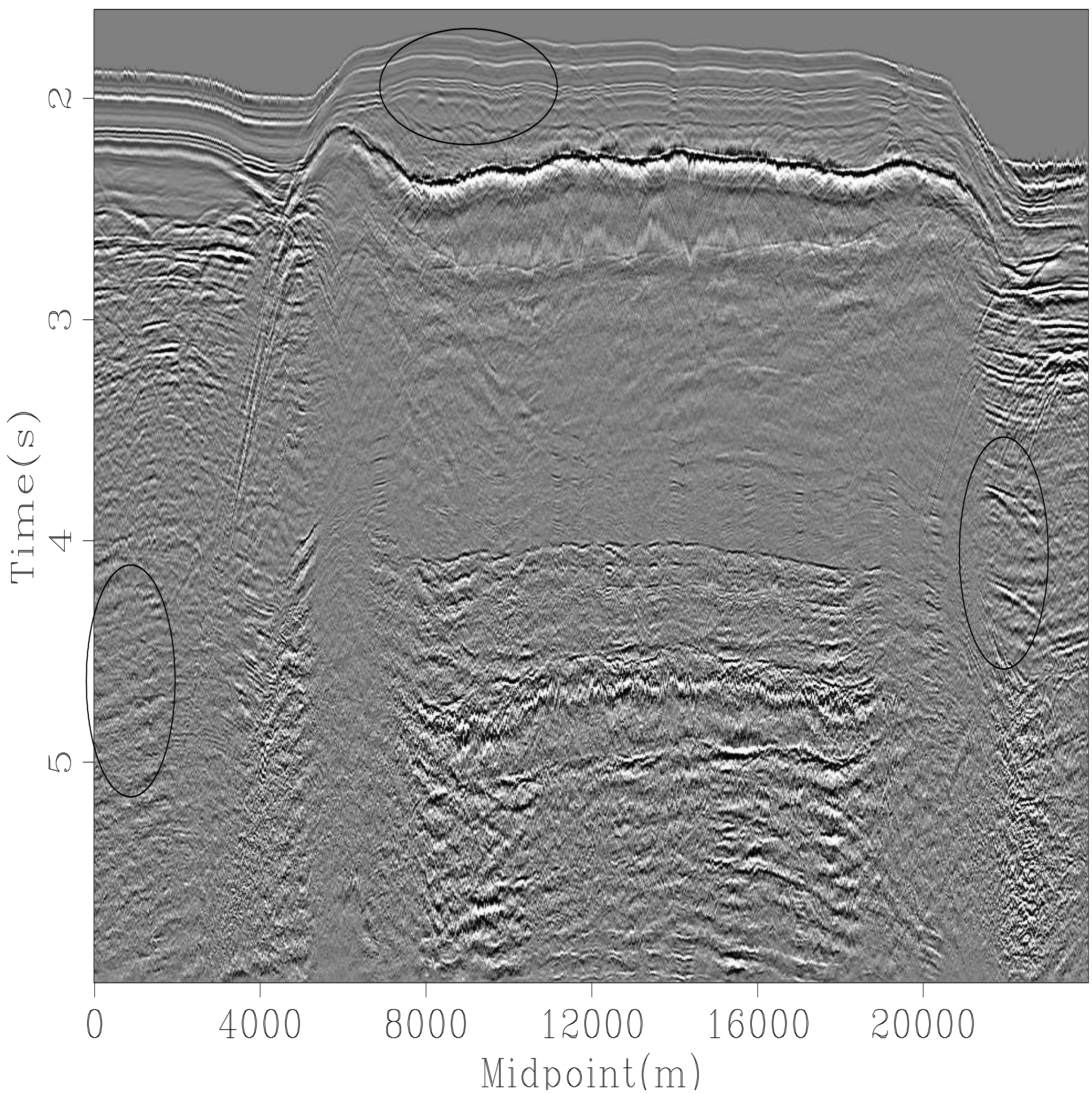

Figure 11: Stacked 2D section using flattened gathers. jesse2-stackflat [CR]

CONCLUSIONS

The analytical flattening method with adjustable regularization flattens data without creating discontinuities. Consequently, the data integrity is preserved allowing for multiple flattening iterations or for undoing the flattening.

There are other ways to achieve a continuous, monotonic flattening result. Smoothing the input dip in depth can achieve similar results. Alternatively, the method that we use to apply the time-shifts can, in principle, be modified so that it is constrained to preserve the relative data order.

The regularized analytical method can be used to find a smooth solution but it cannot fill in missing data. One of the main benefits of regularization is the ability to fill missing data. This requires a weight matrix or mask to discern known data from unknown data. Unfortunately, this weight matrix is singular and the analytical solution would require us to know its inverse. Therefore, in the case of regions of unknown dip, it is necessary to abandon the analytical method and solve the flattening problem in the time domain.

ACKNOWLEDGMENT

We would like to thank WesternGeco for providing the field data used in this paper.

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