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Short Note

Frequency-dependent velocity analysis?

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INTRODUCTION

There are two reasons for exploring imaging and velocity analysis with a frequency-dependent velocity model. First, significant dispersion may occur naturally in the volume of rock investigated by the seismic experiment. Second, the goal of Wave Equation Migration Velocity Analysis (WEMVA, Biondi and Sava (1999)) is image optimization, which some successful approaches (Pratt, 1999) perform by inverting one frequency at a time, from lower to higher frequencies. The velocity model that optimizes the image may vary with frequency within the physical error bar of the velocity analysis solution. In what follows, I will examine only whether dispersion from natural causes is strong enough to be considered in WEMVA.

Frequency dispersion of elastic waves is a frequent occurrence that is more noticeable in surface waves than in body waves (Pedersen et al., 2003). Some authors (Wang, 2001) highlight its importance and the infrequency with which it is explored by rock physics. Other articles (Marion et al., 1994) analyze the conditions in which dispersion appears. Techniques for modeling dispersion (Robinson, 1994), or for migration with a frequency and attenuation-dependent velocity model (Mittet et al., 1995) are currently available.

Dispersive phenomena in the dataset on which FEAVO was originally defined[2] are depicted in Figure 8 of Vlad and Biondi (2002). I further investigate quantitatively whether dispersion plays a large enough role to warrant performing the velocity analysis separately for each frequency, and migrating with a $v(x,z,\omega)$ velocity model. The expense and coding overhead associated with using a frequency-dependent velocity model in wave-equation migration would be negligible, because wave-equation imaging and migration velocity analysis are parallelized over frequencies.

MEASURING DISPERSION

A simple and efficient way to measure the dispersion of plane waves in a shot gather has been described by McMechan and Yedlin (1980; 1981). The method transforms a shot gather from

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[2] Grand Isle (LA), shallow water, courtesy of Dr. Ralph Shuey, Gulf Science and Technology Company.

the original (t,x) coordinate system to a (velocity, frequency) coordinate system, on which velocity can be picked as a function of frequency for each dispersion mode present in the data. The events to be analyzed this way must be linear. Surface waves fulfil this condition. An example of applying this method to a shot gather with ground roll stronger than the reflections is presented in Figure 1. The method consists of two steps: first the shot gather is slant-stacked, then it is Fourier-transformed along the time axis to obtain the power spectrum.

To investigate whether reflected events are dispersive, I transform hyperbolas (from a CMP gather this time, not from a shot gather) into straight lines using a $T^2 - X^2$ stretch. Linear event slopes will correspond to RMS velocities. I then apply the previously described algorithm for transforming the gather into an easy-to-pick (velocity, frequency) panel.

In practice, however, transforming to $T^2 - X^2$ coordinates then slant stacking is equivalent to summing along hyperbolic paths – a bare-bones Radon transform with no amplitude corrections. This cuts the cost in half and is the way I implemented it. Similar hyperbolic Radon transforms were used in velocity analysis, but they have been replaced in common practice by semblance panels. To obtain a semblance panel, instead of summing along hyperbolas, a statistical measure of coherence is computed along them. Feeding such a semblance panel into the dispersion analysis flow did not show improvement. The mathematics of the McMechan and Yedlin (1980) method seem to be geared towards actual summations along velocity-dependent paths. Deconvolution (which usually improves velocity analysis) did not result in visible changes either, which means that predictive deconvolution with the same filter for all traces in the dataset did not affect dispersion.

The results (bandpassed to eliminate DC components) are presented in Figure 2. Dispersion curves are much less visible than in the case of surface waves, but this may be because the number of reflections in the left panel of Figure 2 is considerably greater than the number of distinct linear events in the left panel of Figure 1. This may lead to mutually destructive interference in the dispersion analysis panel. The inferable trend does not warrant a variation in velocity of more than 100m/s across the reflection seismic frequency spectrum. These conclusions from a single Mississippi Delta 2-D line cannot be generalized to all seismic reflection data. However, I did not find any literature or anecdotal evidence pointing to variations in body wave velocity of more than 100 m/s across the useful reflection seismic frequency range.

CONCLUSIONS

For the analyzed dataset, the velocity of seismic reflection events varies less than 100m/s with frequency across the usual exploration spectrum. There is no information available indicating greater variation for other datasets. Future wave-equation migration velocity analysis tools may allow for still greater variation, since the computational expense is not higher, and the inversion is more stable if the output of velocity analysis for low frequencies is taken as a starting model for higher frequencies. The dispersion in reflected waves does not warrant discarding proven velocity analysis and imaging software tools, but is worth considering when trying to build significantly more accurate ones.

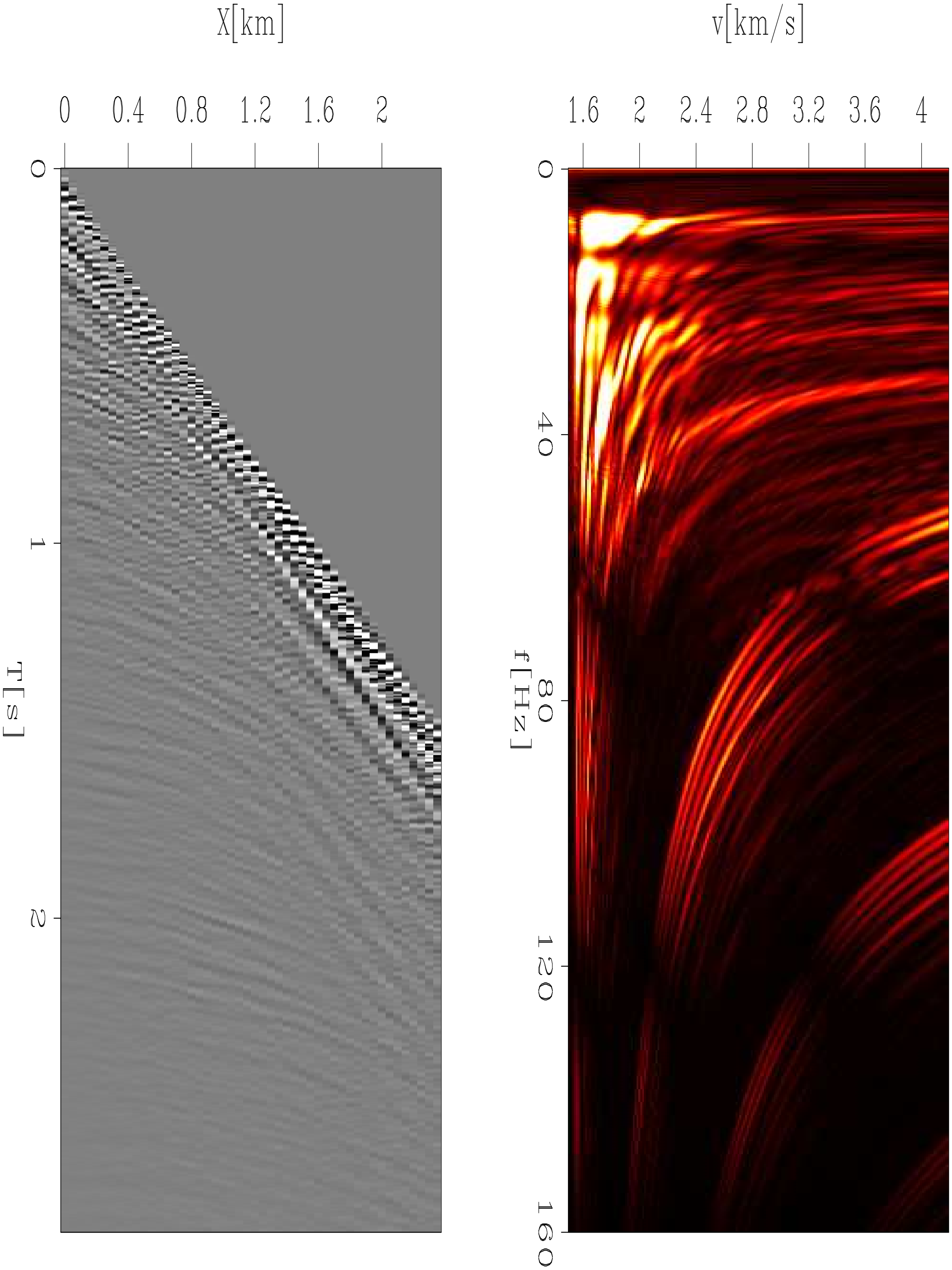

Figure 1: Example of applying the McMechan and Yedlin (1980) dispersion analysis method. **Left:** input shot gather. **Right:** output dispersion panel. Curved events represent various dispersion modes for surface waves. Other events correspond to different arrivals in the data (direct arrival, reflections, etc.). nick2-trytool [CR]

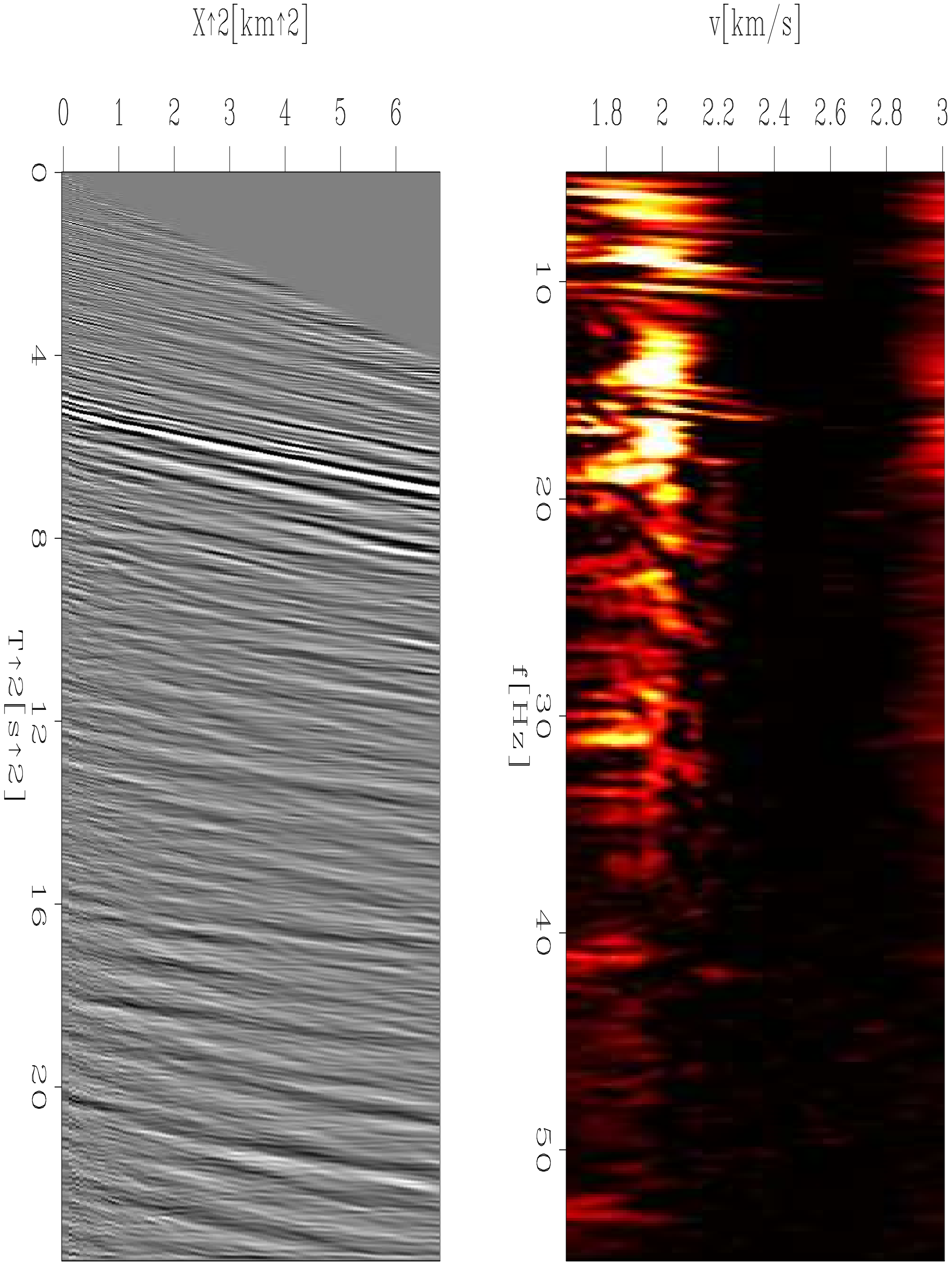

Figure 2: Application of the McMechan and Yedlin (1980) dispersion analysis method to a CMP gather from which the surface waves have been eliminated by muting and f-k filtering. **Left:** virtual input. **Right:** output of the method. The frequency range is different from Figure 1 because the data has been processed to highlight reflections and eliminate surface waves. nick2-petrix [CR]

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