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Short Note

Dip estimation from irregularly-sampled seismic data

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INTRODUCTION

The geometry of reflection seismic experiments rarely conforms to an idealized, or nominal geometry. This is especially true in regions with unavoidable surface obstructions like rivers, towns, or existing offshore oil platforms. Such deviations from nominal geometry render many computer seismic processing applications ineffective. It is therefore of considerable importance to 1) accurately map irregularly-sampled seismic data onto a regular grid with sufficient spatial resolution, and 2) to interpolate any missing trace locations with reasonable values.

Any interpolation method fills missing traces using a prior estimate of the missing data's spatial correlation; obtaining this correlation is often the main challenge. When the data geometry is regular, many existing methods can resample the data onto a finer grid. Sinc interpolation (e.g.: (Bracewell, 1986)) is optimal for the regular resampling of band-limited data. Crawley (2000) estimates a nonstationary prediction-error filter (PEF) on regularly sampled, spatially-aliased seismic data and uses inverse interpolation (Claerbout, 1999) to fill missing traces on a finer grid. Fomel (2001) solves the same problem, but substitutes "plane-wave-destructor" filters, derived from estimated reflector dip, for a PEF.

Unfortunately, irregular data geometry renders most conventional methods, including those mentioned previously, inapplicable. To reliably estimate autoregressive filters, like the PEF, all points in the filter stencil must, but generally do not, fall on known data locations. A multi-scale autoregression technique (Curry and Brown, 2001; Curry, 2002) has yielded some success by estimating the PEF simultaneously from data subsampled to a series of different resolutions. Biondi and Vlad (2001) uses azimuth moveout to transform known data to arbitrary azimuth/offset bins to constrain missing data. Other techniques (e.g: Liu and Sacchi (2001) and Zwartjes and Hindriks (2001)) solve an inverse interpolation problem in the Discrete Fourier Transform domain by regularizing the unknown coefficients. Nonetheless, a industry-standard technique does not yet exist.

In this short note, I present a different way to obtain the correlation between irregularly-sampled traces in three dimensions. Given a pair of traces, as shown in Figure 1, reflector dip can be measured along an arbitrary azimuth, by (for instance) Claerbout's "puck" method

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(Claerbout, 1992). If the dip between a master trace and two or more neighbors is measured along two distinct azimuths, we can solve a simple least-squares problem for the dip in the x- and y-directions at the master trace's location. Both geology and survey geometry often dictate that the reflector dip should vary smoothly in space. Therefore it is both natural and intuitive to extend the estimated reflector dip to missing data locations.

I test my dip estimation scheme on a sparsely and irregularly sampled 3-D synthetic model. I use the estimated dip to compute "steering filters" (Clapp et al., 1997) which regularize an inverse interpolation problem. This choice of regularization leads to a far better result than the spatial gradient operator, which corresponds to the assumption of zero dip between traces.

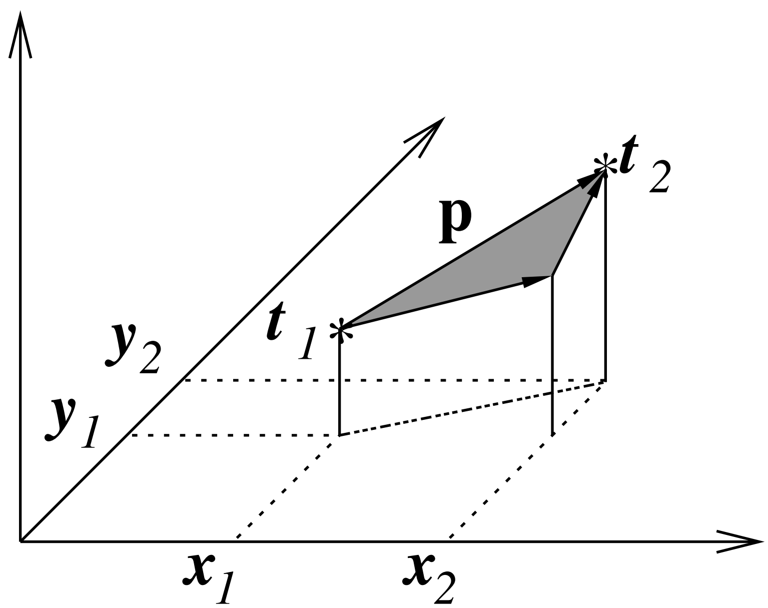

Figure 1: morgan1-puck-irreg-schem [NR]

IRREGULAR-GEOMETRY DIP ESTIMATION METHODOLOGY

Figure 1 illustrates a simplified conceptual model of seismic data. Given two traces, we assume that a seismic event passes through each trace location at times t_1 and t_2, and that the event takes the form of a plane in the neighborhood of the two traces.

Imagine that we wanted to measure $t_2 - t_1$, given the local dip of the plane, p_x and p_y. The total time shift is simply the sum of the time shifts along the x and y planes, going first from x_1 to x_2 and then from y_1 to y_2. We can write an equation directly:

$$t_2 - t_1 = p_x(x_2 - x_1) + p_y(y_2 - y_1) \tag{1}$$

However, in actuality we do not know p_x and p_y, but using Claerbout's puck method, we can measure $t_2 - t_1$. Implemented on a computer, the puck method computes the time shift (in pixels) between two traces that optimally aligns a seismic event on the two traces as a function of time. In other words, the method measures

$$p_{21} = \frac{t_2 - t_1}{\Delta t}, \tag{2}$$

where Δt is the time sampling of the traces. We can now rewrite equation accordingly:

$$p_{21} = \frac{p_x(x_2 - x_1)}{\Delta t} + \frac{p_y(y_2 - y_1)}{\Delta t} \tag{3}$$

Equation (3) describes the linear relationship between the computer dip measured between traces 2 and 1, p_{21}, and the local 3-D reflector dip. We require two equations to obtain a unique estimate of the parameters, but the noise and incoherency inherent to real data make it desirable to use more than two traces and exploit the statistical "smoothing" of least-squares estimation.

If trace 1 is the "master" trace and traces 2 through n its neighbors, let us define the forward modeling operator $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} \frac{x_2 - x_1}{\Delta t} & \frac{y_2 - y_1}{\Delta t} \\ \frac{x_3 - x_1}{\Delta t} & \frac{y_3 - y_1}{\Delta t} \\ \vdots & \vdots \\ \frac{x_n - x_1}{\Delta t} & \frac{y_n - y_1}{\Delta t} \end{bmatrix} \tag{4}$$

and the data vector $\mathbf{d}$

$$\mathbf{d} = \begin{bmatrix} p_{21} \\ p_{31} \\ \vdots \\ p_{n1} \end{bmatrix}. \tag{5}$$

The estimated p_x and p_y at trace location 1 are then the solution to the normal equations:

$$\begin{bmatrix} p_x \\ p_y \end{bmatrix} = \left(\mathbf{A}^T \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{d} \tag{6}$$

Inversion of the 2×2 matrix $\mathbf{A}^T\mathbf{A}$ is trivial. The result of the process is a pair of dip measurements (p_x and p_y) at each trace location. These dip measurements must be interpolated to fill the entire model grid. In this paper, I use an expanding-window smoothing algorithm to accomplish the task. If reflectors are continuous, their dips should be somewhat smooth in space, so to some extent, spatial smoothing is justified.

REVIEW OF INVERSE INTERPOLATION

In least-squares fitting goals, the regularized inverse interpolation problem can be stated as follows:

$$\begin{aligned} \mathbf{Lm} &\approx \mathbf{d} \quad (7) \\ \epsilon_x \mathbf{A}_x \mathbf{m} &\approx \mathbf{0} \quad (8) \\ \epsilon_y \mathbf{A}_y \mathbf{m} &\approx \mathbf{0} \quad (9) \end{aligned}$$

Operator $\mathbf{L}$ maps traces in a gridded model $\mathbf{m}$ to the earth's continuous surface. Operators $\mathbf{A}_x$ and $\mathbf{A}_y$ are "steering filters" (Clapp et al., 1997) in the x and y direction, respectively. The steering filters are initialized with a space-variable dip function and decorrelate events which have that dip, and tend to steer the estimated model along the dip direction. In this fashion we impose our prior model covariance estimate on the missing traces. Scalars ϵ_x and ϵ_y balance the two model residuals [equations (8) and (9)] with the data residual [equation (7)].

TESTS

I test my irregular-geometry dip estimation method on a decimated subset of the "quarter dome" model (Claerbout, 1999), shown in Figure 2. Only 100 out of 1024 input traces are assumed known, for a decimation rate of over 90 percent. The dip estimated by my method will then be used to interpolate the missing trace locations by solving the system (7)-(9). The left

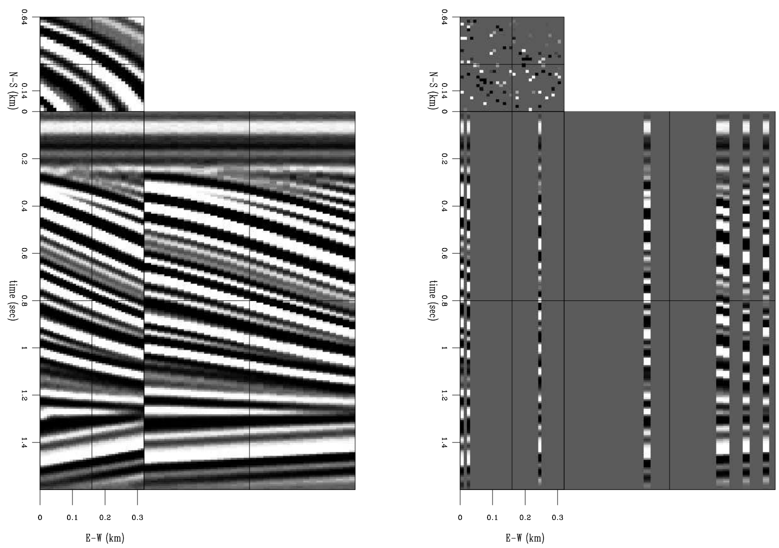

Figure 2: Left: $32x32$-trace subset of the quarter dome model. The subset was selected to be less sensitive to spatial aliasing than the steeper-dipping portions of the model. Right: Missing data test, with 100 known traces and 924 missing traces. morgan1-qdome-known [ER]

panels of Figure 3 show the quarter dome model's "known" dip, which was computed using a variant of Fomel's (2002) dip estimation method. The right panels show the dip estimated from the irregularly-sampled traces shown in Figure 2, using the method described herein. To smooth the dip estimates between master trace locations, I use an expanding-window smoothing program.

While the dip estimates are decidedly imperfect, they nontheless do contain the general trends seen in the known dip fields. Particularly note the unconformity deep in the section. We see that in the more steeply-dipping parts of the section, my method tends to underestimate the reflector dip. Since the decimation is severe, spatial aliasing may arise, even if the dips are not severe, because my method measures dip directly between two arbitrary traces. Claerbout's puck method is known to be sensitive to spatial aliasing. Fomel (2002) discusses ways to de-sensitize dip estimation with respect to spatial aliasing. Figure 4 compares the result of

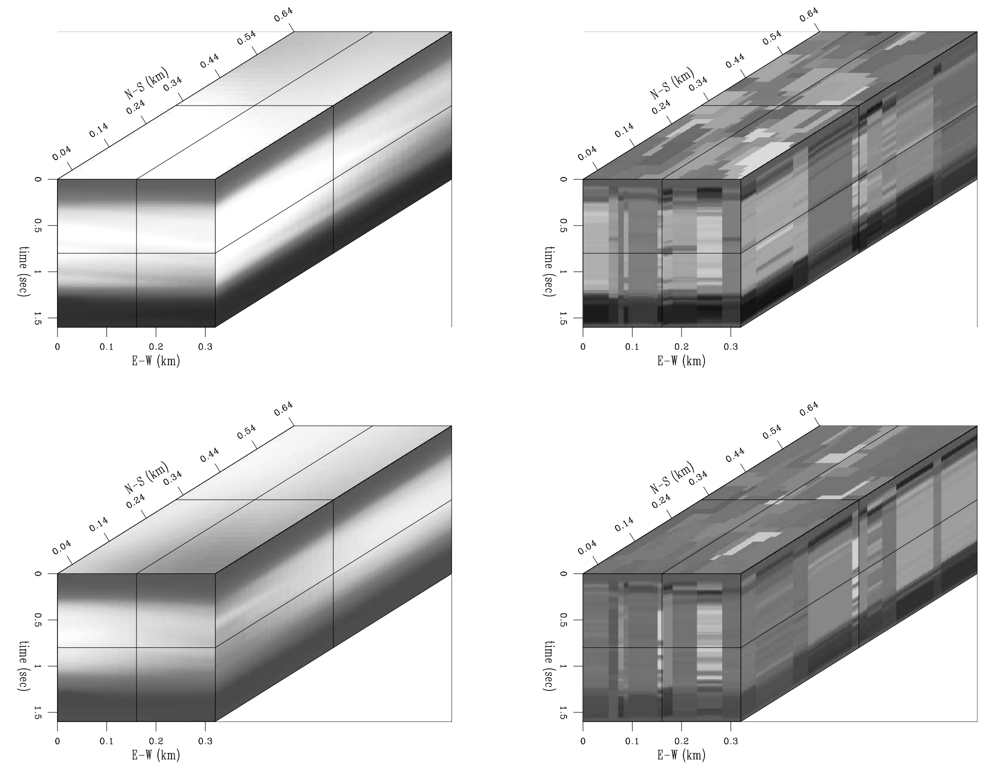

Figure 3: Top row, L-to-R: Known quarter dome x-direction dip; Estimated x-direction dip. Bottom row, L-to-R: Known quarter dome y-direction dip; Estimated x-direction dip. morgan1-qdome-dipcomp [ER]

using: a) the known dip, b) the dip estimated from the irregular data, and c) zero dip in solving equations (7)-(9) for an infilled model. We see that in spite of the imperfections of the dip estimated from the irregular data, that it definitely leads to a better inverse interpolation result that the zero dip result, which just smoothes laterally. All results were computed using 20 iterations of a linear conjugate gradient solver.

CONCLUSIONS

In this short note I presented a methodology to estimate 3-D reflector dip from seismic traces with arbitrary geometry. While the resultant dip estimates are imperfect, I show that they nontheless represent a significant improvement over a zero dip assuption in the desired application: inverse interpolation.

I foresee this method as having some value as a first-order starting guess for nonlinear implementations of Claerbout's "two-stage" inverse interpolation method (Claerbout, 1999), as tested by Curry and Brown (2001). Significant improvements are likely possible, particularly in the method for estimating the initial reflector dip between two arbitrary traces.

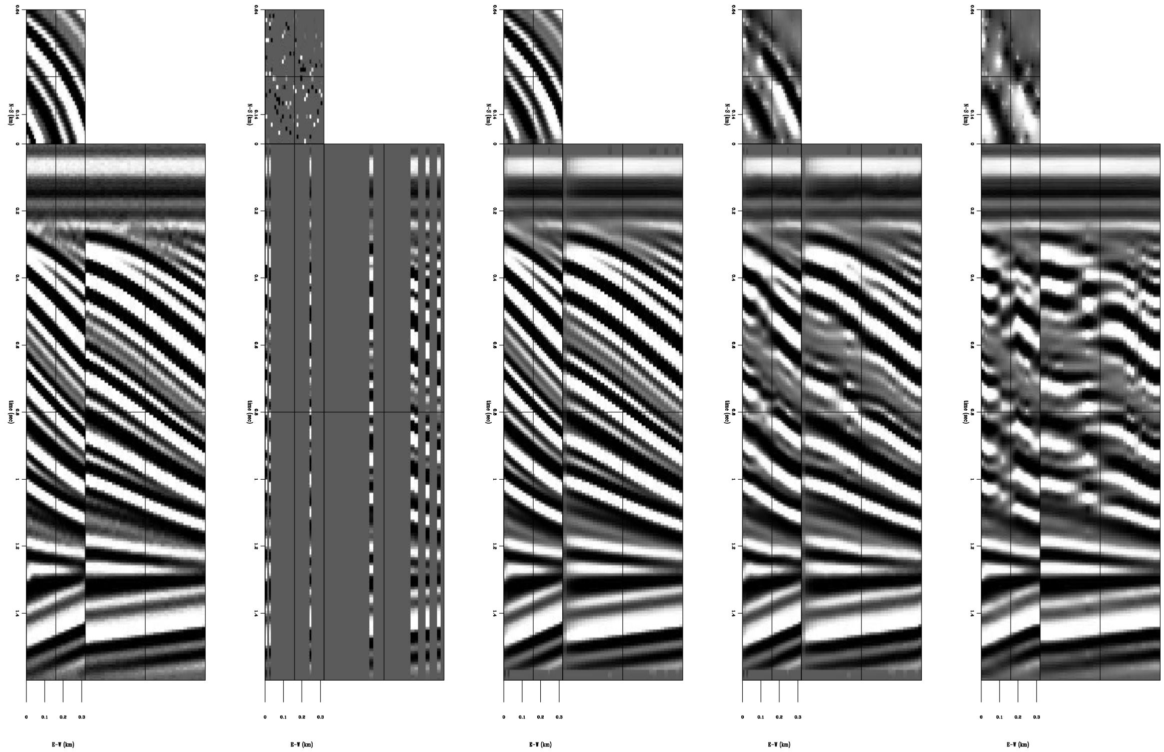

Figure 4: Left to right: Known result; Decimated data; Data filled with known-dip steering filters; Data filled with estimated-dip steering filters; Data filled with spatial gradient only (zero dip). morgan1-qdome-cubecomp [ER]

ACKNOWLEDGMENT

Discussions with Bill Curry initially motivated me to take on this project.

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