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# Velocity sensitivity of subsalt imaging through regularized inversion

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**ABSTRACT**

The effects of inaccurate velocity models on migration are well known. Accurate velocity models are most difficult to obtain in complex areas where iterative inversion can provide a better image than migration. This paper investigates the velocity sensitivity of a regularized inversion scheme that explicitly assumes that the correct velocity is being used. This inversion uses a regularization operator that assumes that there is no moveout along the offset ray parameter axis. Experiments performed with various incorrect velocity models indicate that this assumption is valid for velocity models that can be reasonably produced by common velocity analysis techniques. Velocity models that are very inaccurate cause the inversion process to reject attempts by the regularization to produce an image that is inconsistent with the data.

## INTRODUCTION

The difficulties of imaging below salt edges are compounded by the difficulty of generating an accurate velocity model in these areas. The majority of imaging techniques require an accurate velocity model in order to produce a well focused result. The artifacts seen in a migrated image caused by errors in the velocity model are well known (Claerbout, 1985). In fact, migration can be used as a tool for developing a velocity model (Biondi and Sava, 1999). However, in areas such as those around salt, where it is most difficult to obtain a good velocity model, poor illumination makes it impossible for migration alone to provide a satisfactory image.

In complex areas, imaging can be improved by using a migration operator in an iterative inversion scheme. Unfortunately, if the velocity model is inaccurate the artifacts that are seen in the migration result will affect the inversion result as well. Even more ominously, some iterative inversion techniques make the assumption of correct velocity a critical part of their theory. In particular, imaging using the regularized inversion described by Prucha et al. (2000) and Kuehl and Sacchi (2001) assumes that the correct velocity is being used to justify the choice of regularization operator. The regularization operators they use assume that there is no moveout along the offset ray parameter axis. In this paper, I will examine the sensitivity of this assumption for a variation of Prucha et al. (2000)'s implementation.

I will first explain the manner in which I carry out regularized inversion and the regular-

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ization operator used. Then I will perform migration and regularized inversion on a synthetic dataset using the correct velocity model and two velocity models that have been perturbed in different ways. I will compare these results to make a statement on the validity of the zero-moveout assumption.

## THEORY

My inversion scheme is based on the downward continuation migration explained by Prucha et al. (1999a). To summarize, this migration is carried out by downward continuing the wavefield in frequency space, slant stacking at each depth, and extracting the image at zero time. The result is an image in depth (z), common midpoint (CMP), and offset ray parameter ($p_h$) space.

This migration operator is used in a Tikhonov regularized (Tikhonov and Arsenin, 1977) conjugate-gradient least-squares minimization:

$$min\{Q(\mathbf{m}) \ = \ ||\mathbf{W}(\mathbf{Lm}-\mathbf{d})||^2 + \epsilon^2 \ ||\mathbf{Am}||^2\}. \tag{1}$$

This inversion procedure can be expressed as fitting goals as follows:

$$\begin{aligned} \mathbf{0} &\approx \mathbf{W}(\mathbf{Lm}-\mathbf{d}) \\ \mathbf{0} &\approx \epsilon\mathbf{Am}. \end{aligned} \tag{2}$$

The first equation is the "data fitting goal," meaning that it is responsible for making a model that is consistent with the data. The second equation is the "model styling goal," meaning that it allows us to impose some idea of what the model should look like using the regularization operator **A**. The model styling goal also helps to prevent a divergent result.

In the data fitting goal, **d** is the input data and **m** is the image obtained through inversion. **L** is a linear operator, in this case it is the adjoint of the angle-domain wave-equation migration scheme summarized above and explained thoroughly by Prucha et al. (1999b). In the model styling goal, **A** is, as has already been mentioned, a regularization operator. **W** is a weighting operator. $\epsilon$ controls the strength of the model styling.

Unfortunately, the inversion process described by fitting goals (2) can take many iterations to produce a satisfactory result. I can reduce the necessary number of iterations by making the problem a preconditioned one. I use the preconditioning transformation $\mathbf{m} = \mathbf{A}^{-1}\mathbf{p}$ (Fomel et al., 1997; Fomel and Claerbout, 2002) to give us these fitting goals:

$$\begin{aligned} \mathbf{0} &\approx \mathbf{W}\left(\mathbf{L}\mathbf{A}^{-1}\mathbf{p}-\mathbf{d}\right) \\ \mathbf{0} &\approx \epsilon\mathbf{p}. \end{aligned} \tag{3}$$

$\mathbf{A}^{-1}$ is obtained by mapping the multi-dimensional regularization operator **A** to helical space and applying polynomial division (Claerbout, 1998).

The question now is what the regularization operator **A** is. I built my regularization operator based on the same assumptions as Prucha et al. (2000). First, I assume that the correct velocity is being used in the inversion, therefore there should be no moveout along the offset ray parameter ($p_h$) axis. Second, I assume that the amplitudes of individual events should vary smoothly and any drastic changes in amplitude are caused by illumination problems, which are what we wish to overcome. These assumptions allow me to say that **A** needs to act to minimize amplitude differences horizontally along the $p_h$ axis. Rather than using the derivative operator used by Kuehl and Sacchi (2001) or the steering filter used by Prucha et al. (2000), I have created a symmetrical filter by cascading two steering filters that are mirror images of each other.

## RESULTS

I applied the downward continuation migration and the preconditioned inversion scheme to a synthetic dataset provided to us by SMAART JV, using different velocity models. The correct velocity model can be seen in Figure 1. The result of migration using this model is Figure 2. In the CRP-depth panel, note the sudden decrease in amplitude of the reflectors as they pass beneath the salt edge, particularly within the oval. There are also strong artifacts in the shadow zone beneath the salt (inside the oval) which make it difficult to pick out any true events. In the $p_h$-depth panel, note the "holes" in the events at the mid-range of ray parameters (inside the oval). These holes are caused by the poor illumination under the salt edge. The steeply dipping events in the $p_h$-depth panel are artifacts caused by aliasing along the offset axis in Fourier space.

The result of 3 iterations of conjugate-gradient preconditioned least-squares inversion using the correct velocity model can be seen in Figure 3. Note that the artifacts have been largely cleaned up. It is now possible to reliably pick out events beneath the salt (see inside the oval). In the CRP-depth panel, the amplitude of the events is maintained farther beneath the salt (particularly within the oval). The holes in the $p_h$-depth panel (inside the oval) are being filled in.

To test the sensitivity of the preconditioned inversion, the first incorrect velocity model I tested simply increased the correct velocities by 5%. As expected, the migration result using

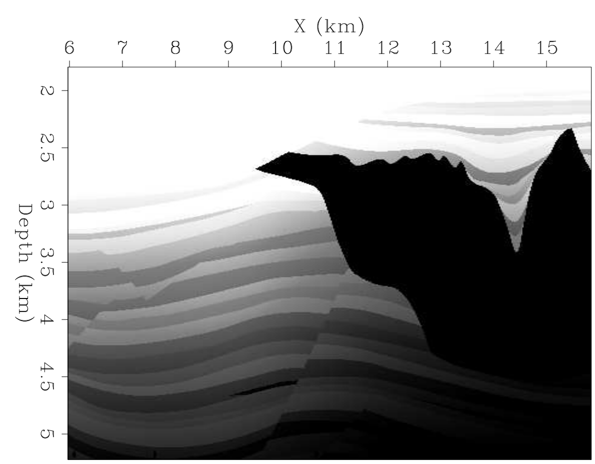


Figure 1: The correct velocity model. marie1-vel.corvel [ER]

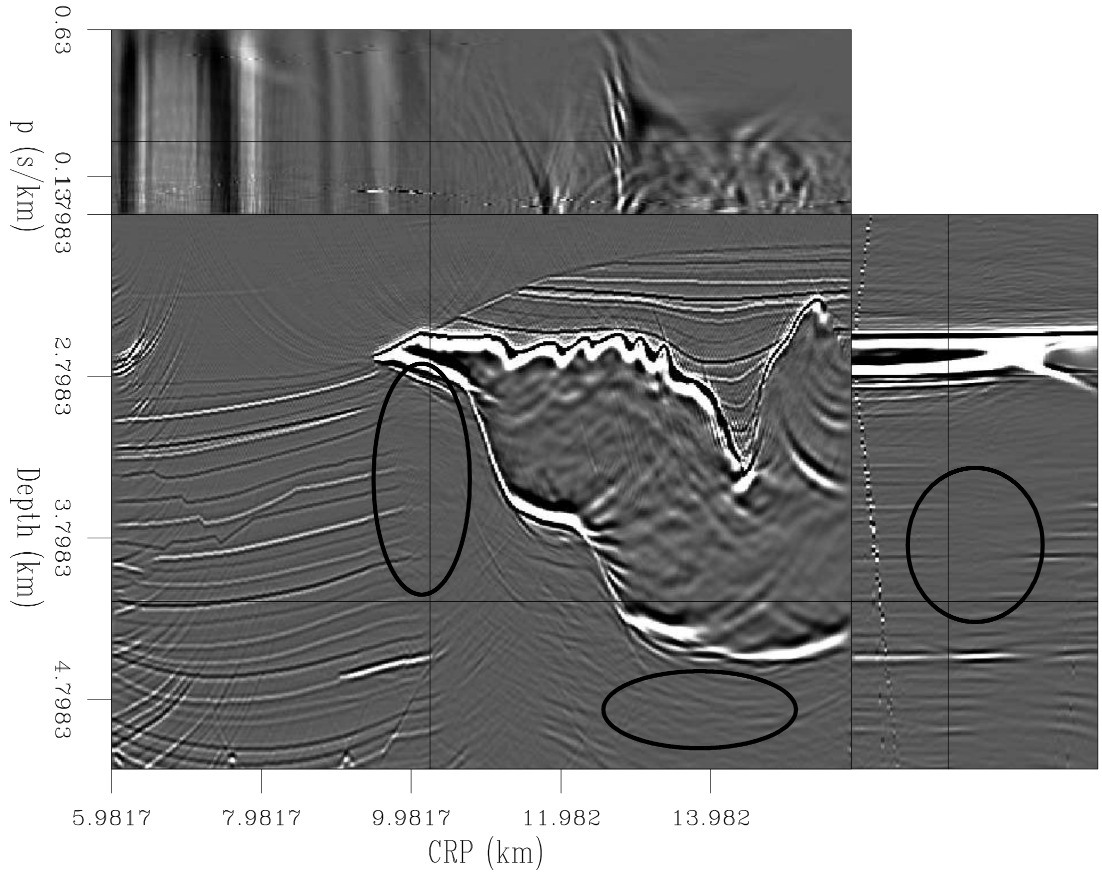


Figure 2: The result of downward continuation migration using the correct velocity model. Note the low amplitudes of events as they pass beneath the salt in the CRP-depth panel and the artifacts obscuring events beneath the salt (indicated by ovals). In the offset ray parameter-depth panel note the holes in the events at the mid-range of offset ray parameters (particularly within the oval). marie1-mig.corvel [CR]

this velocity model (Figure 4) shows the events positioned deeper than they should be and moveout along the offset ray parameter axis. The ovals on this figure are placed in the same absolute positions as the the ovals in Figures 2 and 3, not relative to the events themselves.

Recall that the preconditioning operator acts horizontally along the offset ray parameter axis. It is this sensitivity that we are interested in observing in the result of 3 iterations of preconditioned inversion using the high velocity model (Figure 5). Note that once again the preconditioned inversion has cleaned up many of the artifacts. In the CRP-depth panel, the events extend farther under the salt, in a similar way to the inversion result using the correct velocity (Figure 3). The more interesting result is the $p_h$-depth panel. The inversion is still successfully filling in the holes along the events at the mid-range of offset ray parameters. At large $p_h$, where the moveout is more pronounced, the preconditioning has made some attempt to change the dips to be more horizontal, but the moveout is still visible. This means that this result is most likely not safe to use for velocity analysis, but this preconditioned

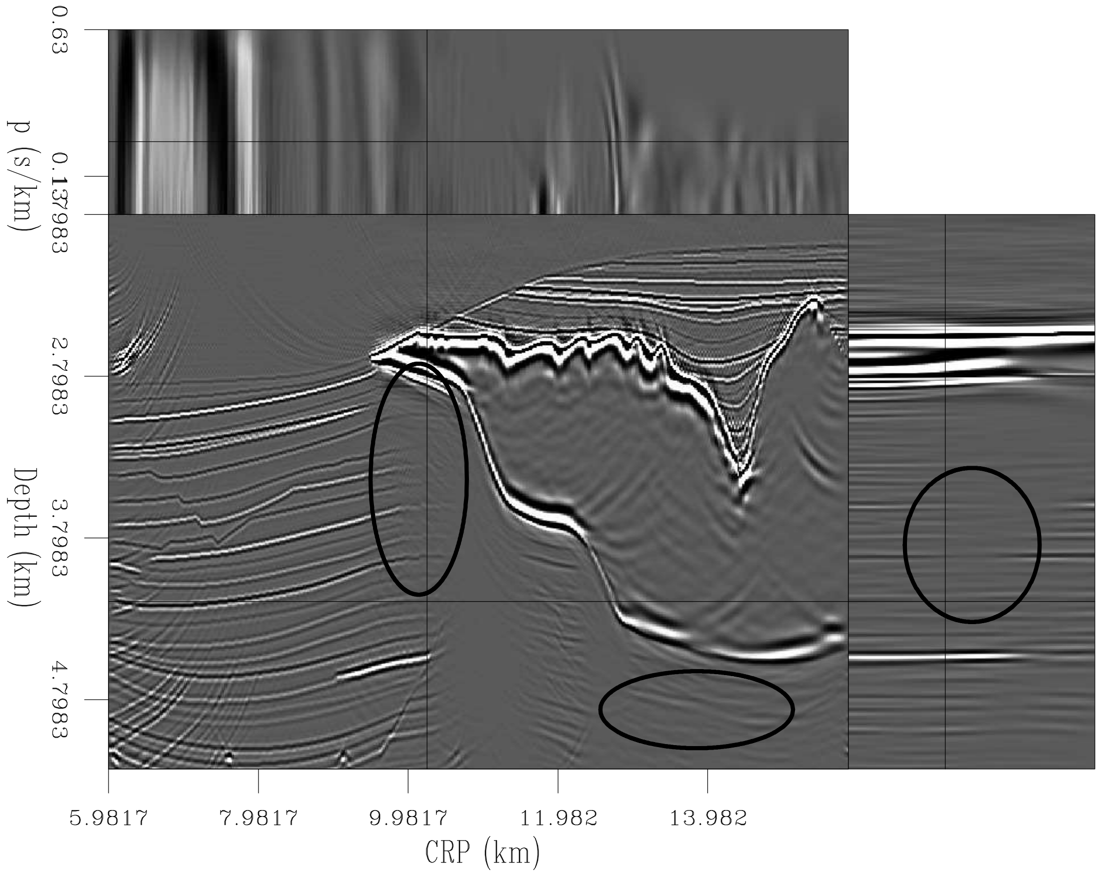


Figure 3: The result of 3 iterations of preconditioned inversion using the correct velocity model. Note the more consistent amplitudes of events as they pass beneath the salt in the CRP-depth panel and the lack of artifacts obscuring events beneath the salt (inside ovals). In the offset ray parameter-depth panel note the filling in of the holes in the events at the mid-range of offset ray parameters (inside ovals). marie1-geop.corvel [CR]

inversion technique was never intended as a velocity tool. Overall, this result indicates that this technique can produce a better image than migration alone, even when the velocity model is incorrect by up to 5%.

A more extreme velocity model I tested was a severely smoothed one (Figure 6). This model has been smoothed so much that the canyon in the top of the salt has disappeared. As expected, the migration result from this model isn't very good (Figure 7). The depth positioning of events is fairly good away from the salt, but becomes poor near the salt. The salt top and bottom are very poorly imaged. The events in the $p_h$-depth panel appear to be mostly random. Once again, the ovals indicate the same absolute regions as the ovals in Figures 2 and 3.

The result of 3 iterations of preconditioned inversion using this smoothed velocity model can be seen in Figure 8. Although many of the artifacts have been cleaned up, overall the image is not any better than the migration result. The events in the $p_h$-depth panel are more

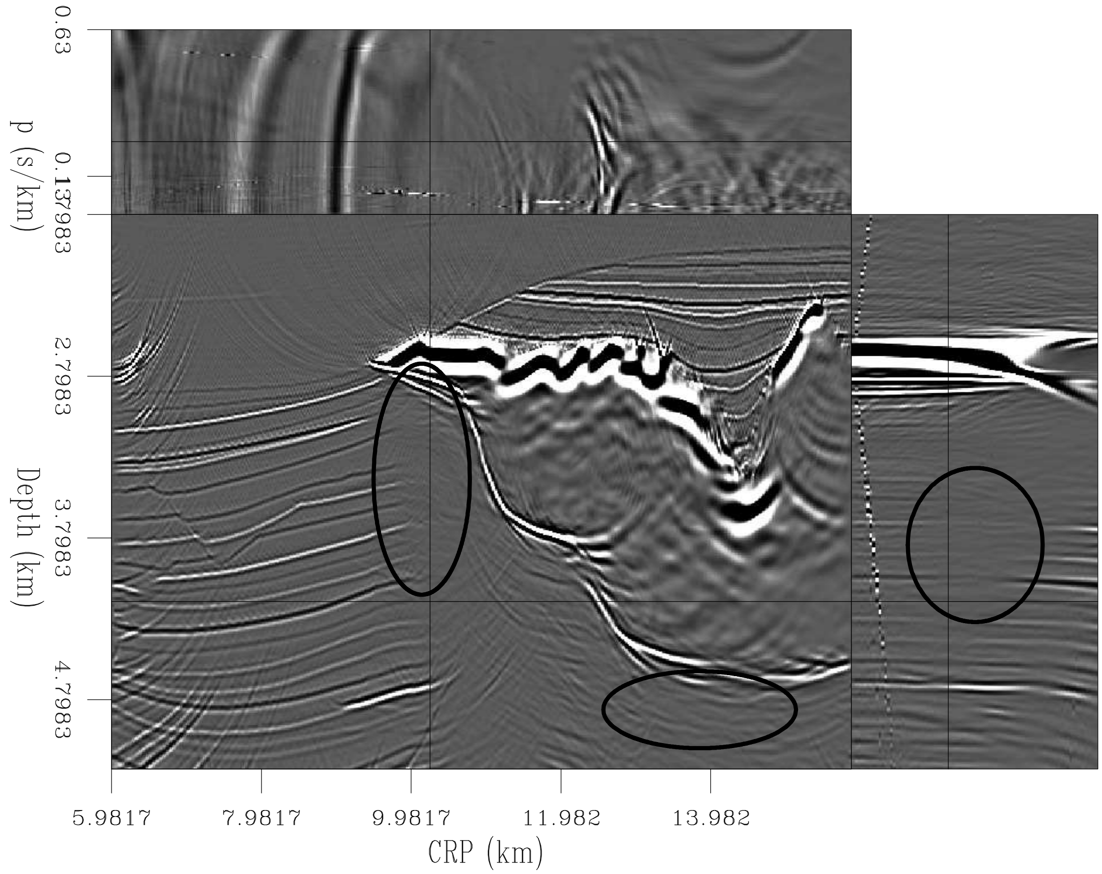


Figure 4: The result of downward continuation migration using a velocity model 5% higher than the correct model. The events are all positioned deeper than they should be and there is moveout along the offset ray parameter axis. The ovals still indicate the loss of amplitudes under the salt edge and the poor imaging beneath the salt in the CRP-depth panel and holes in the events in the $p_h$ panel. marie1-mig.hivel [CR]

horizontal, but they are not more believable than the events in the $p_h$-depth of the migration result. This is a reassuring result, as it indicates that the regularization was not able to artificially introduce events where the data indicated otherwise.

## CONCLUSIONS

The results of preconditioned inversion with incorrect velocity models are encouraging. As long as the velocity model is not too inaccurate, the preconditioning operator behaves as it would for the correct velocity model and produces a better image than migration alone. In the case of a highly inaccurate model, the inversion itself prevents us from producing an image that would conflict with the known data. Overall, as long as the velocity model is reasonably close to correct, the assumption of zero moveout made by the preconditioning operator is acceptable.

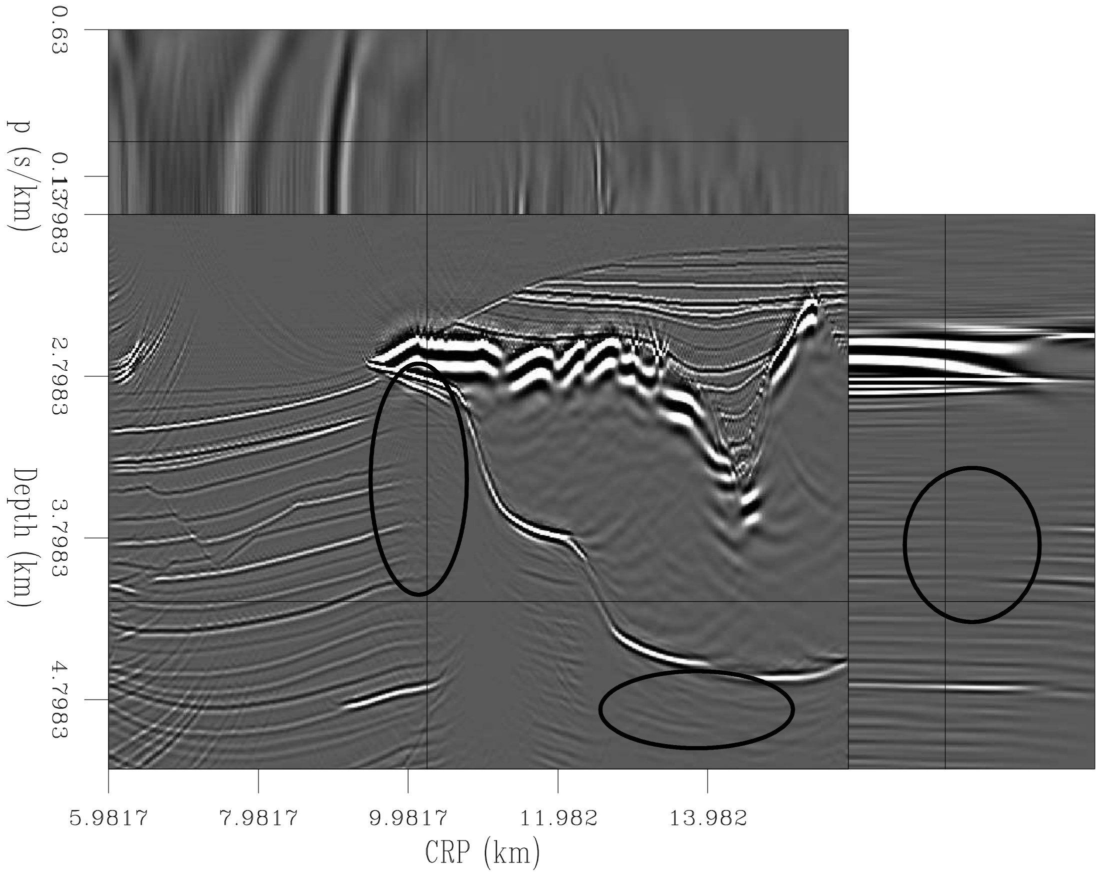


Figure 5: The result of 3 iterations of preconditioned inversion using the 5% too high velocity model. Despite the use of the incorrect velocity model, the image is quite comparable to the result using the correct velocity (Figure 3). In the CRP-depth panel, the events extend farther under the salt and events under the salt can be seen (inside the ovals). In the $p_h$-depth panel, the holes in the events are filled in (inside the oval). marie1-geop.hivel [CR]

## ACKNOWLEDGMENTS

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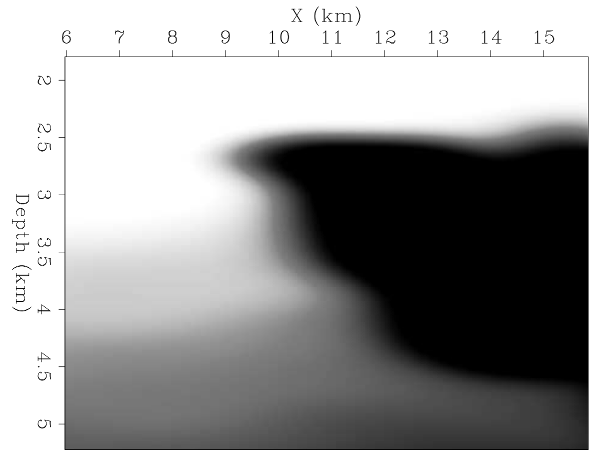


Figure 6: The smoothed velocity model. Note that the canyon in the top of the salt has disappeared. marie1-vel.smoothvel [ER]

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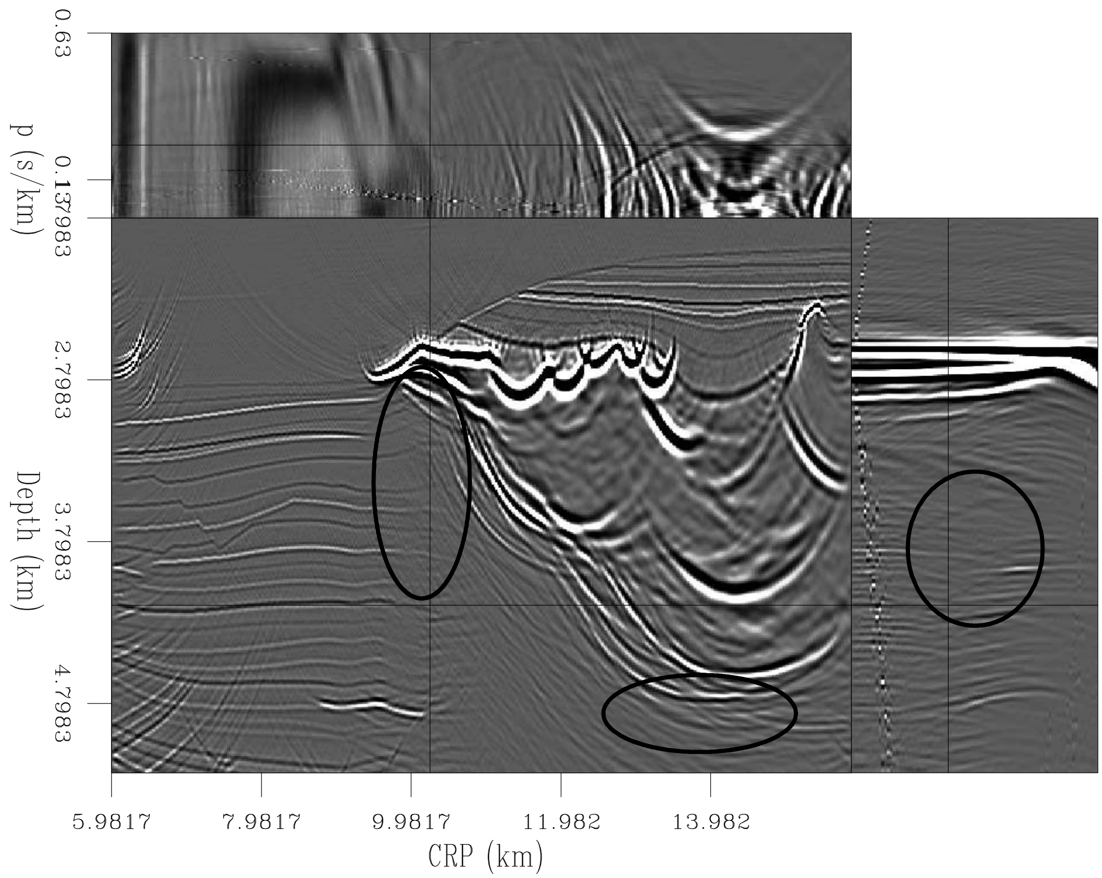


Figure 7: The result of downward continuation migration using a severely smoothed velocity model. The events in the CRP-depth panel are properly imaged away from the salt but are mispositioned near the salt. The offset ray parameter-depth panel is completely uninformative. The ovals indicate the same absolute regions as the ovals in Figures 2 and 3. marie1-mig.smoothvel [CR]

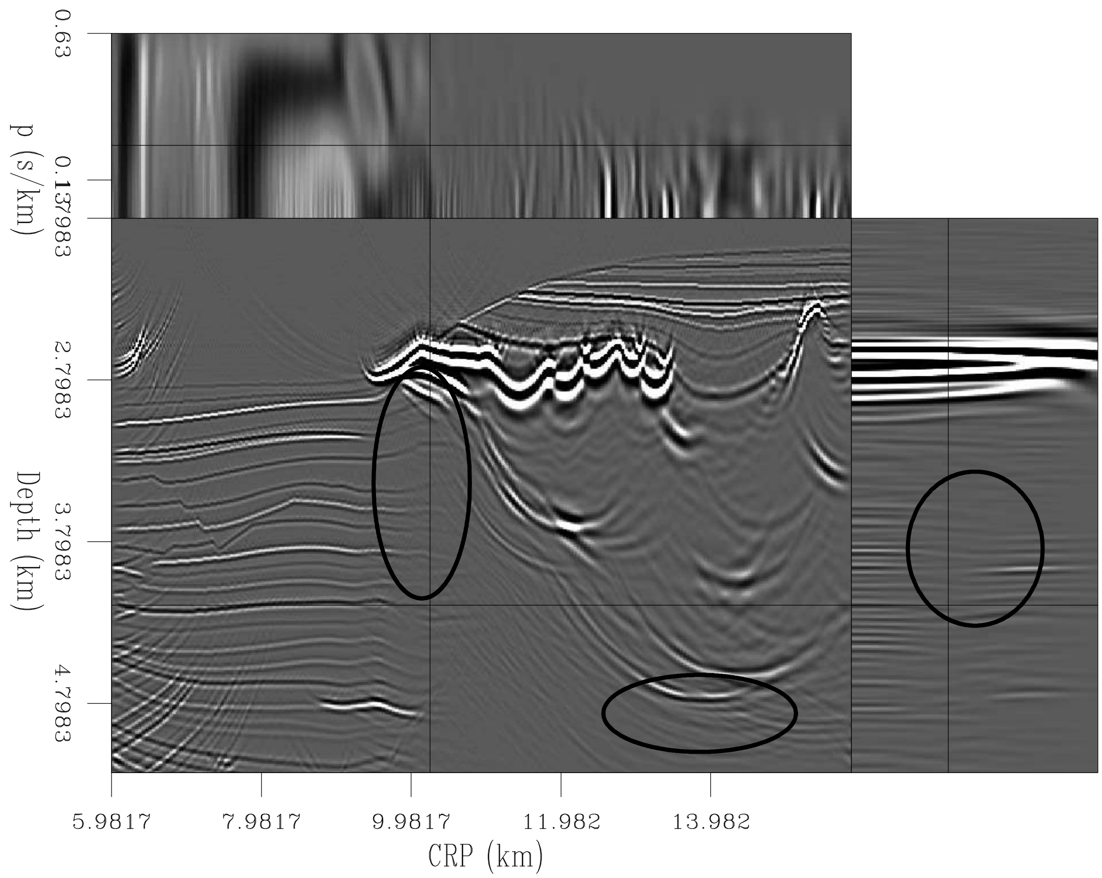


Figure 8: The result of 3 iterations of preconditioned inversion using the smoothed velocity model. The result is cleaner than the migration result, but not more believable. The ovals indicate the same absolute regions as the ovals in Figures 2 and 3 marie1-geop.smoothvel [CR]