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Prediction-error filter estimation on irregular traces

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ABSTRACT

Regularly-sampled data is normally required to estimate a prediction-error filter (PEF). I show how a small PEF can be estimated when the data is irregularly sampled in one dimension. I then estimate a PEF on an irregularly-sampled 2D examples, and discuss extension to 3D.

INTRODUCTION

Data interpolation has been performed with the use of multidimensional prediction-error filters (PEFs) (Spitz, 1991; Claerbout, 1999). However, methods to generate a prediction-error filter require regularly-sampled data. When the data are not regularly sampled, the data is first transformed to a regular grid (or multiple grids), and then the PEF is estimated on those copies of the data.

In the case of interlaced data (where traces are sampled at regular intervals) the PEF is stretched (Claerbout, 1999; Crawley, 2000) such that the coefficients fall on the interlaced traces during convolution. This method can be used to estimate a PEF only in those circumstances, and relies on the assumption of scale invariance, where a stretched filter and an unstretched filter behave in a similar fashion.

In the case of a line of irregularly-sampled traces, a small PEF can be determined by dynamically stretching the filter so that it fits each trace pair. This method does not require the distance between traces to be cleanly divisible by the shortest distance, nor does it even require the data to be gridded in all dimensions. It also does away with many of the parameter choices needed by other PEF estimation methods for irregular data. The method is first tested on a simple plane-wave model, with promising results.

BACKGROUND

A prediction-error filter is estimated by solving a minimization problem where known data is convolved ($\mathbf{Y}$) with an unknown filter ($\mathbf{a}$), so

$$\mathbf{0} \approx \mathbf{r} = \mathbf{Y}\mathbf{a}, \tag{1}$$

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where the first coefficient of $\mathbf{a}$ is constrained to be unity. This can be written as

$$\mathbf{0} \approx \mathbf{r} = \mathbf{YKa} + \mathbf{y}, \tag{2}$$

where $\mathbf{K}$ is a mask for the first PEF coefficient, and $\mathbf{y}$ is simply a copy of the data. Writing out the matrices for a PEF with 3 coefficients and for 7 data samples looks like

$$\mathbf{0} \quad \approx \quad \mathbf{r} = \begin{bmatrix} y_2 & y_1 & y_0 \\ y_3 & y_2 & y_1 \\ y_4 & y_3 & y_2 \\ y_5 & y_4 & y_3 \\ y_6 & y_5 & y_4 \end{bmatrix} \begin{bmatrix} 0 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}. \tag{3}$$

The previous equations have all been for a 1D case. By use of the helical coordinate (Claerbout, 1998), these equations can easily be extended to higher dimensions.

In the case of missing data, a diagonal weight ($\mathbf{W}$) can be introduced that is 0 when a missing data point is in the equation, and 1 where all data are present. This weight can also be used to eliminate edge effects caused by helical convolution.

When data are interlaced, a PEF can be estimated by spacing filter coefficients during convolution, so that they fall on known data. An example of this filter spacing is shown in Figure 2. The problem with this methods is that the data must be regularly sampled in all dimensions.

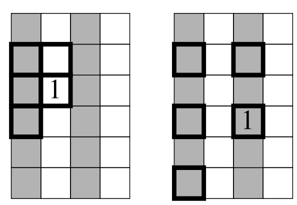

Figure 1: Examples of PEFs on interlaced data. White bins are empty, gray have data. Left: A PEF cannot be estimated due to too much missing data. Right: The spaced PEF can be estimated on interlaced data. bill2-interlace [NR]

IRREGULAR TRACES

Seismic data are recorded as a series of traces that are regularly-sampled in time, but are often spatially distributed in an irregular manner. Since the estimation of a PEF requires a regularly-sampled input, the data are placed on a regular grid. If this grid is too fine to allow a PEF to be estimated, a multi-scale method can be used to introduce more fitting equations (Curry and Brown, 2001; Curry, 2002).

Instead of relying on methods to transform the data onto a regular grid, we can examine if it is possible to manipulate the filter so that it can fit the irregular data. In the case of interlaced data, the filter can be spaced to fit the data. If the data are from a 2D grouping of irregular traces, we can take a small 2-column PEF and dynamically stretch it so that as the filter is convolved with the data, the PEF will always fall on known data.

When looking at 2-column PEF estimation, we can take equation (2) and include a lag matrix **S**, which matches the appropriate data point to filter coefficient. For a simple 2×5 PEF, **S** will look like:

$$\mathbf{S}_{orig} = \begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}. \tag{4}$$

The gap in the diagonal matrix is the length of the n_1 axis (time) minus the filter length in the n_1 dimension. When stretching the data for interlaced traces, the matrix changes, so that

$$\mathbf{S}_{inter} = \begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}. \tag{5}$$

In this case, every second data point is selected by this matrix, and the gap now corresponds to 2 times the length of the n_1 axis, since a row is skipped. Now for a dynamically stretched PEF, the matrix will look somewhat different, as the PEF is not stretched by an integer amount, so if a filter coefficient falls between two data points (as shown in Figure 2) a linear combination of the two data points will be multiplied with the PEF coefficient. For a single trace pair, **S** will now be

$$
\mathbf{S}_{irreg} = \begin{bmatrix}
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 0.5 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 0.5 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 0.5 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 0.5 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0.5 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0.5 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1
\end{bmatrix} \qquad (6)
$$

if the traces were considered to be 1.5 bins apart. For each trace pair, a separate set of

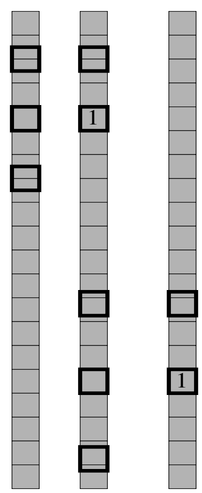

Figure 2: PEFs on irregular traces. For the first trace pair, the PEF is spaced by 1.5 bins, and for the second the PEF is spaced by 2.25 bins. Where a PEF coefficient falls between data points, a linear combination of the two are used. bill2-stretch [NR]

equations where $\mathbf{Y_i}$ corresponds to convolution with the trace pair, $\mathbf{S_i}$ is a filter lag matrix that is a function of the distance between the two traces, $\mathbf{y_i}$ is a copy of the second trace of the pair, and $\mathbf{K}$ and $\mathbf{f}$ remain unchanged. For a three-trace set, the fitting goal would be

$$
\mathbf{0} \quad \approx \quad \mathbf{r} = \begin{bmatrix} \mathbf{Y_1} & \cdot & \cdot \\ \cdot & \mathbf{Y_2} & \cdot \\ \cdot & \cdot & \mathbf{Y_3} \end{bmatrix} \begin{bmatrix} \mathbf{S_1} & \cdot & \cdot \\ \cdot & \mathbf{S_2} & \cdot \\ \cdot & \cdot & \mathbf{S_3} \end{bmatrix} \begin{bmatrix} \mathbf{K} \\ \mathbf{K} \\ \mathbf{K} \end{bmatrix} \mathbf{f} + \begin{bmatrix} \mathbf{y_1} \\ \mathbf{y_2} \\ \mathbf{y_3} \end{bmatrix}. \qquad (7)
$$

In methods where the data are transformed to a regular grid (Curry and Brown, 2001), many parameters (such as scales) need to be chosen. With this method of manipulating the filter to fit the data, the only parameter that needs to be set is the aspect ratio of the PEF, meaning what distance would correspond to a single bin in time.

There are several assumptions being made when estimating a PEF in this manner, most of which are shared with Crawley's (1998) approach. First of all, this method assumes that the filter is scale-invariant, meaning that the stretched filter would behave the same as if it were not stretched, which is valid when dealing with plane waves (Claerbout, 1999). Next, there is an assumption of stationarity. The filter can be made to be non-stationary, but the stretching of the filter assumes that the area that the stretched filter covers is stationary. Finally, since using a linear combination of two data points as a proxy for a data point located between those two is tantamount to linear interpolation, there is an assumption that the data are not aliased in the well-sampled (time) axis.

There are also several limitations to this approach. First of all, in order to dynamically stretch a PEF to fit data along one dimension, all other dimensions must be regularly-sampled and contain no aliasing. This limits the estimation to 2D with seismic data, since the only dimension with the required regularity is the time axis. This problem can be addressed by simultaneously estimating numerous 2D PEFs for other dimensions. Another issue that arises is multiple dips. A non-stationary filter can deal with different dips in different areas, but is not effective if those dips are co-located. A pair of PEFs could be simultaneously estimated, which might work in estimating both dips.

TEST CASE

Figure 3 shows a fully-sampled plane wave previously used as a test case for PEF estimation (Brown et al., 2000). Nine traces (approximately 15 percent of traces) were removed from this model and were used to estimate a PEF. The only inputs to the algorithm are the nine traces and the distances between the traces. The results are promising, as the dip is easily recovered. The inverse impulse response of the PEF, shown in Figure 3c, matches the dip of the original fully-sampled data. Figure 3d contains a re-sampled version of Figure 3b, which was then interpolated with the PEF estimated on the irregularly-sampled traces. The flat areas in that image are due to the re-sampling and not the PEF.

CONCLUSIONS AND FUTURE WORK

By dynamically stretching filter coefficients, a small PEF can be estimated on irregularly-sampled traces. This method works for a 2D test case with a simple plane-wave model. This method is computationally inexpensive, and only requires a single extra parameter in addition to those required for PEF estimation on a regular grid.

Certain limitations need to be addressed with this method: First, extension to three dimensions. One possible approach to this problem is to use a series of PEFs operating over different axes. Another problem that arises is what to do in the presence of multiple dips. Again, using a series of PEFs might address this issue, as well as the use of non-stationary filters.

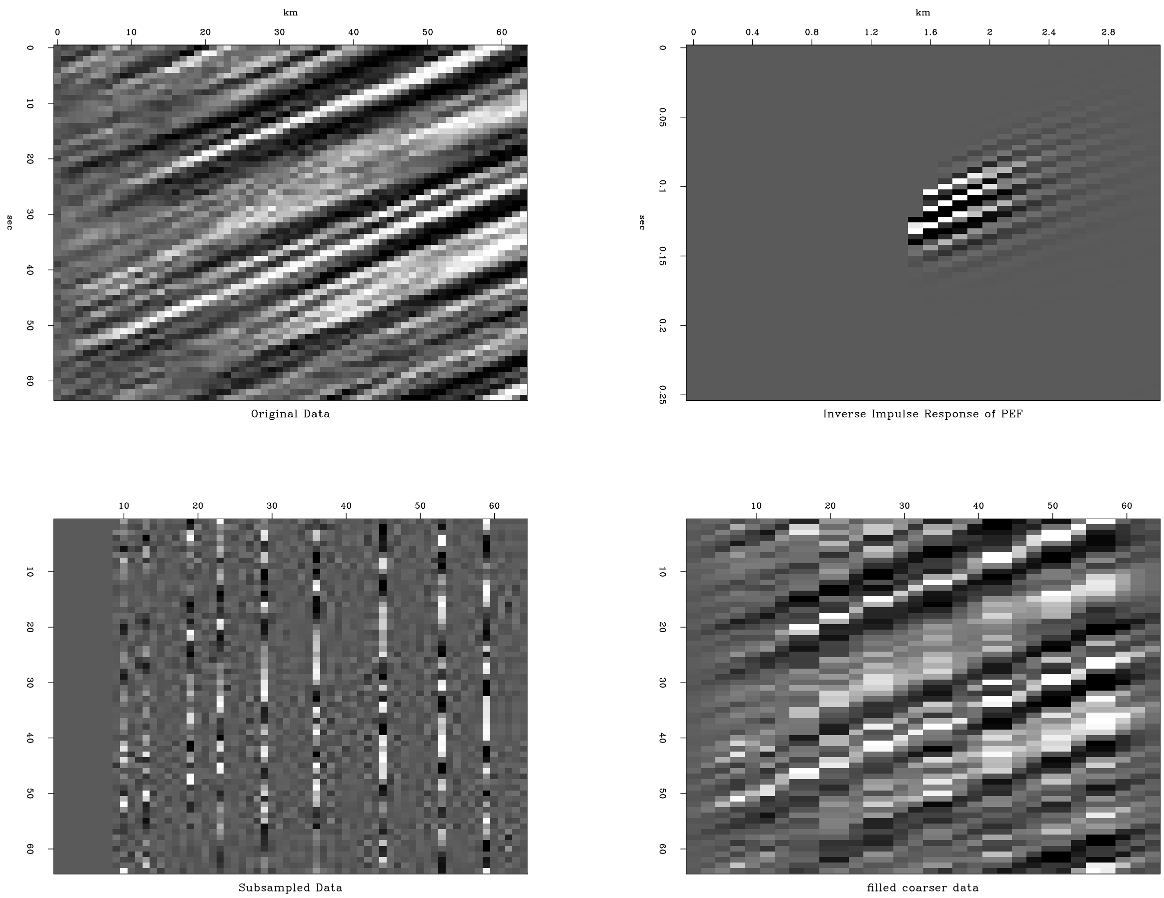

Figure 3: Plane wave test case: (a) is the original fully-sampled data; (b) is the sub-sampled version; (c) is the inverse impulse response of the PEF; and (d) is an interpolated copy of coarsened data. bill2-plane [ER]

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