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Short Note

Deblurring using edge-preserving regularization

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INTRODUCTION

In this short note, we test various edge-preserving regularization schemes in the context of deblurring a text image with random noise. The blurred text image was created by Nagy and O'Leary (2003a) as a test case. Even if the blurring filter is known exactly, as it is in this case, sharp features are nearly in the nullspace of the filter which we must "invert", or deblur. Those eigenvalues of the filter matrix corresponding to edges may be well below the noise level, and thus difficult or impossible to resolve.

We know that letters should be homogeneous for intervals (piecewise constant), thus it makes sense to impose model smoothness using a regularization operator. But letters also have abrupt discontinuities, thus using a regularization operator that imposes model smoothness considerably degrades our ability to discern the letter edges.

We present three separate strategies which allow the model regularization operator to preserve edges in the deblurring process. All are implemented as regularization operators in an unconstrained least-squares deblurring problem.

The data

The data **g** (figure 1B) (Nagy and O'Leary, 2003a) were generated by taking the original image **f** (figure 1A), posted in the answer paper (Nagy and O'Leary, 2003b), multiplying it by a non-stationary convolution matrix **K** and then adding random noise. The noise prevents us from completely recovering the initial image.

REGULARIZATION SCHEMES

In Nagy and O'Leary (2003a), regularization is used to make the least-squares deblurring problem less sensitive to the noise. Figures 2 and 3 show the result of using the identity operator for regularization. This is comparable to the results presented in Nagy and O'Leary

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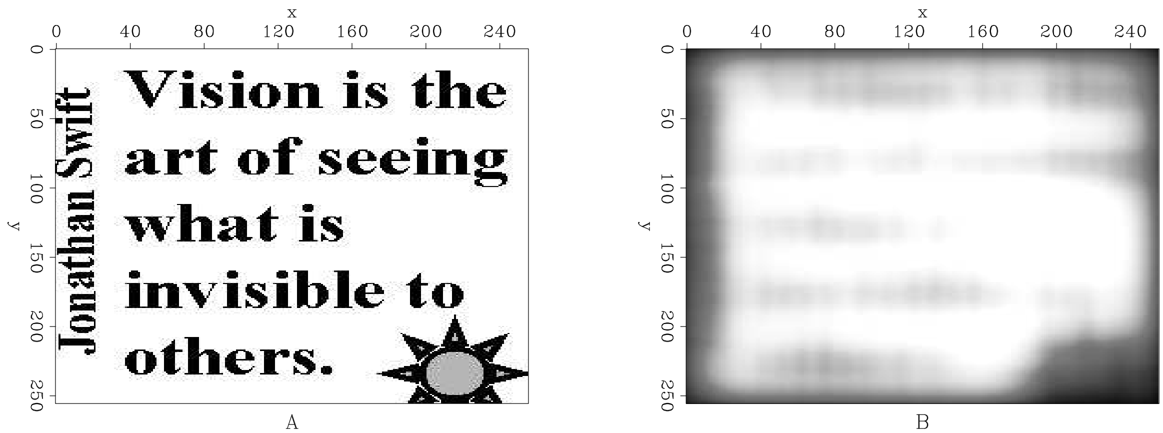

Figure 1: A) Original message, B) Blurred message (data). alejandro2-start [ER]

(2003b). We go a step further by using regularization to impose *a priori* information on the solution of the problem. We exploit the fact that letters should be homogeneous for intervals (piecewise constant) with abrupt discontinuities between them.

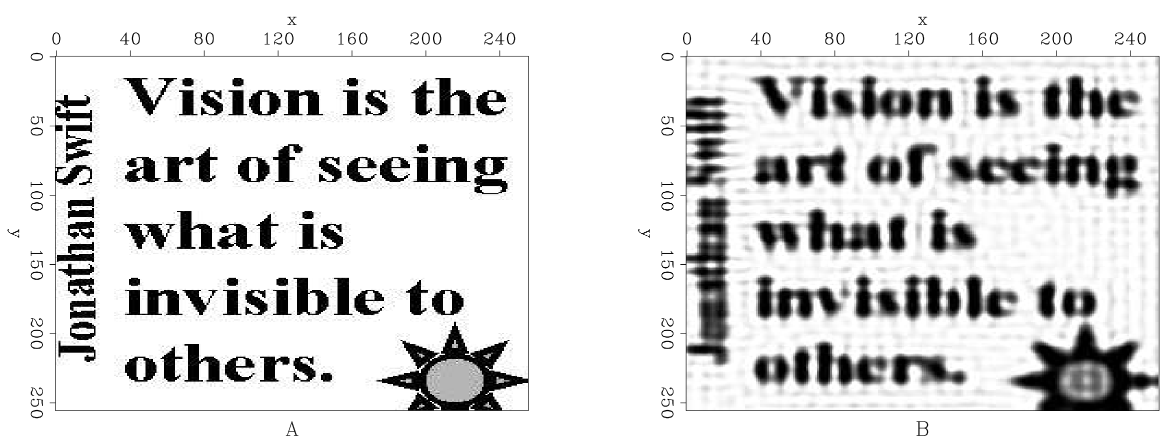

Figure 2: A) Original image, B) Deblurred image using LS with damped regularization. alejandro2-comp_damp [CR]

First order derivative regularization

A way of forcing a function to be piecewise constant is forcing its first order derivatives to be sparse. Thus, using a first order derivative regularization operator and forcing the model residuals to follow a Cauchy distribution should make the letters “blocky” and preserve the letter edges (Youzwishen, 2001). Obtaining model residuals following a Cauchy distribution can be achieve posing the inverse problem as Iterative Reweighed Least Squares (IRLS) (Darche, 1989).

The Cauchy norm first order derivative edge-preserving regularization fitting goal was

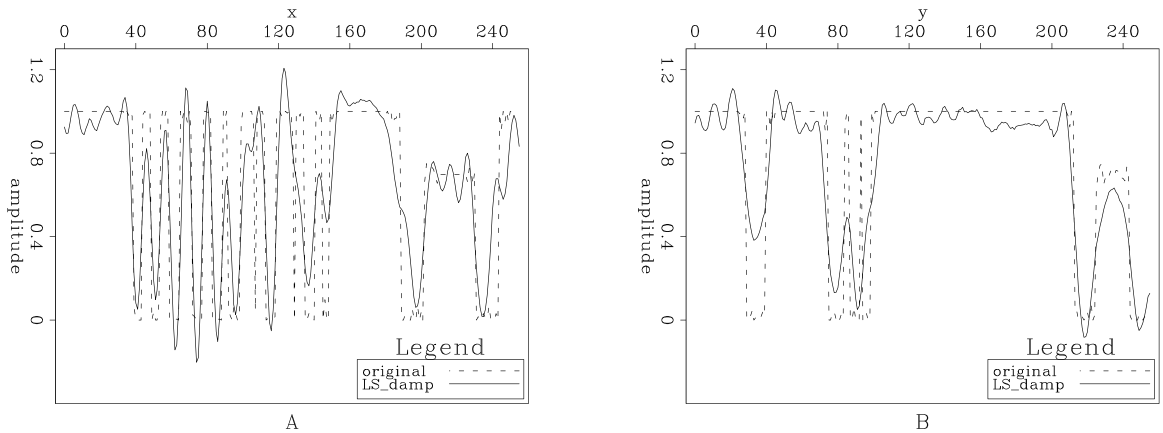

Figure 3: Comparison between Figures 2A and 2B; A) Slice $y = 229$ and B) Slice $x = 229$. alejandro2-comp_damp_graph [CR]

set following the nonlinear iterations: starting with $\mathbf{Q_x}^0 = \mathbf{Q_y}^0 = \mathbf{I}$, at the k^{th} iteration the algorithm solves

$$\begin{aligned} \mathbf{K}\mathbf{f}^k - \mathbf{g} &\approx \mathbf{0} \\ \epsilon \mathbf{Q_x}^{k-1}\mathbf{D_x}\mathbf{f}^k &\approx \mathbf{0} \\ \epsilon \mathbf{Q_y}^{k-1}\mathbf{D_y}\mathbf{f}^k &\approx \mathbf{0} \end{aligned} \tag{1}$$

where

$$\begin{aligned} \mathbf{Q_x}^{k-1} &= \frac{1}{\left[1+\left(\frac{\mathbf{D_x}\mathbf{f}^{k-1}}{\alpha}\right)^2\right]^{\frac{1}{2}}}, \\ \mathbf{Q_y}^{k-1} &= \frac{1}{\left[1+\left(\frac{\mathbf{D_y}\mathbf{f}^{k-1}}{\alpha}\right)^2\right]^{\frac{1}{2}}}. \end{aligned} \tag{2}$$

$\mathbf{K}$ is a non-stationary convolution matrix, $\mathbf{f}^k$ is the result of the k^{th} nonlinear iteration, $\mathbf{Q_x}^{k-1}$ and $\mathbf{Q_y}^{k-1}$ are the $(k-1)^{th}$ diagonal weighting operators, and $\mathbf{D_x}$ and $\mathbf{D_y}$ are the first order derivative operator in the x and y directions, $\mathbf{I}$ is the identity matrix, the scalar α is the trade-off parameter controlling the discontinuities in the solution, and the scalar ϵ balances the relative importance of the data and model residuals.

We were successful in obtaining what we designed the algorithm to produce. The result is blocky in the x and y directions (Figures 4 and 5). However, the derivatives in the x and y directions do not produce an isotropic result. We know that letters often have round shapes. Thus, the problem could benefit from using a more isotropic operator to calculate the diagonal weights.

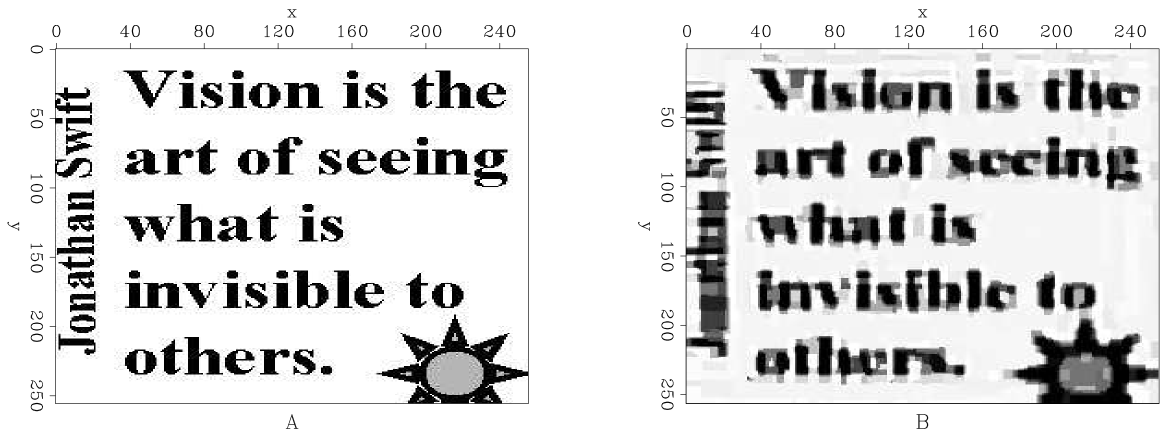

Figure 4: A) Original image, B) Deblurred image using LS with the first order derivative edge-preserving regularization alejandro2-comp_images_2d [CR]

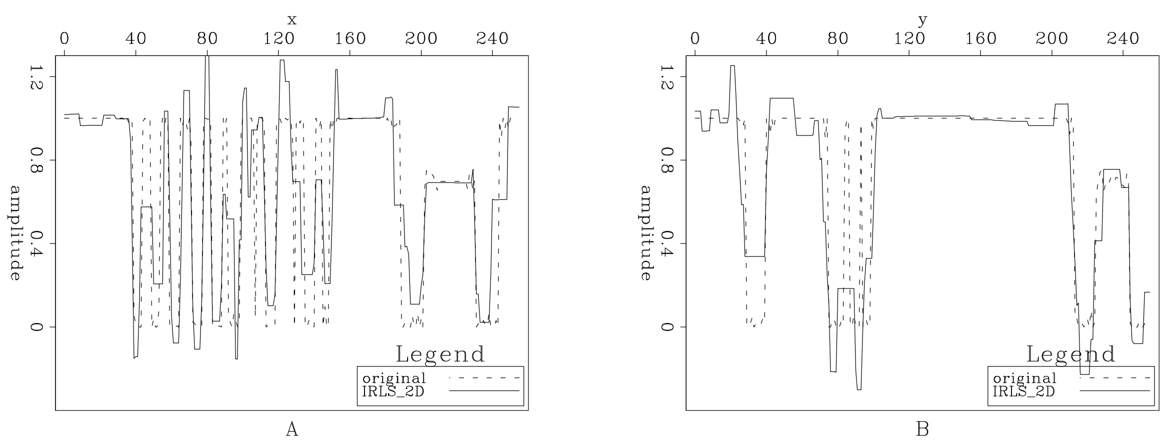

Figure 5: Comparison between Figures 4A and 4B; A) Slice $y = 229$ and B) Slice $x = 229$. alejandro2-comp_graph_2d [CR]

Gradient magnitude and Laplacian regularization

In the previous section we changed the norm of the minimization problem to prevent the roughener from smoothing over the edges of the model. In this subsection we shift from a statistical to a more mechanical approach to attain the same goal.

The gradient magnitude ($||\nabla||$) is an isotropic edge-detection operator that can be used to calculate the diagonal weights. Unfortunately, it is a nonlinear operator thus couldn't be used for regularization. Instead we used the Laplacian, which is a regularization operator used in several SEP applications (Claerbout and Fomel, 2001).

The gradient magnitude edge-preserving regularization fitting goal was set following the nonlinear iterations: starting with ${\mathbf{Q}_{||\nabla||}}^0 = \mathbf{I}$, at the k^{th} iteration the algorithm solves

$$\begin{aligned} \mathbf{K}\mathbf{f}^k - \mathbf{g} &\approx \mathbf{0} \\ \epsilon {\mathbf{Q}_{||\nabla||}}^{k-1} \nabla^2 \mathbf{f}^k &\approx \mathbf{0} \end{aligned} \tag{3}$$

where

$${\mathbf{Q}_{||\nabla||}}^{k-1} = \frac{1}{1 + \frac{||\nabla \mathbf{f}^{k-1}||}{\alpha}}. \tag{4}$$

$\mathbf{K}$ is a non-stationary convolution matrix, $\mathbf{f}^k$ is the result of the k^{th} nonlinear iteration, ${\mathbf{Q}_{||\nabla||}}^{k-1}$ is the $(k-1)^{th}$ diagonal weighting operators, $\mathbf{I}$ is the identity matrix, $||\nabla||$ is the gradient magnitude, ∇^2 is the Laplacian operator, the scalar α is the trade-off parameter controlling the discontinuities in the solution, and the scalar ϵ balances the relative importance of the data and model residuals.

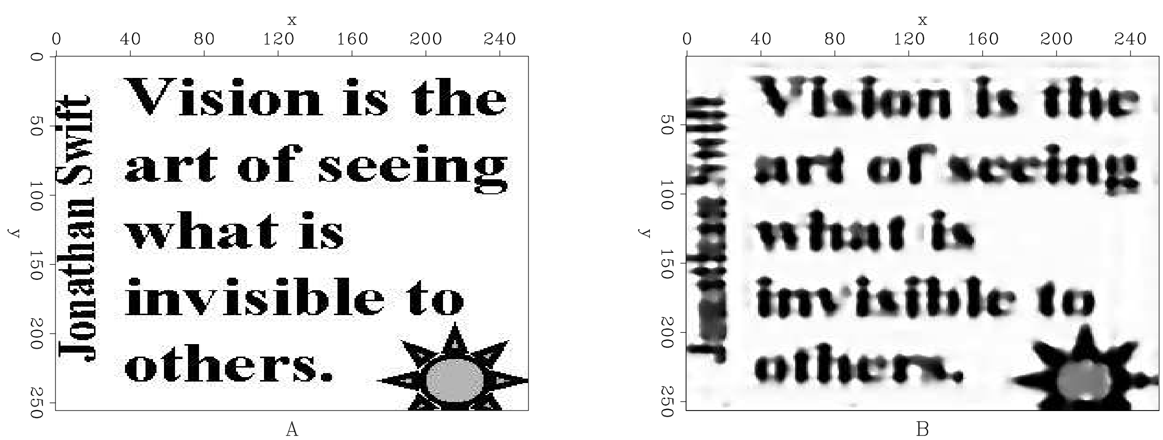

Figure 6: A) Original image, B) Deblurred image using LS with the gradient magnitude edge-preserving regularization ($||\nabla||$) with laplacian edge-preserving regularization alejandro2-comp_images_laplac_2d [CR]

Figures 6 and 7 show a considerable improvement over Figures 4 and 5. They are noise free but keep the round features of the original image. However, since we are not imposing blockiness on the model but rather on the derivative of the model (using the Laplacian as the regularization operator), the edges are not as sharp as the previous case.

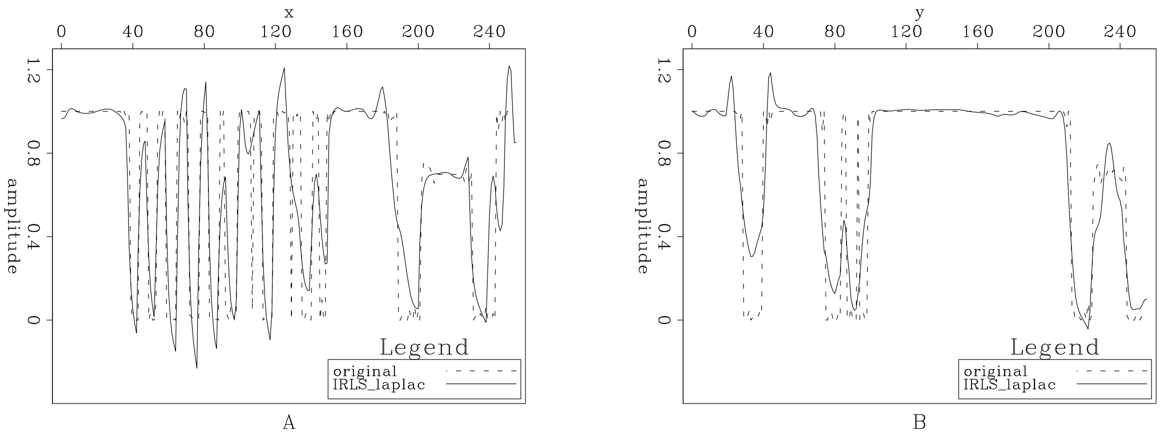

Figure 7: Comparison between Figures 6A and 6B; A) Slice $y = 229$ and B) Slice $x = 229$. alejandro2-comp_graph_2d_laplac [CR]

Gradient magnitude and smooth regularization

Until now we have used a roughener as the regularization operator to take into account the prior knowledge that letters are piecewise constant. Turning it off at the edges and on away from the edges. To do the opposite also makes sense, using a smoothing operator at the edges and turning it off away from the edges doesn't make the model smoother away from the edges but increases the sharpness of the model at the edges.

The gradient magnitude with triangular smoothing edge-preserving regularization fitting goal was set following the nonlinear iterations: starting with $\mathbf{W^0} = \mathbf{I}$, at the k^{th} iteration the algorithm solves

$$\begin{aligned} \mathbf{K}\mathbf{f}^k - \mathbf{g} &\approx \mathbf{0} \\ \epsilon \mathbf{W}^{k-1} \Delta \mathbf{f}^k &\approx \mathbf{0} \end{aligned} \tag{5}$$

where

$$\mathbf{W}^{k-1} = ||\nabla \mathbf{f}||^{k-1}. \tag{6}$$

$\mathbf{K}$ is a non-stationary convolution matrix, $\mathbf{f}^k$ is the result of the k^{th} nonlinear iteration, $\mathbf{W}^{k-1}$ is the $(k-1)^{th}$ diagonal weighting operators, $\mathbf{I}$ is the identity matrix, Δ is triangular smoother operator, and the scalar ϵ balances the relative importance of the data and model residuals.

Figures 8 and 9 show what we were expecting, i.e. the edges look sharp, but away from the edges the noise dominates. However, we can see in figure 8 some details that we are not visible in Figures 6, 4, and 2. Looking that a combination of the styling goals in equations (3) (smoothing away from the edges) and (5) (roughening at the edges) should give a good result.

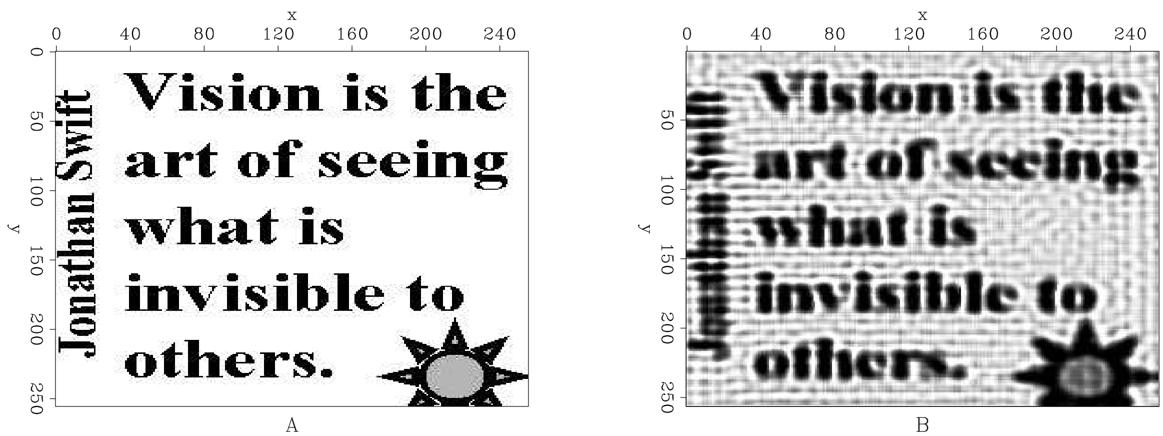

Figure 8: A) Original image, B) Deblurred image using LS with the edge-preserving regularization gradient magnitude with triangular smoothing alejandro2-comp_images_smooth_2d [CR]

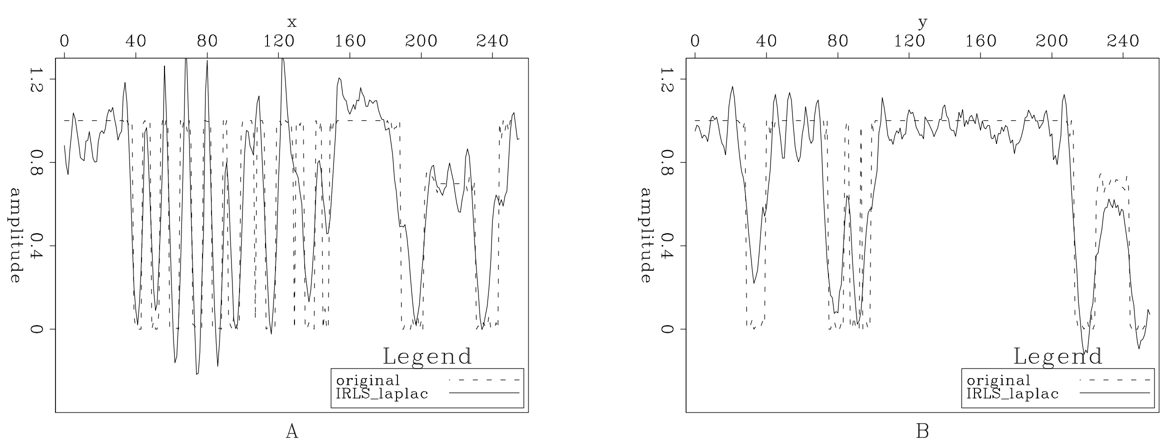

Figure 9: Comparison between Figures 8A and 8B; A) Slice $y = 229$ and B) Slice $x = 229$of. alejandro2-comp_graph_2d_smooth [CR]

CONCLUSIONS

Introducing an edge-preserving regularization helps to take into account prior knowledge about letter statistics into the least-squares deblurring problem. The proposed gradient magnitude and Laplacian regularization was the better option for getting rid of the noise and preserving the round sharp features present in the original model.

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