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Interval velocity estimation using edge-preserving regularization

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ABSTRACT

We test two edge-preserving forms of model regularization in a least-squares implementation of Dix formula that leads to interval velocities with sharp edges in the (τ, x) plane. This characteristic in the interval velocity may be desirable in geologic environments with abrupt changes in velocity, like: carbonate layers, salt bodies, and strong faulting.

INTRODUCTION

Interval velocity estimation is a central problem in reflection seismology (Claerbout, 1999). Without an estimate of seismic velocity, we would be unable to transform prestack seismic data into an interpretable image. Advanced velocity estimation techniques (Clapp, 2001; Biondi and Sava, 1999) have been developed to estimate interval velocity in complex geological environments, though the cost of these methods is often considerable.

In the early stages of prospect evaluation, an inexpensive interval velocity estimate is often desired. The Dix equation (Dix, 1952) analytically inverts root-mean-square (RMS) velocity for interval velocity as a function of time. In addition to many physical shortcomings (assumption of a stratified $v(z)$ earth), Dix inversion suffers from numerical problems that lead to poor velocity estimates. Dix inversion is unstable when RMS velocities vary rapidly, and may produce interval velocities with unreasonably large and rapid variations. For this reason, the problem is often cast as a least-squares problem, with is regularized in time with a differential operator to penalize such rapid variations and to produce a smooth result (Clapp et al., 1998).

While temporal smoothness may often be justified from a geologic stand point ($v(z)$), in some cases velocity can change abruptly (e.g., carbonate layers, salt bodies, strong faulting). In these cases we desire a regularization technique that gives smooth velocity in most places, but which preserves sharp "geologic" interval velocity contrasts when they are present, without requiring pre-defined edges to be supplied.

In this paper we present two automatic edge-preserving regularization methodologies for the least-squares implementation of Dix formula. The first uses IRLS to effectively change the norm of the problem to permit a "spiky" or "sparse" model residual, which leads to a "blocky" velocity model. The second uses an isotropic edge detector, the gradient magnitude,

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in a nonlinear scheme to compute a measure of the edges of the model. This edge measure is then used as a model residual weight, along the lines of Clapp et al. (1998) and Lizarralde and Swift (1999).

DIX EQUATION AS A LEAST-SQUARES PROBLEM

The Dix Equation states the nonlinear relationship between RMS velocity and interval velocity. Harlan (1999) linearizes the relationship and solves the problem using a Gauss-Newton nonlinear iteration. However, the problem is linear in the square of the interval velocity. This "linear" problem was solved by Clapp et al. (1998). They apply a preconditioned least squares optimization to "invert" Dix equation, with spatial smoothness constraints.

Let us rewrite for completeness the Dix equation as a least-squares fitting goal:

$$\mathbf{W}(\mathbf{C}\mathbf{u}-\mathbf{d}) \approx \mathbf{0} \tag{1}$$

where $\mathbf{u}$ is the unknown model, a vector of squared interval velocities. $\mathbf{d}$ is the known data, a vector of squared RMS velocities multiplied by time. $\mathbf{C}$ is the causal integration operator. $\mathbf{W}$ is a data residual weighting function, which is proportional to our confidence in the RMS velocities.

Fitting goal (1) is notoriously unstable to high frequency variations in RMS velocity, and moreover, it is under-determined in the sense that only strong reflections really qualify as "data". Therefore, Clapp et al. (1998) supplement the system with a regularization term which penalizes "wiggliness". In our case we use first order derivatives, but as we will see later, other rougheners can be used:

$$\mathbf{W}(\mathbf{C}\mathbf{u}-\mathbf{d}) \approx \mathbf{0} \tag{2}$$

$$\epsilon_\tau \mathbf{D}_\tau \mathbf{u} \approx \mathbf{0} \tag{3}$$

$$\epsilon_x \mathbf{D}_\mathbf{x} \mathbf{u} \approx \mathbf{0} \tag{4}$$

where $\mathbf{D}_\tau$ and $\mathbf{D}_\mathbf{x}$ are first-order finite differences derivatives in time and midpoint, respectively, and the scalars ϵ_τ and ϵ_x balance the relative importance of the two model residuals.

In hard rock environments like carbonates, velocities tend to be homogeneous for intervals, with abrupt discontinuities at changes in lithology. There, the desire for a blocky interval velocity model is well-justified.

"BLOCKY" MODELS

In the following subsections we introduce two schemes to weight the model residuals in equations (3) and (4) to preserve sharper edges in the estimated $\mathbf{u}$.

Edge preserving regularization with the Cauchy norm

Imagine that the model residuals in equations (3) and (4) consisted of spikes separated by relatively large distances. Then the interval velocity **u** would be piecewise smooth with jumps at the spike locations, which is what we desire. However in solving (2)–(4) we use the least-squares criterion – minimization of the ℓ_2 norm of the residual. Any spikes in the residual will be attenuated. To do this, the solver smooths the velocity across the spike location.

It is known that the ℓ_1 norm is less sensitive to spikes in the residual (Claerbout and Muir, 1973; Darche, 1989; Nichols, 1994). ℓ_1 norm minimization makes the assumption that the residuals have an exponential distribution, a "long-tailed" distribution relative to the Gaussian. Here, we compute nonlinear model residual weights which force a Cauchy distribution, another long-tailed distribution which approximates an exponential distribution (Youzwishen, 2001).

We perform the following non linear iterations: starting with ${\mathbf{Q}_\tau}^0 = {\mathbf{Q}_\mathbf{x}}^0 = \mathbf{I}$, at the k^{th} iteration the algorithm solves

$$\begin{aligned} \mathbf{W}(\mathbf{C}\mathbf{u}^k - \mathbf{d}) &\approx \mathbf{0} \\ \epsilon_\tau {\mathbf{Q}_\tau}^{k-1} \mathbf{D}_\tau \mathbf{u}^k &\approx \mathbf{0} \\ \epsilon_x {\mathbf{Q}_\mathbf{x}}^{k-1} \mathbf{D}_\mathbf{x} \mathbf{u}^k &\approx \mathbf{0} \end{aligned} \tag{5}$$

where

$${\mathbf{Q}_\tau}^{k-1} = \frac{1}{\left[1 + \left(\frac{\mathbf{D}_\tau \mathbf{u}^{k-1}}{\alpha_\tau}\right)^2\right]^{\frac{1}{2}}}, \tag{6}$$

$${\mathbf{Q}_\mathbf{x}}^{k-1} = \frac{1}{\left[1 + \left(\frac{\mathbf{D}_\mathbf{x} \mathbf{u}^{k-1}}{\alpha_x}\right)^2\right]^{\frac{1}{2}}}, \tag{7}$$

and $\mathbf{u}^k$ is the result of the k^{th} nonlinear iteration, ${\mathbf{Q}_\tau}^{k-1}$ and ${\mathbf{Q}_\mathbf{x}}^{k-1}$ are the $(k-1)^{th}$ diagonal weighting operators, $\mathbf{D}_\tau$ and $\mathbf{D}_\mathbf{x}$ are the first order derivatives in time and midpoint, $\mathbf{I}$ is the identity matrix, the scalars α_τ and α_x are the trade-off parameters controlling the discontinuities in the solution, and the scalars ϵ_τ and ϵ_x balance the relative importance of the two model residuals.

Edge preserving regularization with the gradient magnitude

In the previous section we changed the norm of the minimization problem to prevent the roughener from smoothing over the edges of the model. In this subsection we shift from a statistical to a more mechanical approach to attain the same goal.

To preserve the edges of the model Clapp et al. (1998) propose adding a weight to zero the model residual at the edges. Lizarralde and Swift (1999) implement a similar approach for the inversion of VSP data for interval velocity. This approach requires human intervention for reflector picking. We want to design a weight which de-weights edges in the model residual, but which is estimated automatically.

The 2-D gradient magnitude is a good isotropic edge-detection operator that can be used to calculate the diagonal weights. As we show in the deblurring problem (Valenciano et al., 2003), using the gradient magnitude we can iteratively obtain sharp edges.

We perform the following non linear iterations: starting with ${\mathbf{Q}_{||\nabla||}}^0 = \mathbf{I}$, at the k^{th} iteration the algorithm solves

$$\begin{aligned} \mathbf{W}(\mathbf{C}\mathbf{u}^k - \mathbf{d}) &\approx \mathbf{0} \\ \epsilon {\mathbf{Q}_{||\nabla||}}^{k-1} \nabla^2 {\mathbf{u}}^k &\approx \mathbf{0} \end{aligned} \tag{8}$$

where

$${\mathbf{Q}_{||\nabla||}}^{k-1} = \frac{1}{1 + \frac{||\nabla \mathbf{u}^{k-1}||}{\alpha}}, \tag{9}$$

and $\mathbf{u}^k$ is the result of the k^{th} nonlinear iteration,${\mathbf{Q}_{||\nabla||}}^{k-1}$ is the $(k-1)^{th}$ diagonal weight operator, $||\nabla||$ is the gradient magnitude, ∇^2 is the Laplacian operator, $\mathbf{I}$ is the identity matrix, the scalar α is the trade-off parameter controlling the discontinuities in the solution, and the scalar ϵ balances the relative importance of model and data residuals.

REAL DATA RESULTS

We used 125 CMP's from a 2-D prestack dataset acquired in the Gulf of Mexico. This data is suitable for using Dix equation, since the main reflectors are flat. The area is heavily faulted which may imply strong lateral velocity variations with sharp edges to preserve.

First, we performed velocity analysis on each CMP, and then use an auto-picker to pick the maximum stacking power that corresponds to the best RMS velocity at each CMP location. The value of the stacking power at the auto-picked RMS velocity was used as a quality measure of the data, and used as the data residual weight (**W**) in equations (2), (5), and (8). Clapp (2003) shows an alternative way to calculate the data residual weights based on a multiple realization of the RMS velocity. His approach looks promising since the the RMS velocity average is used as the data and the RMS velocity variance is used as the data residual weight. Figure 1 shows the auto-picked RMS velocity and a stack of the CMP's.

Figures 2, 3, and 4 show the interval velocities resulting from solving the inverse problems stated in equations (2), (5), and (8) respectively. We also show in figure 5 a graph comparing the three methods and the RMS velocity used as input data at two CMP locations.

The resulting interval velocity models show what the regularization was designed to do. In figure 2 the resulting interval velocity is smooth in time and space. Figure 3 shows sharp edged rectangular shapes all over the image, looking reasonable in the faults but in general geological unappealing. Figure 4 shows sharp objects with more geological appeal.

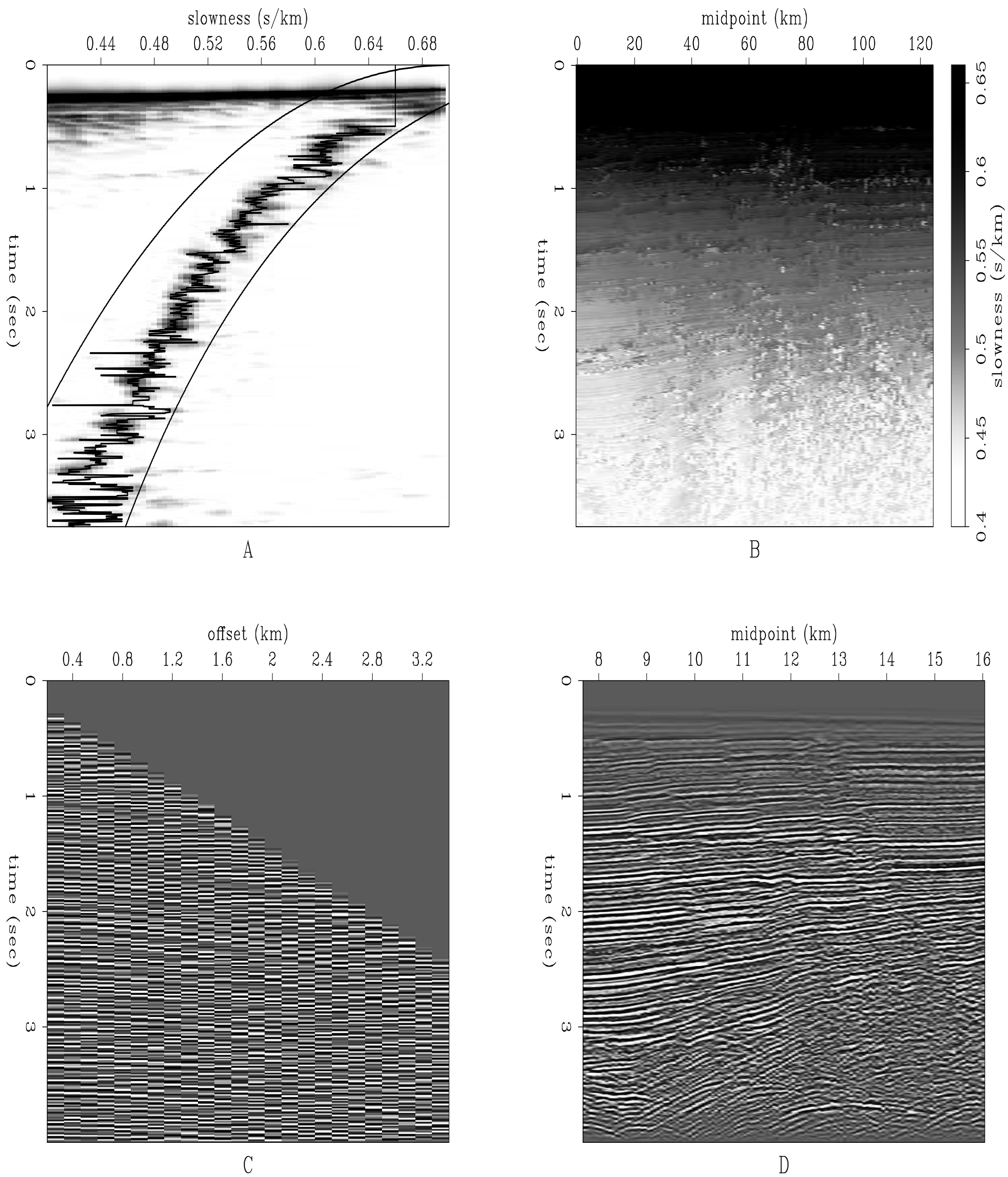

Figure 1: A) Auto-picked RMS velocity of one CMP from a 2-D prestack dataset, B) raw RMS velocity, C) the CMP in question, D) and the stacked data using the raw RMS velocity. alejandro1-vrms2d [CR]

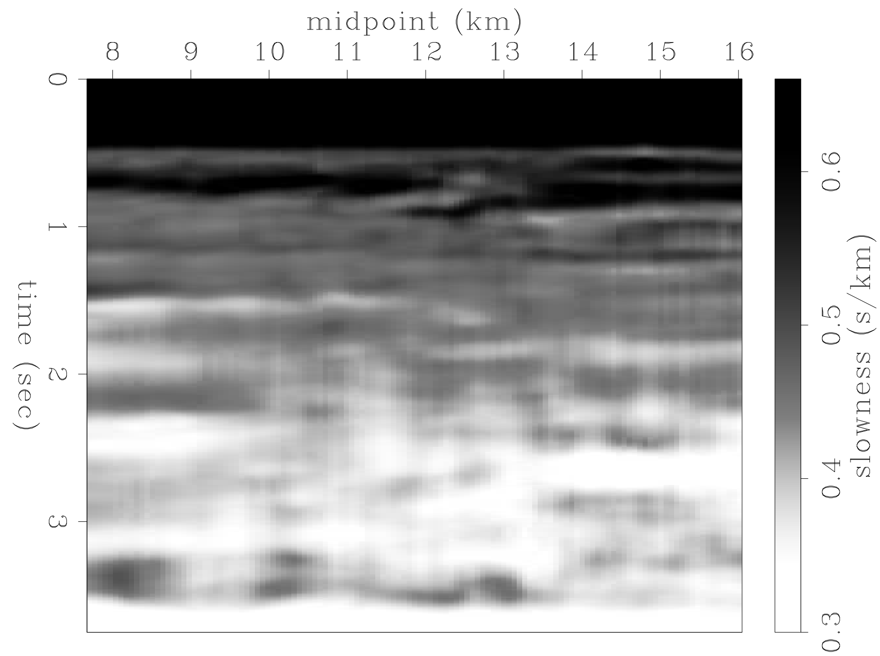

Figure 2: Interval velocity computed by 2-D inversion of the RMS velocity (equation (2)). alejandro1-vint2d_2fit [CR,M]

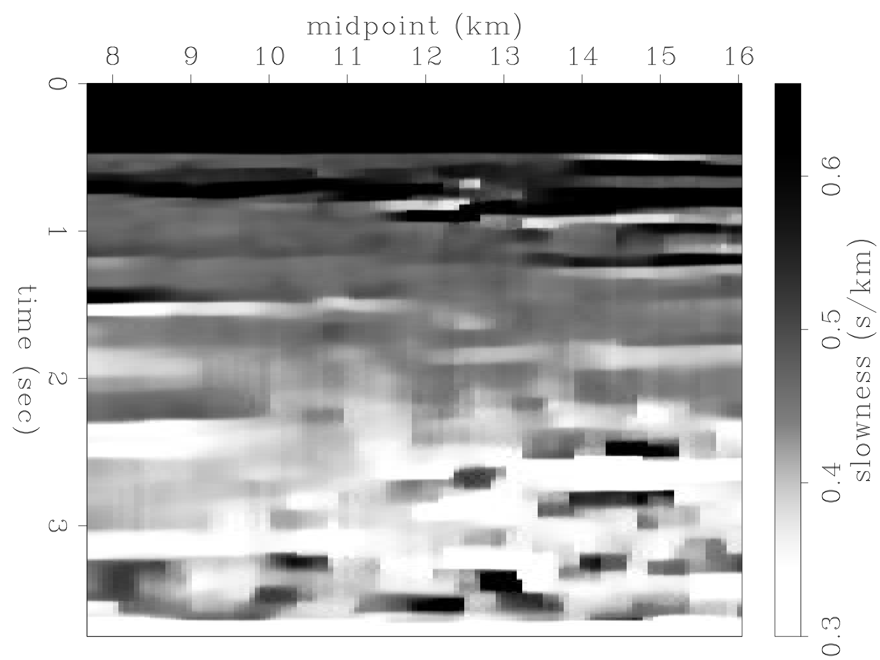

Figure 3: Interval velocity computed by 2-D inversion of the RMS velocity using Cauchy norm (equation (5)). alejandro1-vint2d_2fit_L1 [CR,M]

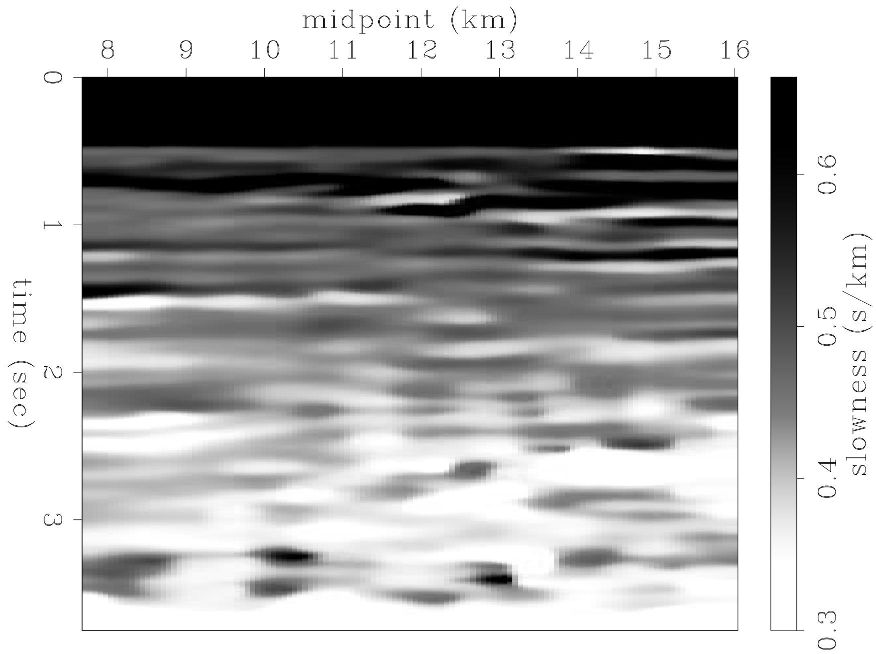

Figure 4: Interval velocity computed by 2-D inversion of the RMS velocity using gradient magnitude in the weighting function (equation (8)) . alejandro1-vint2d_GradMag [CR,M]

The preferential shapes can also be seen in the diagonal weight operator. Figures 6 and 7 show $\mathbf{Q}_{\tau}{}^{N}$ and $\mathbf{Q}_{\mathbf{x}}{}^{N}$, the last nonlinear iteration diagonal weight operator in equation (7). Notice the two preferential directions in what the edges are preserved. Figure 8 shows $\mathbf{Q}_{||\nabla||}{}^{N}$, the last nonlinear iteration diagonal weight operator in equation (9). Notice the isotropic behavior of the diagonal weight calculated using the gradient magnitude operator.

CONCLUSIONS

Dix formula can be implemented in a nonlinear least-squares inversion scheme to attain interval velocities with sharp edges in the (τ, x) plane. In this paper we present two automatic edge-preserving regularization methodologies to achieve this goal.

The first uses IRLS to effectively change the norm of the problem to permit a “spiky” or “sparse” model residual, which leads to a “blocky” velocity model. The second uses an isotropic edge detector, the gradient magnitude, in a nonlinear scheme to compute a measure of the edges of the model. This edge measure is then used as a model residual weight.

Both methods give the expected results when applied in a 2-D real data set acquired in the Gulf of Mexico. Even though, the gradient magnitude method shows sharp objects with more geological appeal than the “blocky” method.

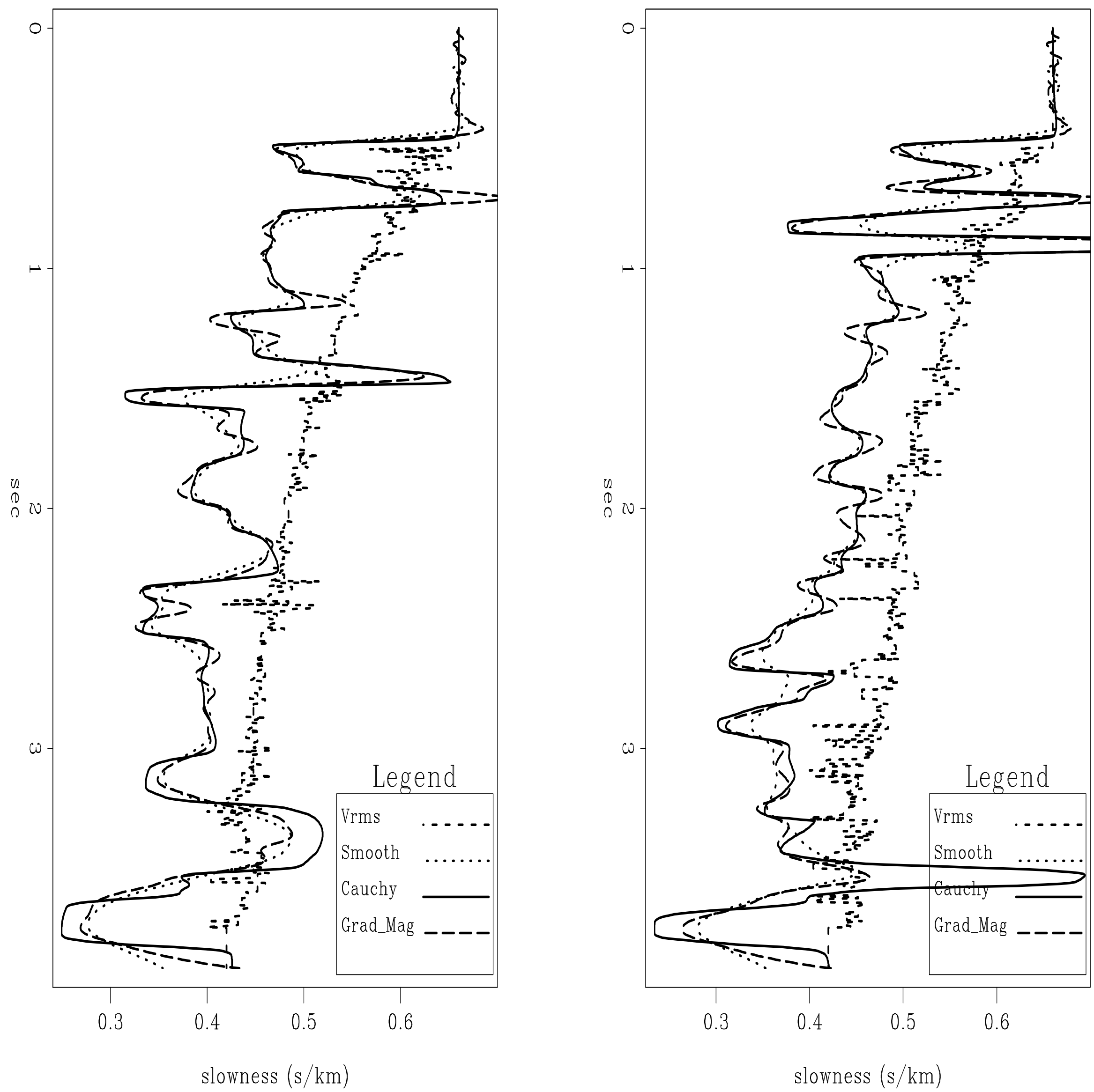

Figure 5: Comparison of the results of solving the inverse problems stated in equations (2) Smooth, (5) Cauchy norm, (8) Gradient Magnitude, and the RMS velocity at the midpoint positions 8.04 and 12.194 km. alejandro1-comp1 [CR]

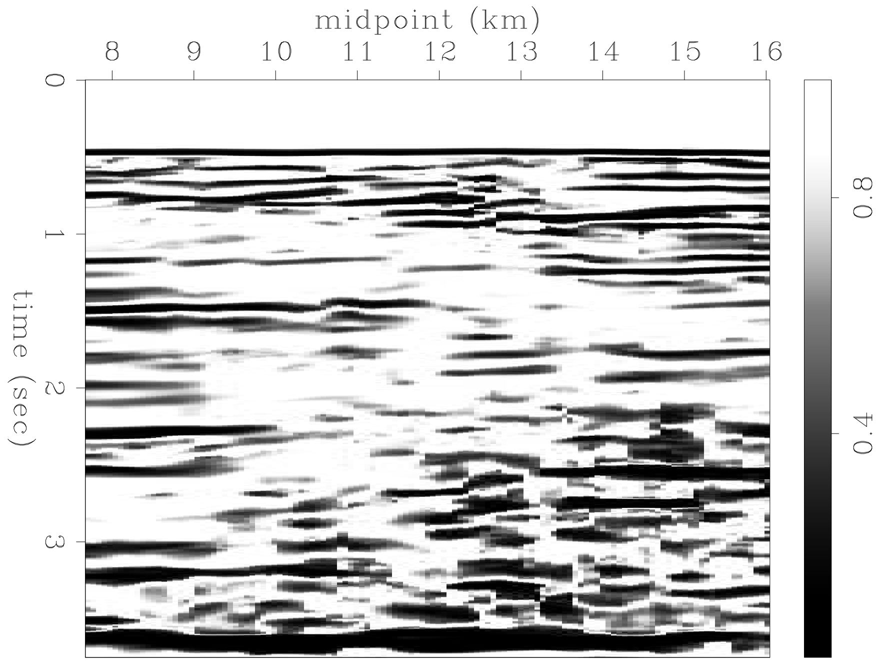

Figure 6: $\mathbf{Q}_{\tau}{}^{N}$ is the last nonlinear iteration diagonal weight operator in equation (6). alejandro1-qx_2fit [CR]

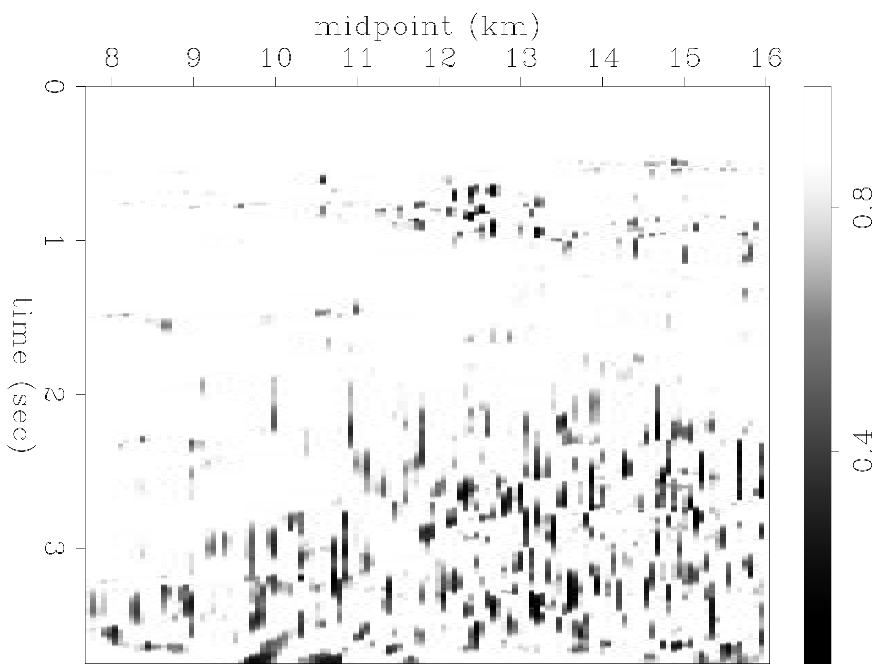

Figure 7: $\mathbf{Q}_{\mathbf{x}}{}^{N}$ is the last nonlinear iteration diagonal weight operator in equation (7). alejandro1-qz_2fit [CR]

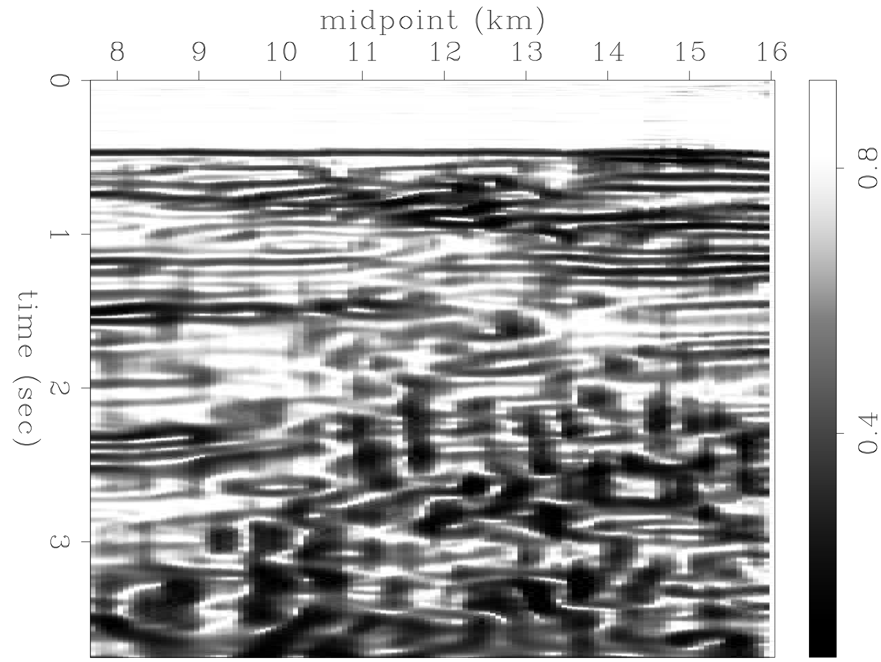

Figure 8: $\mathbf{Q}_{||\nabla||}{}^N$ is the last nonlinear iteration diagonal weight operator in equation (9). alejandro1-q_GradMag [CR]

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