Stanford Exploration Project, Report 111, June 9, 2002, pages 289–293

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Short Note

Amplitude inversion for three reflectivities

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INTRODUCTION

Seismic amplitudes can provide information about rock properties if the amplitudes can be properly processed (Backus, 1983; Castagna and Backus, 1993). This is very difficult to do given the nonlinearity of the Zoeppritz equation. The linearized Bortfeld approximation (Bortfeld, 1961) allows us to invert the data to obtain the reflectivities of events. Unfortunately, most forms of the Bortfeld approximation involve the primary and shear wave interval velocities which are difficult to obtain without knowing the rock properties. This paper will first look at the basic form of the Bortfeld approximation then explain a new form that eliminates the need for the interval velocities. Finally, the new form will be used on a simple synthetic.

BORTFELD'S 3 TERM REFLECTIVITY EQUATION

Bortfeld's three term reflectivity equation is a linearized form of the Zoeppritz equation. It predicts the amplitude (R) at various reflection angles (θ) given three reflectivity terms. These three terms are the zero-offset reflectivity (R_O), the P-wave reflectivity (R_P), and a gradient term (R_{sh}). This is its basic form:

$$R(\theta_i) \;=\; R_O + R_{sh}\sin^2(\theta_i) + R_P\tan^2(\theta_i)\sin^2(\theta_i) \tag{1}$$

where

$$R_P \;=\; \frac{\Delta V_p}{2V_p} \qquad R_\rho \;=\; \frac{\Delta\rho}{2\rho} \qquad R_O \;=\; R_P + R_\rho \tag{2}$$

$$R_{sh} = \frac{1}{2}\left(\frac{\Delta V_p}{V_p} - k\frac{\Delta\rho}{\rho} - 2k\frac{\Delta V_s}{V_s}\right) \qquad k \;=\; \left(\frac{2V_s}{V_p}\right)^2$$

This form is easily obtained algebraically from the form derived in Aki and Richards (1980). Both forms are well known and frequently used, but may not be accurate in areas where the interval velocities are not well known. Therefore, we choose to look at a different form.

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The Stack-Constrained form

The basic form just described depends on a $\frac{V_s}{V_p}$ ratio, meaning that to get the information on the rock properties, you first need a fairly accurate interval velocity model. Fred Herkenhoff of Chevron realized that by substituting the value of a stacked trace into the problem, he could constrain the result to have values similar to those of the stack and remove the dependence on the velocity ratio. The stack amplitude can be calculated from the basic form:

$$S \;=\; R_O + R_{sh}\sin^2(\theta_{S1}) + R_P\tan^2(\theta_{S2})\sin^2(\theta_{S1}) \tag{3}$$

Here, $\sin^2\theta_{S1}$ and $\tan^2\theta_{S2}$ are the averages of $\sin^2\theta$ and $\tan^2\theta$ over the range of input angles. This stack equation is then used as a substitute for the $\frac{V_s}{V_p}$ ratio:

$$R(\theta_i) - S\frac{\sin^2(\theta_i)}{\sin^2(\theta_{S1})} \;=\; R_O\left(1-\frac{\sin^2(\theta_i)}{\sin^2(\theta_{S1})}\right) + R_P\left(\tan^2(\theta_i) - \frac{\tan^2(\theta_i)\sin^2(\theta_i)}{\sin^2(\theta_{S1})}\right) \tag{4}$$

This form can now be inverted for the zero-offset reflectivity (R_O) and the P-wave reflectivity (R_P) without needing the interval velocities. Once those reflectivities are obtained, the stack equation (Eqn. 3) can be used to find the gradient term. This inversion is demonstrated in the next section.

RESULTS

At this time I am testing this formulation on synthetic data. I created a ten trace angle gather (Figure 1) using the basic form of Bortfeld's equations. Figure 1 also contains the stacked trace of this angle gather. There are five events with different reflectivities. The results of the inversion are in Figure 2 and are fairly accurate. The answers are:

Event		Correct R_O	Est. R_O		Correct R_{sh}	Est. R_{sh}		Correct R_P	Est. R_P
1		0.023	0.0230		0.0	-0.00139		0.023	0.0257
2		0.035	0.0350		-0.01	-0.0132		0.023	0.0251
3		0.01	0.0100		0.01	0.00572		0.03	0.0342
4		-0.03	-0.0300		0.0	-0.00476		0.03	0.0367
5		0.02	0.0200		-0.02	-0.0175		-0.02	-0.0229

The solutions for R_O have been found correctly. There is some error in the solution for R_P and this in turn has produced some error in the calculated values for R_{sh}. However, the errors are small so it seems that this form can be successfully inverted. The fact that it is not dependent on interval velocities makes it preferable to other forms of the Bortfeld approximation.

ACKNOWLEDGMENTS

I would like to thank Chevron and my group leader, John Toldi, for the experience of working on this project as an intern at Chevron and for giving me permission to publish it in this report.

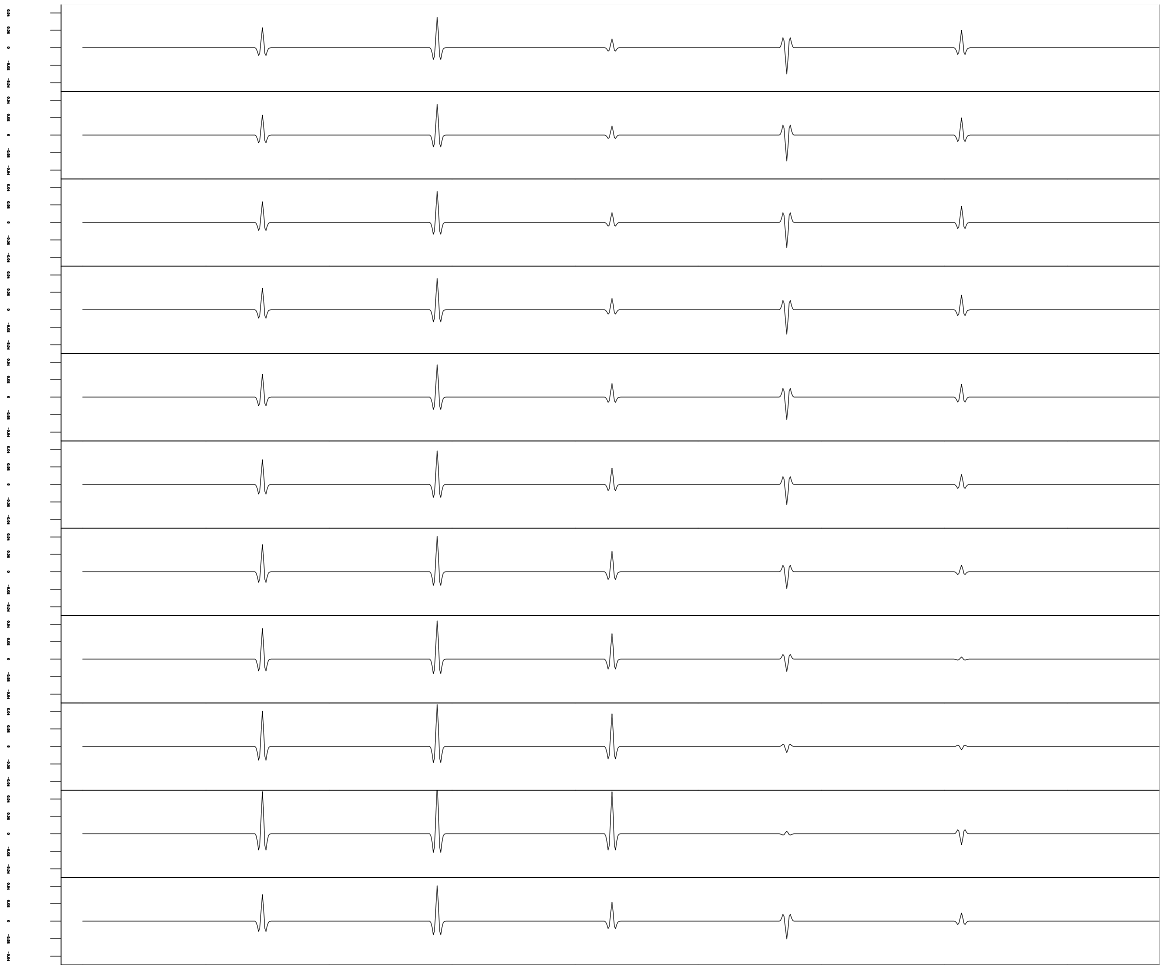

Figure 1: Input angle gather. The bottom trace is the stacked trace. marie2-input [ER]

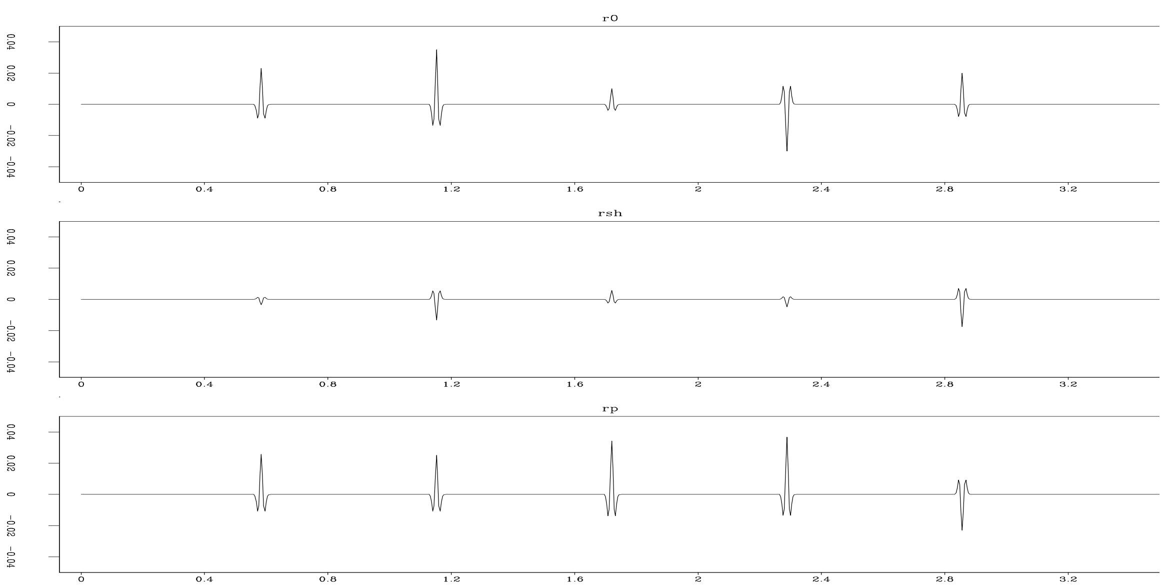

Figure 2: Output reflectivities. Top: R_O, middle: R_{sh}, bottom: R_P. marie2-output [ER]

Many Chevron employees, most notably Rich Alford and Jeff Wright, were also instrumental in this work.

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