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Converted wave azimuth moveout

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ABSTRACT

Accurate prestack partial migration operators are important in seismic exploration. The development of different technologies, like the use of PS converted wave data, suggests the extension of applications of already successful operators and techniques for PP data. Azimuth moveout (AMO) is a partial migration operator that transforms prestack data into equivalent data with arbitrary offset and azimuth. We introduce a new, more accurate prestack partial migration operator for converted wave data. This operator has promising future applications in the regularization of ocean bottom seismic data.

INTRODUCTION

Stacking is an important process to the seismic exploration industry. It is an effective way to reduce the size of data sets and to enhance reflections while attenuating noise. However, the validity of stacking multiple-coverage data is questionable in the case of PS converted wave data because, even for a horizontal reflection in a constant velocity media, raypaths in a CMP gather strike different reflection points.

Prestack partial migration operators are useful tools in reducing the size of seismic data. Dip moveout (DMO) is the most common prestack partial migration operator. Rosales (2002) comments on a series of DMO operators for PS data. The operators differ in numerical approximations of the moveout equation, processing domain and implementation domain. He also introduces a more accurate PS-DMO operator in the log-stretch *f-k* domain that gives an appropriate amplitude distribution.

Biondi et al. (1998) introduce a more general prestack partial migration operator called Azimuth Moveout (AMO). AMO has the advantage of transforming prestack data into equivalent data with arbitrary offset and azimuth, moving events across midpoints according to their dip. Several advantages have been described for the AMO operator. Among them are: 1) partial stacking of prestack data, in order to create regularly sampled common offset-azimuth cubes (Chemingui and Biondi, 1997; Chemingui, 1999; Biondi, 2000) and 2) data regularization of irregular sampled data which preserves amplitudes (Biondi and Vlad, 2001).

This work presents the equivalent of the PP-AMO operator for converted wave data. We explain the geometrical interpretation of our PS-AMO operator, in which the concept of CCP transformation is important since it is the base for event movement according to its dip. Our

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PS-AMO operator is a cascade operation of PS-DMO and inverse PS-DMO. We exploit the knowledge of the fast and accurate PP-AMO in the log-stretch frequency-wavenumber domain (Vlad and Biondi, 2001) by selecting the PS-DMO operator in the log-stretch frequency-wavenumber domain introduced by Xu et al. (2001), reformulated and improved by Rosales (2002).

The PS-AMO operator has a significant future application, the regularization of ocean bottom seismic (OBS) data. The presence of already existing platforms produces holes in the data. This information can be safely regularized with an appropriate operator, in this case a PS-AMO operator.

PS-AMO

Azimuth moveout is a prestack partial migration operator that transforms 3D prestack data with a given offset and azimuth into equivalent data with a different offset and azimuth.

PP-AMO is not a single trace to trace transformation. It is a partial migration operator that moves events across **midpoints** according to their dip. Due to the nature of PS-data, where multiple coverage is obtained through common conversion point gathers (CCP), the PS-AMO operator moves events across **common conversion points** according to their geological dip.

Theoretically, the cascade of any imaging operator with its corresponding forward-modeling operator generates an AMO operator (Biondi, 2000). Our PS-AMO operator is a cascade operation of PS-DMO and inverse PS-DMO.

The 2D PS-DMO smile (Harrison, 1990; Xu et al., 2001; Rosales, 2002) extends to 3D by replacing the offset and midpoint coordinates for the offset and midpoint vectors, respectively. The factor D, responsible for the CMP to CCP transformation, also transforms to a vector quantity. The PS-DMO smile in 3D takes the form of:

$$\frac{t_0^2}{t_n^2} + \frac{\|\vec{y}\|^2}{\|\vec{H}\|^2} = 1, \tag{1}$$

where,

$$\|\vec{y}\|^2 = \|\vec{x} + \vec{D}\|^2, \tag{2}$$

$$\vec{H} = \frac{2\sqrt{\gamma}}{1+\gamma}\vec{h}, \tag{3}$$

and

$$\vec{D} = \left[1 + \frac{4\gamma\|\vec{h}\|^2}{v_p^2 t_n^2 + 2\gamma(1-\gamma)\|\vec{h}\|^2}\right]\frac{1-\gamma}{1+\gamma}\vec{h}$$

Here, $\vec{x}$ is the midpoint position vector, $\vec{h}$ is the offset vector and $\gamma = \frac{v_p}{v_s}$ ratio. The PS-AMO operator, a cascade operator of PS-DMO and inverse PS-DMO, takes the form of:

$$t_2^2 = t_1^2 \frac{\vec{H}_{02}^2}{\vec{H}_{10}^2} \left\{ \frac{\vec{H}_{10}^2 \sin^2(\theta_1 - \theta_2) - \vec{x}^2 \sin^2(\theta_2 - \Delta\phi) - \mathbf{D}_1}{\vec{H}_{02}^2 \sin^2(\theta_1 - \theta_2) - \vec{x}^2 \sin^2(\theta_1 - \Delta\phi) - \mathbf{D}_2} \right\}, \quad (4)$$

where

$$\begin{aligned} \mathbf{D}_1 &= \vec{D}_{10}^2 \sin^2(\theta_1 - \theta_2) + 2\vec{x}\,\vec{D}_{10} \sin(\theta_2 - \Delta\phi) \sin(\theta_1 - \theta_2) \cos\lambda, \\ \mathbf{D}_2 &= \vec{D}_{02}^2 \sin^2(\theta_1 - \theta_2) + 2\vec{x}\,\vec{D}_{0i} \sin(\theta_1 - \Delta\phi) \sin(\theta_1 - \theta_2) \cos\lambda. \end{aligned}$$

This operator reduces to the traditional expression of PP-AMO (Biondi et al., 1998) for $\vec{D} = \vec{0}$ (i.e. $\gamma = 1$).

Although the PP-AMO operator is velocity independent, this independence doesn't propagate for the PS-AMO operator. The PS-AMO operator depends on the P velocity and the $\frac{v_p}{v_s}$ ratio. We assume that the velocity of the new trace position is the same as in the previous position.

Geometrical interpretation of PS-AMO

A trace with input offset vector $\vec{h}_1$ and midpoint position $\vec{x}$ is first transformed to its corresponding CCP position and zero offset. By defining the new offset and azimuth position and by applying inverse PS-DMO, we transform the data to a new CCP position and its corresponding CMP position.

Here, we follow the same procedure as Fomel and Biondi (1995); Biondi et al. (1998) for the derivation of the PS-AMO operator.

First, we refer to equations (1) and (2) in order to understand the relationship between CMP and CCP for the 3D case. We rewrite equation (2) as

$$\|\vec{y}\|^2 = \|\vec{x}\|^2 + \|\vec{D}\|^2 + 2\|\vec{x}\|\,\|\vec{D}\| \cos\lambda,$$

where λ is the angle between the midpoint vector ($\vec{x}$) and the transformation vector ($\vec{D}$).

We can then rewrite equation (1) as

$$\frac{t_0^2}{t_n^2} + \frac{\|\vec{x}\|^2}{\|\vec{H}\|^2} + \frac{\|\vec{D}\|^2 + 2\|\vec{x}\|\,\|\vec{D}\| \cos\lambda}{\|\vec{H}\|^2} = 1. \quad (5)$$

$\vec{D}$ is an extension of $\vec{h}$ and lies in the CCP space. Figure 1 shows both $\vec{h}$ and $\vec{D}$ in the same plane. Since the vectors are parallel, the angle between $\vec{x}$ and $\vec{D}$ is the same as the angle between $\vec{x}$ and $\vec{h}$. If the coordinate system is aligned with the midpoint coordinates, then the angle λ is the same as the azimuth ($\lambda = \theta$). λ changes after and before PS-AMO. This variation is responsible for the event movement along the common conversion point.

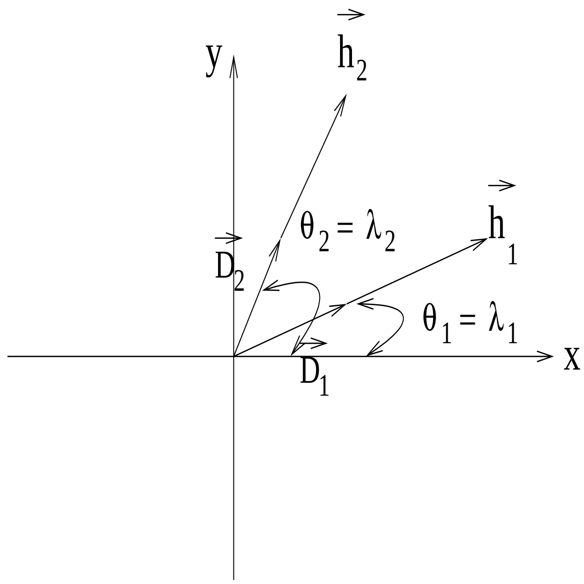

Figure 1: Definition of offset vector $\vec{h}$ and transformation vector $\vec{D}$, before and after PS-AMO daniel1-rot [NR]

Figure 2 shows how event movement along CCP changes with depth. This is due to the dependence of $\vec{D}$ with respect to v_p, γ and t_n. This variance with depth will persist even in a constant velocity media. Figure 2 also illustrates that the time after PS-AMO (t_2) has a new $\vec{h}$ and $\vec{D}$, therefore, a new CCP position.

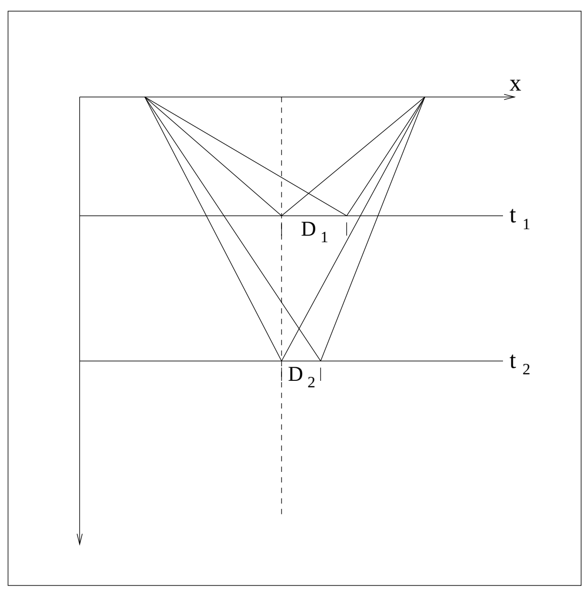

Figure 2: Comparison between the CMP and CCP position in the PS-AMO operator daniel1-plane2 [NR]

Continuing with the procedure presented by Fomel and Biondi (1995) to obtain the PS-AMO operator, we cascade PS-DMO [equation (5)] with its inverse. Figure 3 shows a scheme of the PS-AMO transformation. A trace with input offset vector $\vec{h}_1$ and midpoint at the origin is transformed into equivalent data with output offset vector $\vec{h}_2$ and midpoint position $\vec{x}$. The data is first transformed to its corresponding CCP position and $\vec{D} = \vec{0}$. Subsequently, the inverse PS-DMO repositions the data to a new midpoint position $\vec{x}$ with a new offset vector $\vec{h}_2$. The new trace position is defined by

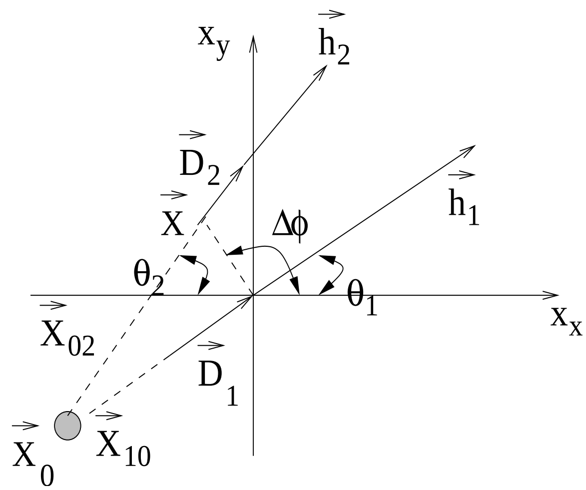

Figure 3: CMP-CCP plane, PS-AMO geometrical interpretation. daniel1-plane [NR]

$$t_2^2 = t_1^2 \frac{\vec{H}_{02}^2}{\vec{H}_{10}^2} \left(\frac{\vec{H}_{10}^2 - \vec{x}_{10}^2 - \vec{D}_{10}^2 - 2\vec{x}_{10}\vec{D}_{10}}{\vec{H}_{02}^2 - \vec{x}_{02}^2 - \vec{D}_{02}^2 - 2\vec{x}_{02}\vec{D}_{02}} \right). \tag{6}$$

Both $\vec{x}_{10}$ and $\vec{x}_{02}$ can be expressed as terms of the final midpoint position $\vec{x}$ by using the rule of sines in the triangle ($\vec{x}$,$\vec{x}_{10}$,$\vec{x}_{02}$) in Figure 3 as

$$\begin{aligned} \vec{x}_{10} &= \vec{x}\frac{\sin(\theta_2 - \Delta\phi)}{\sin(\theta_1 - \theta_2)} \\ \vec{x}_{02} &= \vec{x}\frac{\sin(\theta_1 - \Delta\phi)}{\sin(\theta_1 - \theta_2)}. \end{aligned}$$

The final expression takes the form of

$$t_2^2 = t_1^2 \frac{\vec{H}_{02}^2}{\vec{H}_{10}^2} \left\{ \frac{\vec{H}_{10}^2 \sin^2(\theta_1 - \theta_2) - \vec{x}^2 \sin^2(\theta_2 - \Delta\phi) - \mathbf{D}_1}{\vec{H}_{02}^2 \sin^2(\theta_1 - \theta_2) - \vec{x}^2 \sin^2(\theta_1 - \Delta\phi) - \mathbf{D}_2} \right\}, \tag{7}$$

where

$$\begin{aligned} \mathbf{D}_1 &= \vec{D}_{10}^2 \sin^2(\theta_1 - \theta_2) + 2\vec{x}\vec{D}_{10} \sin(\theta_2 - \Delta\phi)\sin(\theta_1 - \theta_2)\cos\theta_1, \\ \mathbf{D}_2 &= \vec{D}_{02}^2 \sin^2(\theta_1 - \theta_2) + 2\vec{x}\vec{D}_{0i} \sin(\theta_1 - \Delta\phi)\sin(\theta_1 - \theta_2)\cos\theta_2. \end{aligned}$$

This expression represents the azimuth rotation in both the CCP domain and the CMP domain.

PS-AMO IN THE *F-K* LOG-STRETCH DOMAIN

In order to implement a fast azimuth moveout operator, we use the PS-DMO operator in the frequency-wavenumber log-stretch domain (Xu et al., 2001; Rosales, 2002). The *f-k* log-stretch operator for PS-DMO in 3D takes the form

$$P(\Omega,\vec{k},\vec{h}) = P(\Omega,\vec{k},\vec{h})F(\Omega,\vec{k},\vec{h})e^{i\vec{k}\cdot\vec{D}}. \tag{8}$$

Following Vlad and Biondi's (2001) approach, we construct the PS-AMO operator in this domain. The operator takes the form of

$$P(\Omega,\vec{k},\vec{h}_2) = P(\Omega,\vec{k},\vec{h}_1)\frac{F(\Omega,\vec{k},\vec{h}_1)}{F(\Omega,\vec{k},\vec{h}_2)}e^{i\vec{k}\cdot(\vec{D}_1-\vec{D}_2)}, \tag{9}$$

where

$$F(\Omega,\vec{k},\vec{h}_i) = \begin{cases} 0 & \text{for } \vec{k}\cdot\vec{h}_i = 0 \\ 0 & \text{for } \Omega = 0 \\ e^{\frac{i}{2}\Omega \log\frac{1}{2}\left(\sqrt{\left(\frac{2\vec{k}\cdot\vec{H}_i}{\Omega}\right)^2+1}+1\right)} & \text{otherwise,} \end{cases} \tag{10}$$

and $\vec{k}$ is the spatial frequency vector for the midpoints coordinates (for this case, the vectors $\vec{D}$ and $\vec{H}$ are the same as presented in the previous section).

Rosales (2002) discusses a more accurate PS-DMO operator in this domain. This new PS-DMO operator distributes the amplitudes correctly along strong dip events. This operator is just the extension of Zhou et al. (1996) for PS data. Using the improved operator presented by Rosales (2002) the filter $F(\Omega,\vec{k},\vec{h}_i)$ takes the form:

$$F(\Omega,\vec{k},\vec{h}_i) = \begin{cases} 0 & \text{for } \vec{k}\cdot\vec{h}_i = 0 \\ \vec{k}\cdot\vec{H}_i & \text{for } \Omega = 0 \\ e^{\frac{i}{2}\Omega\left\{\sqrt{1+\left(\frac{2\vec{k}\cdot\vec{H}_i}{\Omega}\right)^2}-1-\log\frac{1}{2}\left[\sqrt{\left(\frac{2\vec{k}\cdot\vec{H}_i}{\Omega}\right)^2+1}+1\right]\right\}} & \text{otherwise.} \end{cases} \tag{11}$$

NUMERICAL EXAMPLES

Figure 4 compares the PP-AMO impulses response obtained with the filter in both equation (10) (top) and equation (11) (bottom). Both are obtained with a value of $\gamma = 1$ for the two cases. Both impulse responses are kinematicly equal. However, the dynamically behavior is different, the amplitudes distribution with the filter in equation (11) is more accurate. The impulse response with the filter in equation (11) and $\gamma = 1$ is exactly the same as Vlad and Biondi (2001).

Figure 5 presents a similar comparison to Figure 4, for the case of converted waves. Here, we use $\gamma = 1.2$ and $v_p = 2.0$Km/s. As in the previous case, the same kinematic behavior occurs in both operators, but the response with the filter in equation (11) is dynamically correct.

Figure 6 shows the comparison between the PP-AMO impulse response and the PS-AMO impulse response. The PS-AMO not only has the same saddle shape as the PP-AMO operator, but it also exhibits a lateral movement. This lateral displacement correspond to the asymmetry of the raypaths or the CCP transformation. The displacement is toward the lower-left part of the cube.

A variation of the PS-AMO impulse response with respect to depth is also observable in Figure 7. This behavior is due to the dependence of the operator on $\frac{v_p}{v_s}$ ratio It is possible to

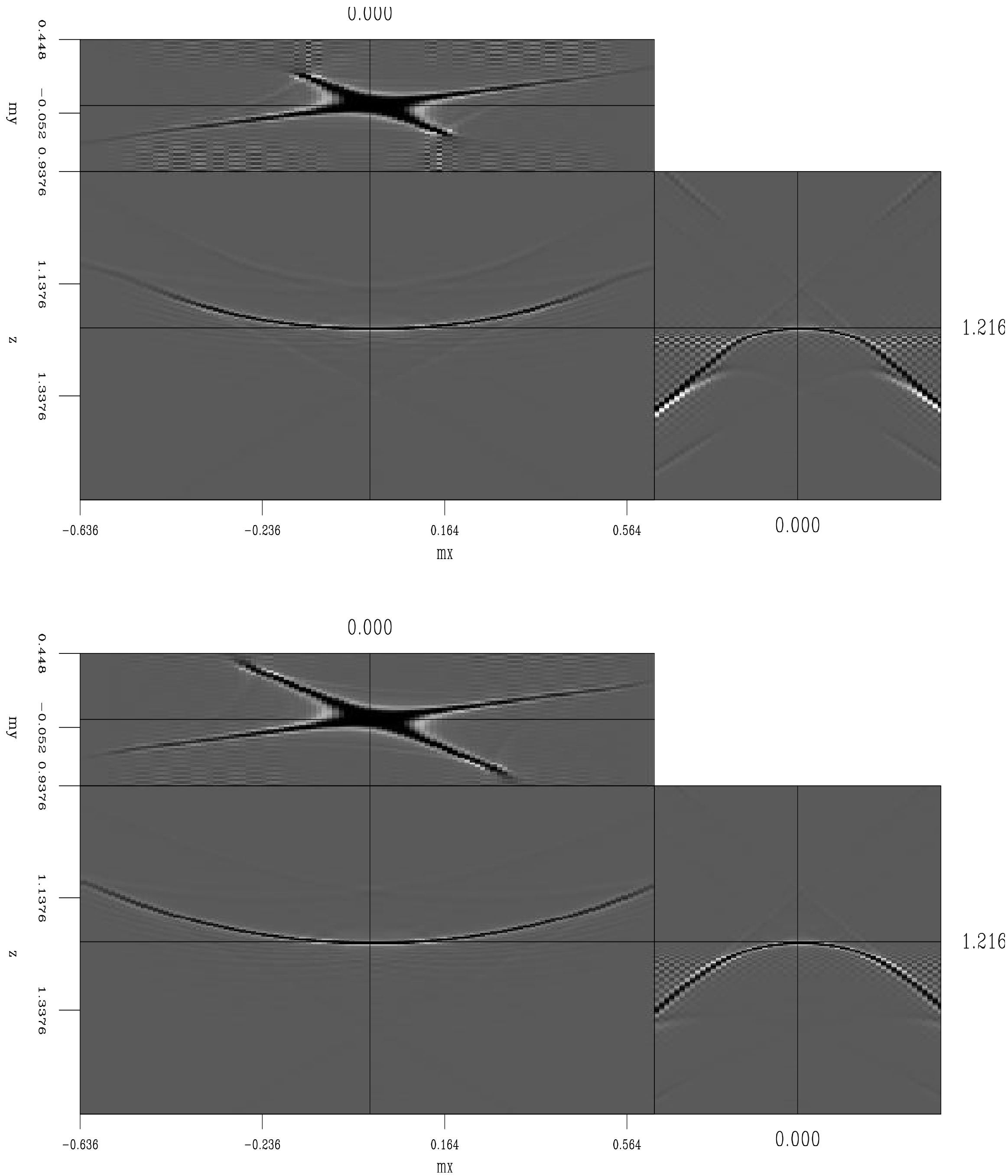

Figure 4: PP-AMO impulse response comparison, filter in equation (10) (top) and filter in equation (11) (bottom), with $\gamma = 1$. daniel1-both [ER,M]

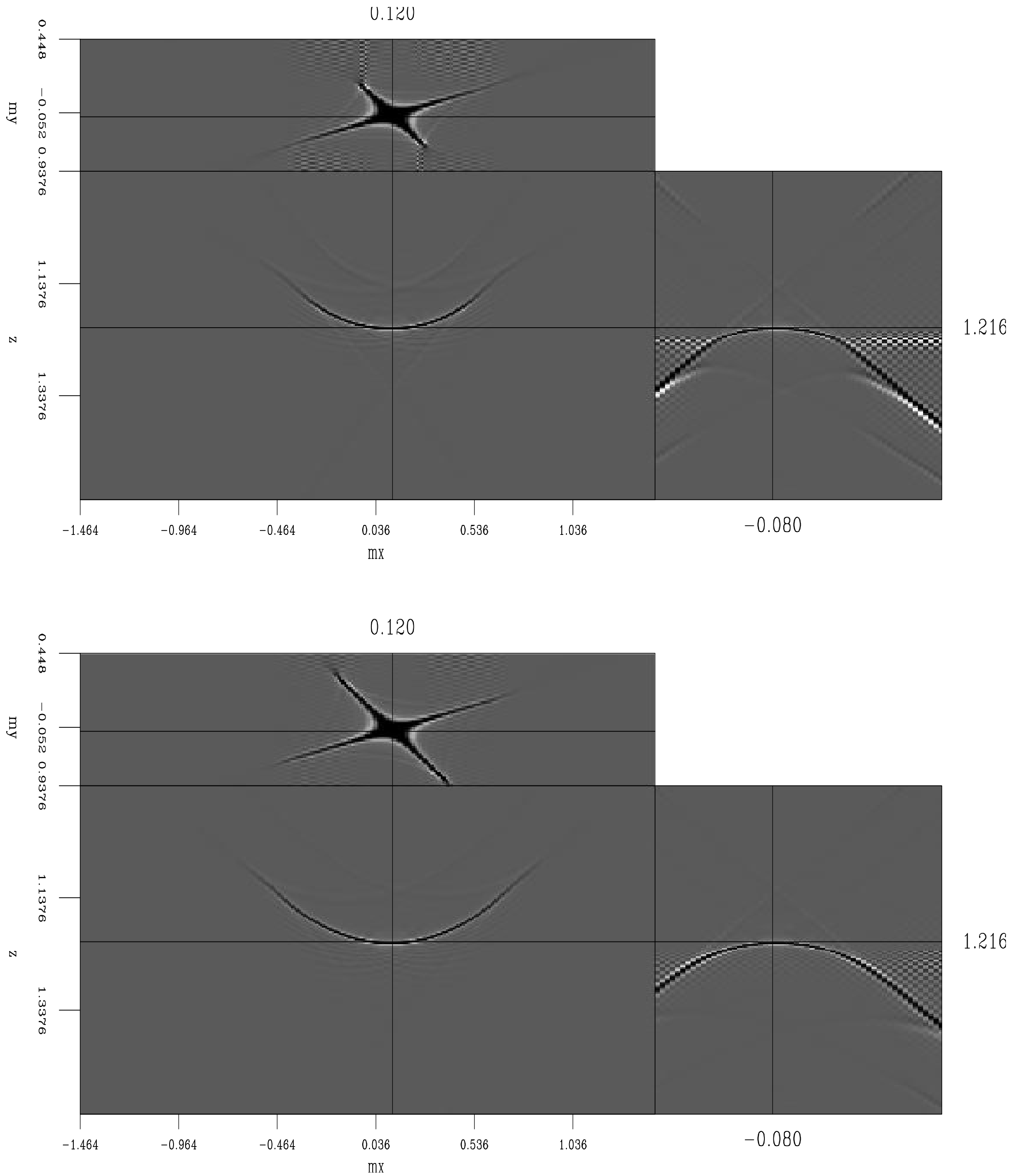

Figure 5: PS-AMO impulse response comparison, filter in equation (10) (top) and filter in equation (11) (bottom), with $\gamma = 1.2$ and $v_p = 2.0$km/s. daniel1-both2 [ER,M]

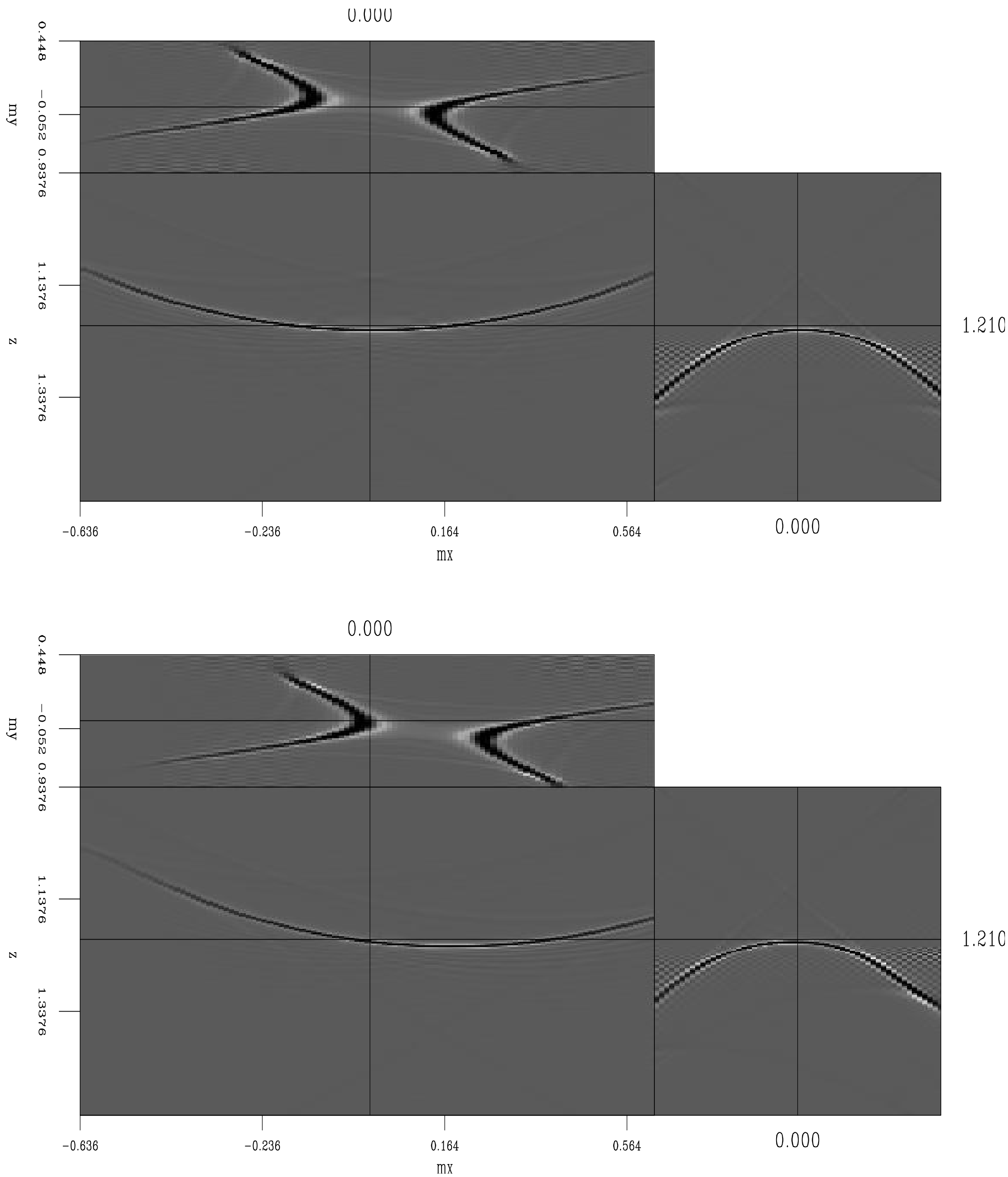

Figure 6: PP-AMO (top) vs. PS-AMO (bottom) impulse response comparison daniel1-both3 [ER,M]

observe how the impulses responses movement toward the left is stronger for shallower events. It is also possible to detect how the response change along the crossline coordinate.

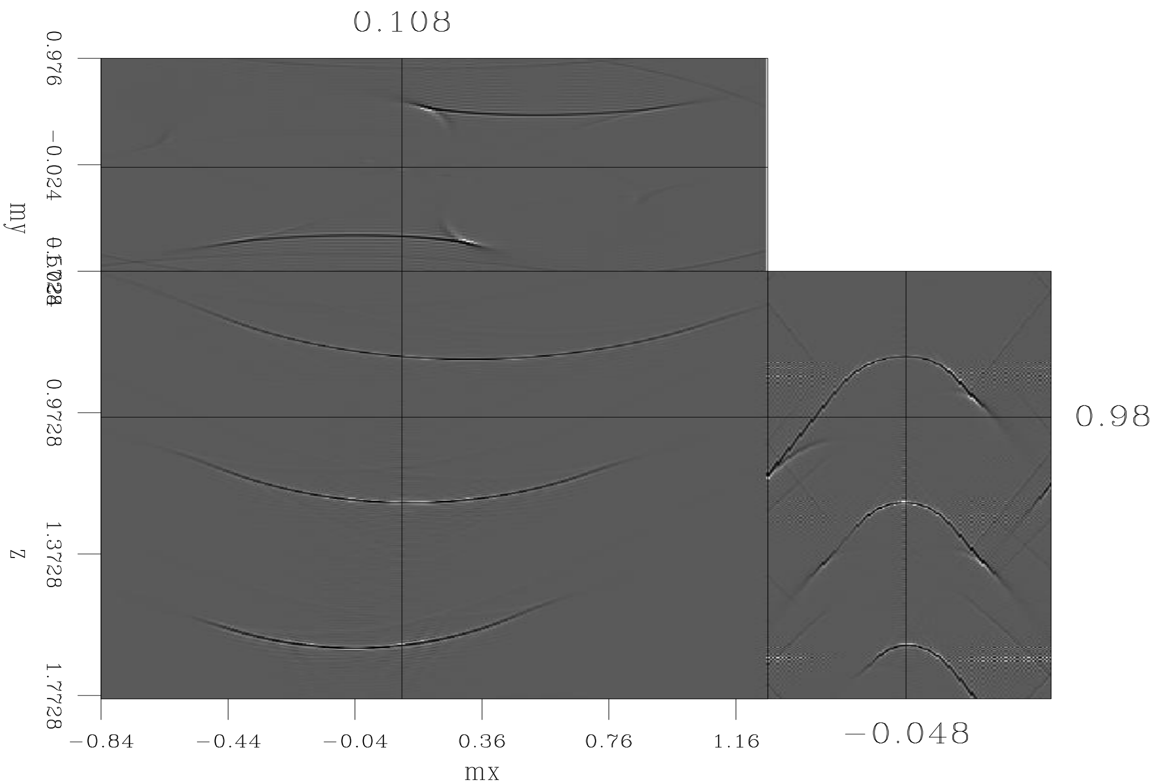

Figure 7: PS-AMO impulses response variation with τ daniel1-amo2 [ER]

DISCUSSION AND CONCLUSION

We present a partial prestack migration operator for converted wave, the PS-AMO operator. This operator is a cascade operator of PS-DMO and inverse PS-DMO. We implement the PS-AMO operator in the log-stretch frequency-wavenumber domain.

Our operator is able to handle the amplitudes correctly, since it uses an accurate PS-DMO operator for this purpose. The PS-AMO operator is velocity dependent, and we assume constant velocity. Therefore, the velocity in the new position is the same as the velocity in the previous position.

The PS-AMO operator transforms data to an arbitrary offset and azimuth. This transformation also considers the CCP binning transformation characteristic of PS data. Therefore, a priori CCP binning is not necessary before applying azimuth moveout to converted wave data.

Our operator can be applied to the regularization of OBS data, where the knowledge of our seismic image is of crucial importance. This is the next step in our project.

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