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Short Note

Matching dips in velocity estimation

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INTRODUCTION

Accurate velocity estimation is essential to obtain a good migrated image and accurate resevoir attributes (Claerbout, 1999). The problem is that tomographic velocity estimation is an underdetermined problem. We can reduce the null space of the tomographic process by adding additional constraints, or more accurate goals, to the estimation. In early work (Clapp et al., 1997; Clapp and Biondi, 1999; Clapp, 2001b,a) I discussed one such constraint: encouraging velocity follows dip. Often we have an added constraint; although we may be unsure of reflector position (due to anisotropy, etc.) or we may have a good estimate of reflector dip (either from well logs, geologic models, etc). By incorporating this information into the inversion we can better constrain the inversion process. This method is tested on a fairly complicated synthetic dataset.

THEORY

Tomography is a non-linear problem that we linearize around an initial slowness model. In this discussion I will be talking about the specific case of ray based tomography but most of the discussion is valid for other tomographic operators. We can linearize the problem around an initial slowness model and obtain a linear relation $\mathbf{T}$ between the change in travel times $\Delta\mathbf{t}$ and change in slowness $\Delta\mathbf{s}$ and reflector position $\Delta\mathbf{r}$. We break up our tomography operator into its two parts, changes due to slowness along the ray $\mathbf{T_{ray}}$ and changes due to reflector movement $\mathbf{T_{ref}}$:

$$\Delta\mathbf{t} \approx \mathbf{T_{ray}}\Delta\mathbf{s} - \mathbf{T_{ref}}\Delta\mathbf{r}. \tag{1}$$

Inverting for both $\Delta\mathbf{s}$ and $\Delta\mathbf{r}$ is an unstable process. We can improve stability by introducing another operator $\mathbf{H}$ which maps slowness changes to reflector changes,

$$\Delta\mathbf{t} \approx \mathbf{T_{ray}}\Delta\mathbf{s} - \mathbf{T_{ref}}\mathbf{H}\Delta\mathbf{s}. \tag{2}$$

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We can approximate the change in travel time due to a change in reflector movement by

$$\mathbf{T_{ref}} = \mathbf{v_{ref}} \cos\theta \cos\phi, \tag{3}$$

where $\mathbf{v_{ref}}$ is the velocity at the reflector, ϕ is the reflector dip, and θ is the reflection angle (Stork, 1992).

We can approximate the change in reflector position due to a change in slowness by assuming movement normal to the reflector and integrating along the normal ray,

$$\mathbf{H} = \int_{\mathbf{ray}} \mathbf{dl}. \tag{4}$$

If we note that the travel time of the normal ray is independent of velocity we can write

$$\begin{aligned} t_0 &= t_1 \\ r_0 s_0 &= (r_0 + \delta r)(s_0 + \delta s) \\ 0 &= \delta r s_0 + \delta r \delta s + r_0 \delta s, \end{aligned} \tag{5}$$

where t_0 is the travel time in the initial model and t_1 is the travel time through the new model. If we ignore the second order term,

$$\delta r \approx -\frac{r_0}{s_0} \delta s. \tag{6}$$

The reason for this review is that our mapping of slowness change to reflector movement leads to a way to approximate reflector dip in the post-tomographic domain.

For simplicity let's concern ourselves with the 2-D problem, though it's easily extendible to 3-D. Imagine that θ_k represents our *a priori* reflector dip, $\mathbf{D}$ is a derivative operator, $\mathbf{r}$ is our final reflector dip, $\mathbf{r_0}$ is the initial reflector position, and $\Delta\mathbf{r}$ is our change in reflector position. We can derive a fairly simple fitting goal relating reflector dip and $\Delta\mathbf{s}$,

$$\begin{aligned} \theta_{\mathbf{k}} &\approx \mathbf{D r} \\ \theta_{\mathbf{k}} &\approx \mathbf{D}(\mathbf{r_0} + \Delta\mathbf{r}) \\ \theta_{\mathbf{k}} - \mathbf{D r_0} &\approx \mathbf{D} \Delta\mathbf{r} \\ \theta_{\mathbf{k}} - \mathbf{D r_0} &\approx \mathbf{D H} \Delta\mathbf{s}. \end{aligned} \tag{7}$$

If we combine this new fitting goal with our tomographic fitting goal and our regularization fitting goal we get,

$$\begin{aligned} \Delta\mathbf{t} &\approx \mathbf{T} \Delta\mathbf{s} \\ \mathbf{0} &\approx \epsilon_0 \mathbf{A} \Delta\mathbf{s} \\ \theta_{\mathbf{k}} - \mathbf{D r_0} &\approx \epsilon_1 \mathbf{D H} \Delta\mathbf{s}. \end{aligned} \tag{8}$$

EXAMPLE

To test the methodology I decided to use a synthetic 2-D dataset generated by BP based on a typical North Sea environment, Figure 1. To avoid tomography's problem with sharp velocity contrasts I chose to assume an accurate knowledge of the velocity structure down to 1.8 km. For the remaining initial velocity structure I smoothed the correct velocity. Figure 2 shows the initial velocity model and initial migration. I then performed two different series of

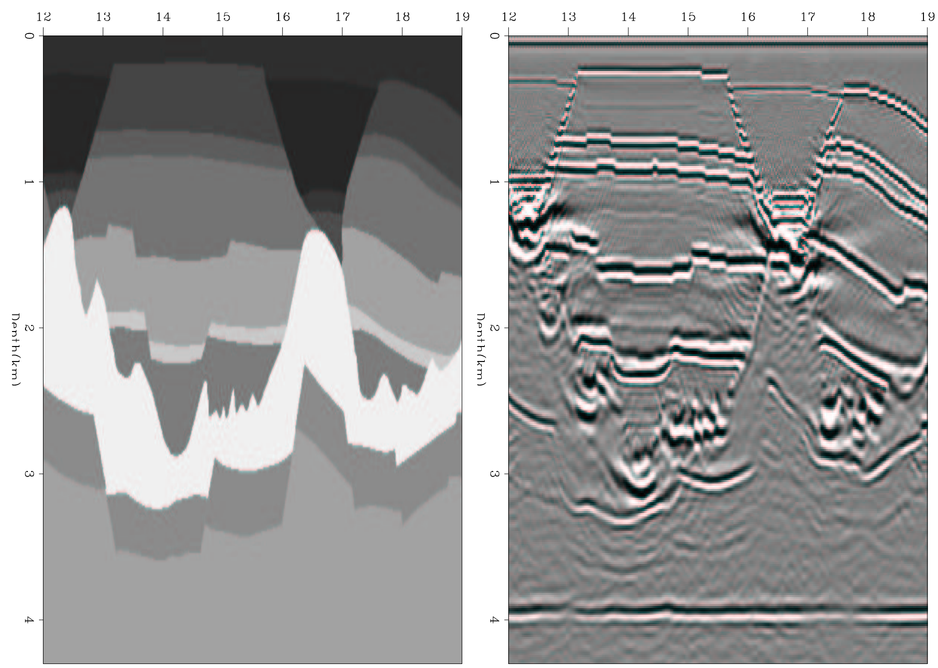

Figure 1: The left panel shows the correct velocity model. The right panel shows the result of migrating with this velocity model. bob5-amoco-vel-cor [CR,M]

tomography loops. In the first case I used a standard approach, without the constraint on dip of the basement reflector at $4km$. Figure 3 shows the initial migration with my pick of the reflector position overlaid ($\mathbf{r_0}$ in fitting goals (8)). Figures 4 and 5 show the velocity and migration result after a single non-linear iteration of tomography using both approaches. In the first iteration the velocity structure looks somewhat more accurate without the dip constraint. The image tells a different story. Note how the bottom reflector is much flatter using the dip constraint condition (Figure 5) and the overall image positioning is a little better. After four iterations, we see a more dramatic difference. Without the dip constraint condition (Figure 6) the velocity model is having trouble converging, especially along the right edge. The bottom reflector is quite discontinuous and misplaced. The overall image quality is disappointing. With the dip constraining condition (Figure 7) the velocity model is correctly finding the salt boundaries. The bottom reflector is fairly flat, consistent, and well positioned. The overall image quality is better than the result without the dip constraint.

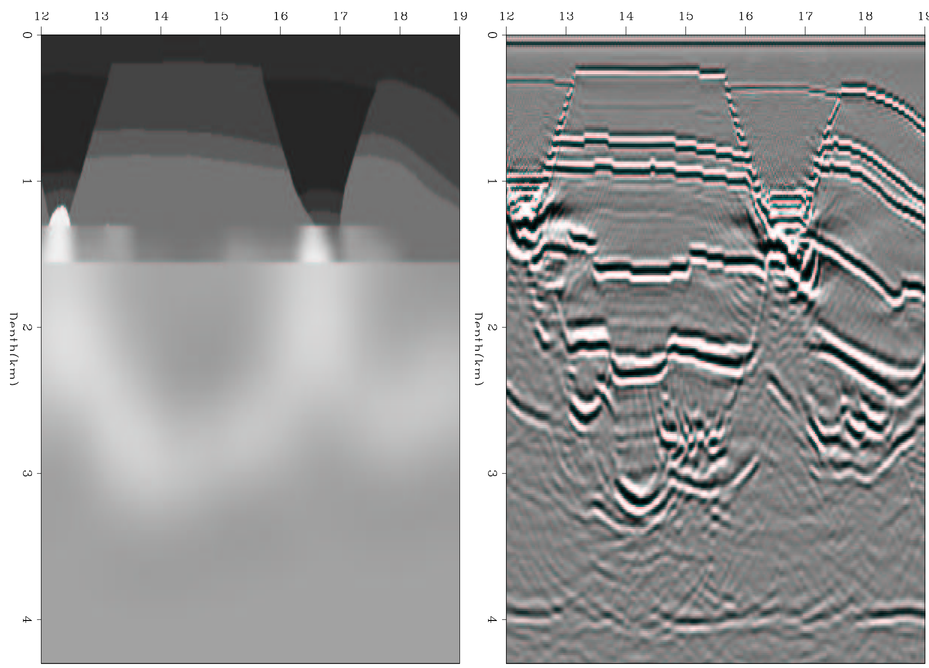

Figure 2: The left panel shows the initial velocity model. The right panel shows the result of migrating with this velocity model. bob5-amoco-vel0 [CR]

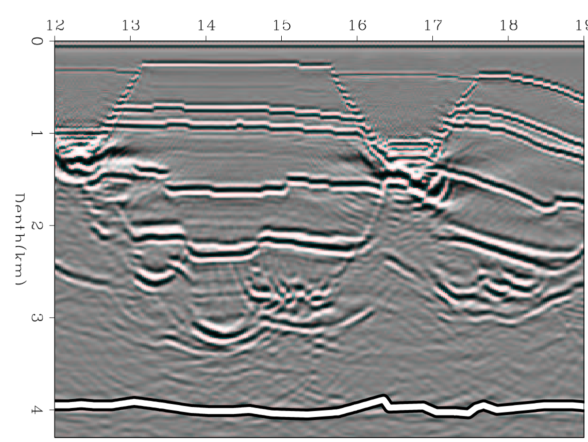

Figure 3: The initial migrated model overlaid by the picked initial reflector position. bob5-picked [CR]

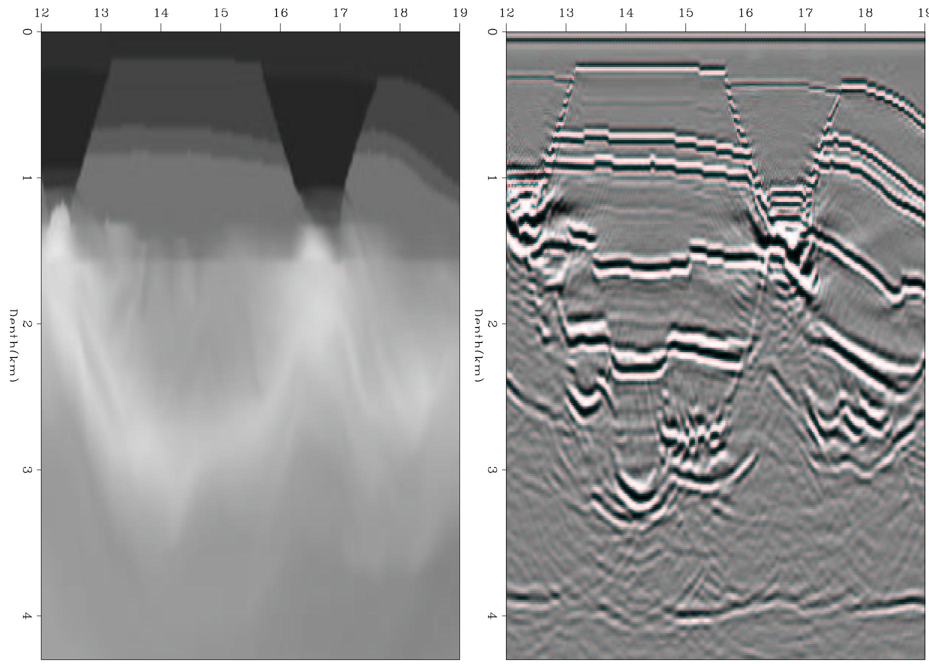

Figure 4: The left panel shows the velocity model after one iteration of 'conventional' tomography. The right panel shows the result of migrating with the velocity model in the left panel. bob5-amoco-vel1.steer [CR,M]

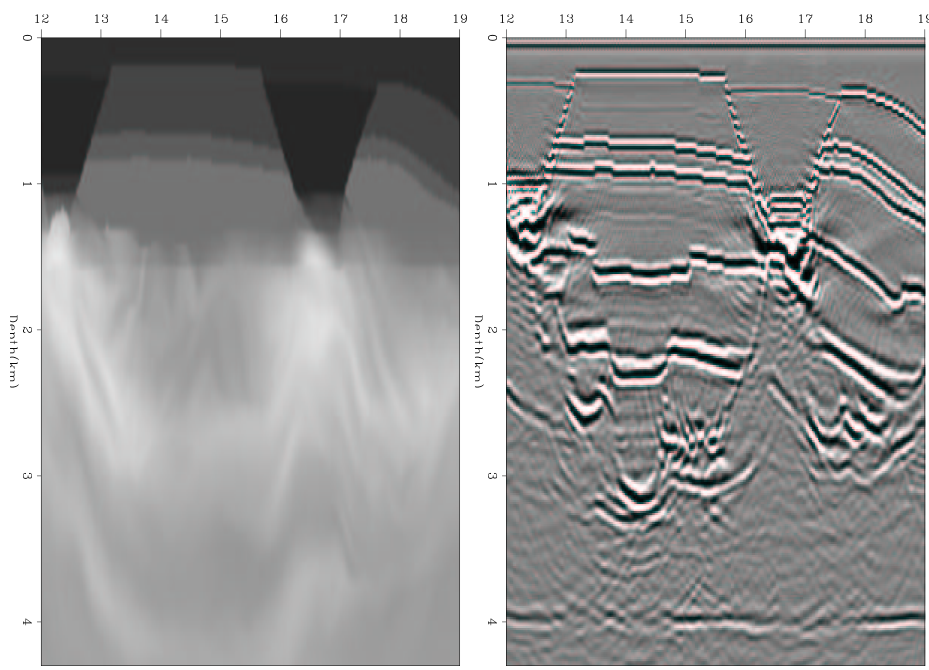

Figure 5: The left panel shows the velocity model after one iteration of tomography with a dip constraint. The right panel shows the result of migrating with the velocity model in the left panel. Note the more continuous nature of the bottom reflector (compared to Figure 4. bob5-amoco-vel1.steer-ref [CR,M]

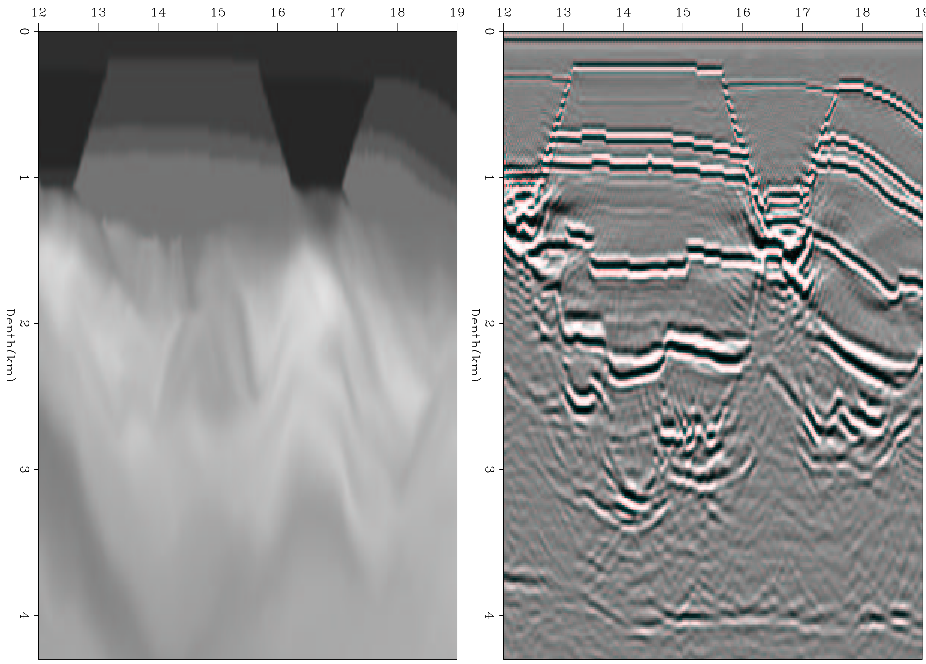

Figure 6: The left panel shows the velocity model after four iteration of ‘conventional’ tomography. The right panel shows the result of migrating with the velocity model in the left panel. bob5-amoco-vel4.steer [CR,M]

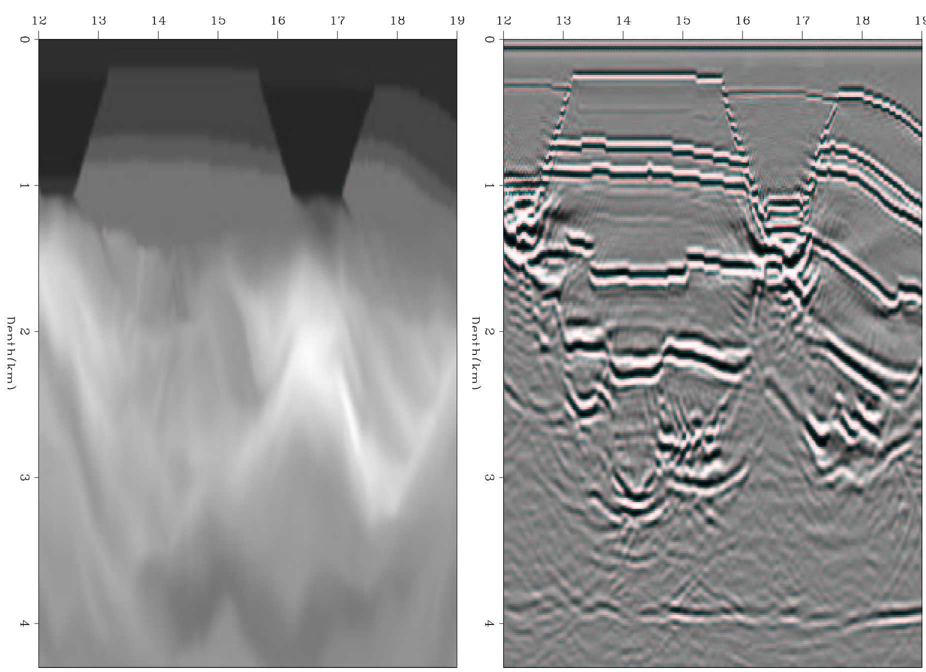

Figure 7: The left panel shows the velocity model after four iteration of tomography with a dip constraint. The right panel shows the result of migrating with the velocity model in the left panel. Note the more continuous nature of the bottom reflector, better constraining of the salt boundaries, and overall more accurate imaging focusing and positioning compared to the result without the added constraint (Figure 6. bob5-amoco-vel4.steer-ref [CR]

CONCLUSIONS

The proposed method for constraining reflector dip in tomography worked well on the complex North Sea synthetic. The estimated velocity model was more accurate with the dip constraint. The migrated image showed overall better image quality and the selected reflector was more continuous and better positioned.

ACKNOWLEDGMENTS

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