

Short Note

Reference velocity selection by a generalized Lloyd method

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INTRODUCTION

Wave equation migration methods are quickly becoming the standard for high-end imaging. These methods are more effective in handling complex wave behavior than traditional Kirchhoff methods (Workshop, 2001). The downside of these methods, specifically ones based upon the one-way wave equation, are that they are generally more expensive than Kirchhoff methods and are only accurate for $v(z)$ media. To overcome the latter weakness extensions such as phase screen (Le Rousseau and de Hoop, 1998), split-step (Stoffa et al., 1990), and Fourier Finite Difference (FFD) (Ristow and Ruhl, 1994) have been introduced. These methods generally rely on downward continuing the wave-field with multiple reference velocities and then applying an approximate correction to the wavefield for the true velocity. The larger the deviation of the reference from the true velocity the less accurately the wavefield will be modeled. The trade-off is that the cost of migration increases almost proportionally with the number of reference velocities. Many different methods are currently employed to select the reference velocities, each with varying levels of success, dependent on the input velocity.

Electrical engineers and image processors face similar problems. In speech compression it is important to accurately describe a signal in as few bytes as possible. In image processing it is often important to reduce the number of colors in image with as little loss in image quality as possible. These problems have led to the field known as quantization. One family of method often employed is based on Lloyd's method (Lloyd, 1982), an iterative technique that allows for variable rate quantization.

In this paper I propose a new method for selecting reference velocities based on a generalized Lloyd's method. I show that the method is effective in choosing appropriate reference velocities for a large variety of models. On two synthetics I demonstrate that it produces a higher quality image with fewer reference velocities.

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THEORY

Selecting reference velocities can be thought of as a non-linear problem. Choosing a reference velocity to use at a given location is a function of which reference velocity is actually closest to the given velocity, which makes posing the problem in a traditional linear framework difficult. When compressing signals, electrical engineers face a similar problem. They try to describe a signal with as few bytes as possible. To do this they develop a codebook. Each member of the codebook represents a region described by a single value.

LLOYD'S METHOD

Lloyd's method attempts to find the best codebook by a simple iterative procedure. Lloyd's method starts from a given initial codebook, then it follows a fairly simple algorithm:

- 1 Find the nearest centroid for all points breaking up the points into cells.
- 2 Recalculate the centroids for each cell.
- 3 Remove any empty cells.

Steps 1-3 are repeated until the solution no longer changes significantly.

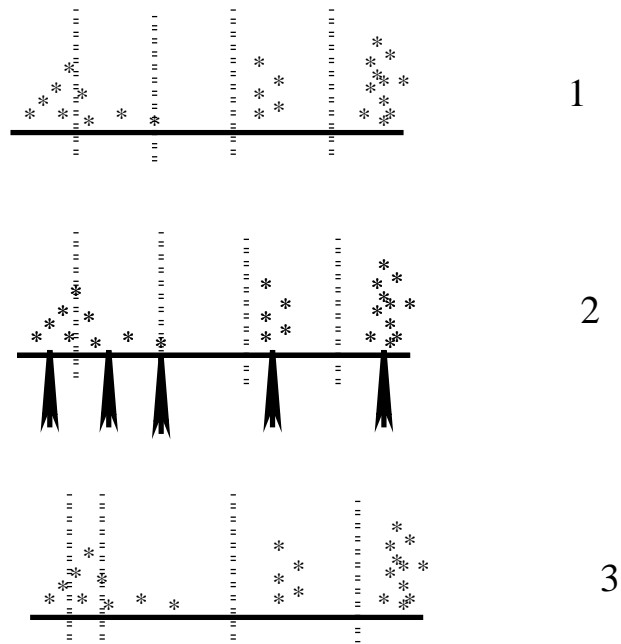


Figure 1: In step 1, the data is dividing into cells. In step2, the centroid of each cell is calculated. In the third step new cell boundaries are calculated between the centroids. If a cell contains no points, the cell is removed. [bob4-ref](#) [NR]

Lloyd's method can be effective but relies on a good initial codebook and can get stuck in local minima. To avoid prejudicing the solution the initial model is often a random set of vectors. To solve the second problem, several methods have been developed (Linde et al., 1980). These methods often rely on replacing empty cell by splitting regions with high variance.

Selecting reference velocities

Lloyd's methods translate fairly easily for a reference velocity selection problem. The values become velocities, the centroids become the reference velocities, and the cells are regions with the same reference velocity. For selecting the initial reference velocities, I used evenly spaced quantiles of the velocity. At early iterations I replace step **3** of Lloyd's method by something a bit more complex.

- 3a** Remove cells whose centroids are closer than a certain percentage from its neighbors (typically 3-8%).
- 3b** Remove cells with fewer than a given number of points (typically 1-3%).
- 3c** Given the removed points in steps 3a and 3b, find the regions with the highest variance that are a given distance away from its neighbors. If no regions meeting these criteria are found, remove them.

The purpose of modifying Lloyd's method is to attempt to avoid local minima. Regions whose centroid are close to its neighbor, or are sparsely populated, are replaced by splitting up regions that seem to be least accurately described by their current centroid.

VELOCITY SELECTION TESTS

To test the methodology I applied it to several different velocity models. The first model is a 2-D synthetic, provided by BP, attempting to emulate North Sea geology (Figure 2). The model is composed of several constant velocity layers. For this test I used the entire velocity model as an input, even though when doing migration each depth step will be analyzed independently. Figure 3 shows a histogram of the velocity function overlaid by the initial reference velocities (0) and those after five (5) and twenty iterations (*). Note how all of the major velocities are identified by the method. For the second example I smooth the velocity in Figure 2 to obtain

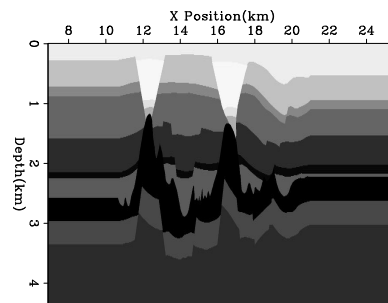


Figure 2: A 2-D synthetic velocity model made up of constant velocity layers. `bob4-amoco-vel` [ER]

the velocity in Figure 4. Figure 5 shows the selected velocities. Note how the initial velocities were clustered around 3.6km/s . The final velocities are generally more spread out, closer in velocity ranges with high count. Also note that in each region the velocity that is most common is chosen as a reference. To test the method's ability on functions with bimodal distributions

Figure 3: A histogram of the velocity function overlaid by the initial reference velocities (0) and those after five (5) and twenty iterations (*).

`bob4-amoco-hist-overlay` [ER]

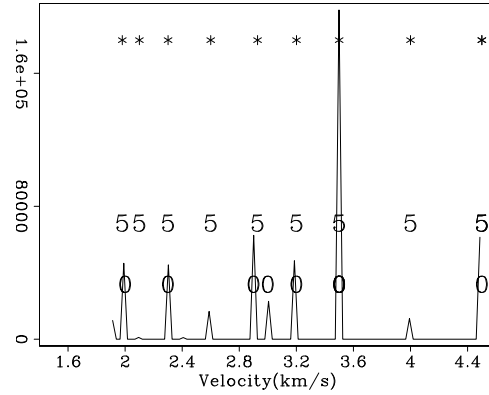


Figure 4: A smoothed version of the model in Figure 2.

`bob4-amsm-vel`

[ER]

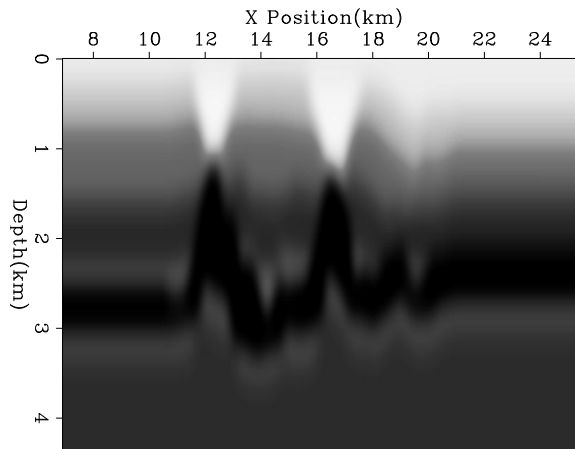
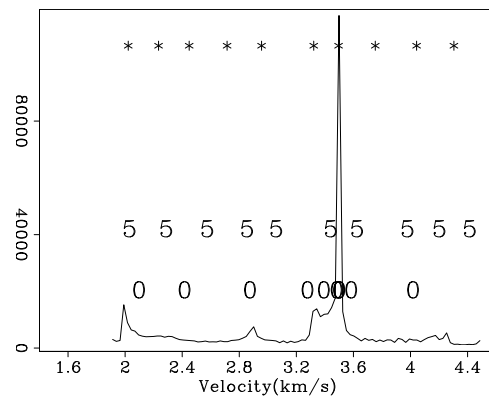


Figure 5: A histogram of the velocity function in Figure 4 overlaid by the initial reference velocities (0) and those after five (5) and twenty iterations (*).

`bob4-amsm-hist-overlay`

[ER]



(such as regions with salt), I used the upper (Figure 6) and lower (Figure 8) portion of the SMAART JV Sigsbee synthetic. Figures 7 and 9 show the selected velocities. Note how in each example the method determined that fewer reference velocities were needed. In each case a single reference velocity between the two modes of the distribution (this reference velocity was kept because some velocities still fall in this region).

Figure 6: The top portion of the Sigsbee synthetic. `bob4-zig1-vel` [ER]

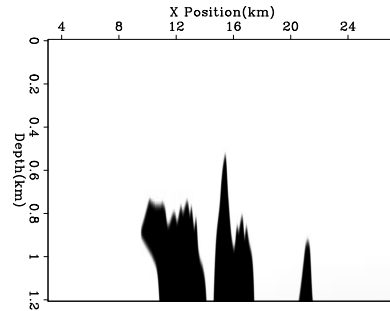


Figure 7: A histogram of the velocity function in Figure 6 overlaid by the initial reference velocities (0) and those after five (5) and twenty iterations (*). `bob4-zig1-hist-overlay` [ER]

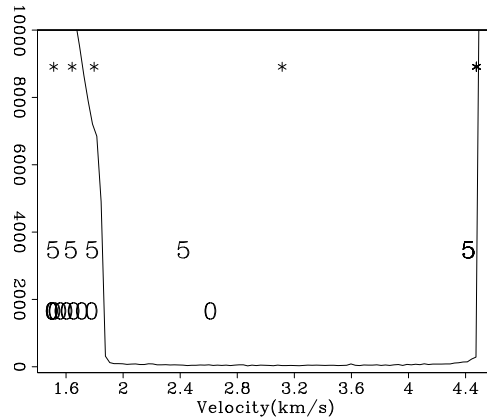
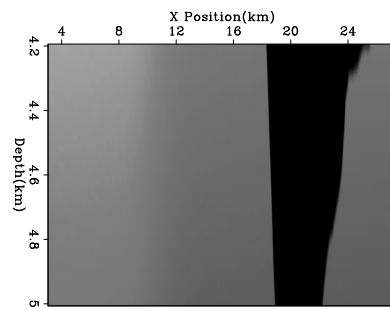


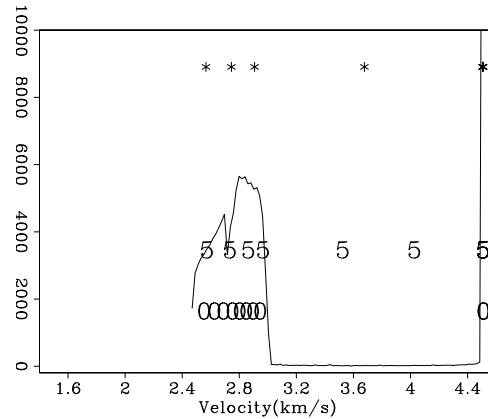
Figure 8: The bottom portion of the Sigsbee synthetic. `bob4-zig2-vel` [ER]



MIGRATION TESTS

To test the migration result I chose the same two synthetics used in the previous section. Figure 10 shows the result of migrating the North Sea synthetic using at maximum four reference velocities. The top picture shows the result of using a conventional approach with four reference velocities plus salt. The bottom picture shows the result of using Lloyd's method with

Figure 9: A histogram of the velocity function shown in Figure 8 overlaid by the initial reference velocities (0) and those after five (5) and twenty iterations (*). `bob4-zig2-hist-overlay` [ER]



a maximum of four velocities. The Lloyd result reduced the number of reference velocities needed at some depth levels, reducing the overall migration 30% faster and higher quality than the result with four reference velocities at each depth. At location 'A' the left edge of the salt structure is better placed. At locations 'B' and 'C' the bottom of the salt is better focused.

Figure 11 shows the correct (left) and selected (right) velocity for the Sigsbee synthetic. Note how the salt velocity was correctly selected and the rest of the model well represented. Figure 12 shows the result of migrating the Gulf of Mexico synthetic. In this case for the standard approach I chose five+salt reference velocities. The Lloyd's algorithm selection method resulted in a 35% cost saving in compute time. The top picture shows the conventional approach while the bottom picture shows the result of using Lloyd's method for velocity selection. The bottom of the salt reflection, 'A', is better focused. At 'B' the right canyon is more accurately positioned and better focused. At location 'C' we see how the point diffractors are collapsed using the Lloyd velocity selection method while using the more standard approach they are not collapsed.

CONCLUSIONS

Reference velocity selection shares similar characteristics with speech compression. By using a modified version of Lloyd's method to select velocities the velocity at a given depth can be more accurately described, often with fewer velocities, than conventional approaches. The resulting migration images can be computed faster, with higher image quality compared to conventional methods.

ACKNOWLEDGMENTS

I would like to thank BP and SMAART JV for the synthetic datasets used in this paper.

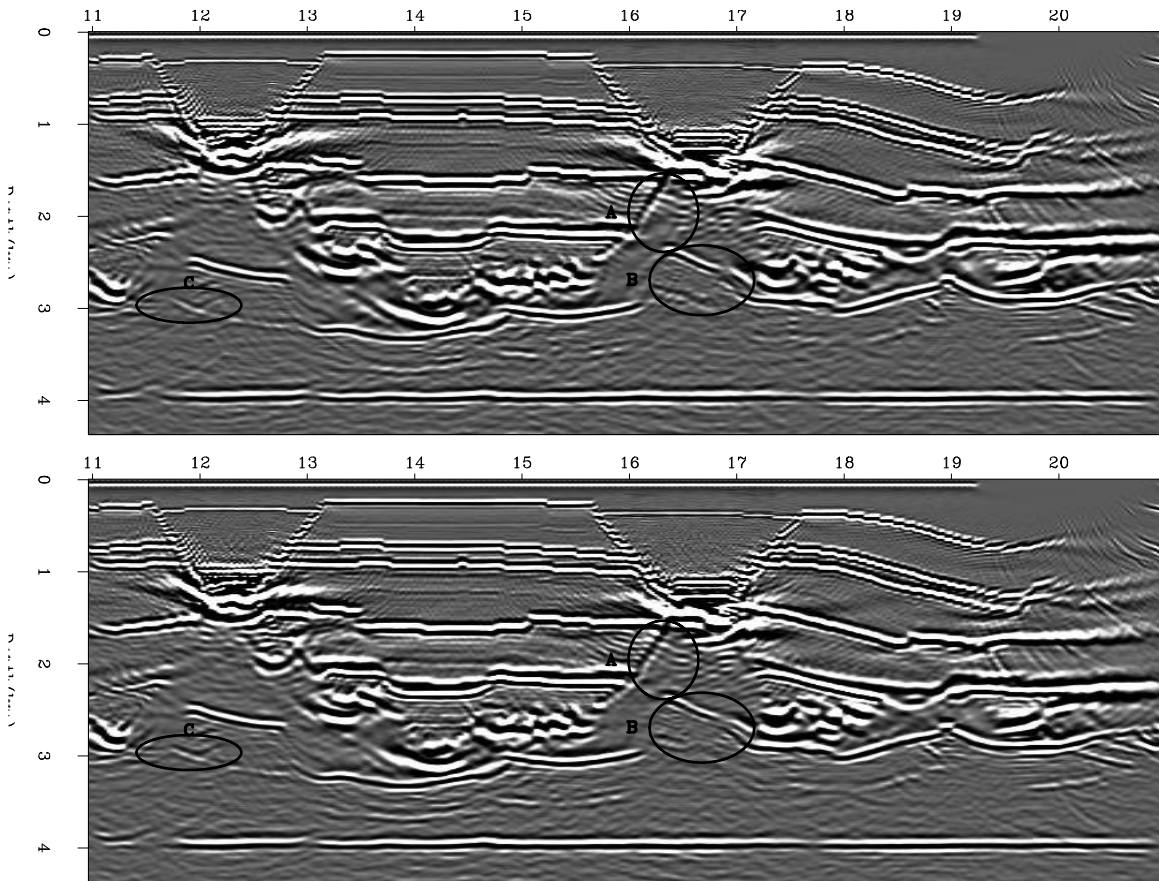


Figure 10: Comparison between regularly sampling the velocity range (top panel) and using Lloyd’s algorithm for velocity selection. At location ‘A’ the left edge of the salt structure is better placed. At locations ‘B’ and ‘C’ the bottom of the salt is better focused. bob4-amoco-mig [CR,M]

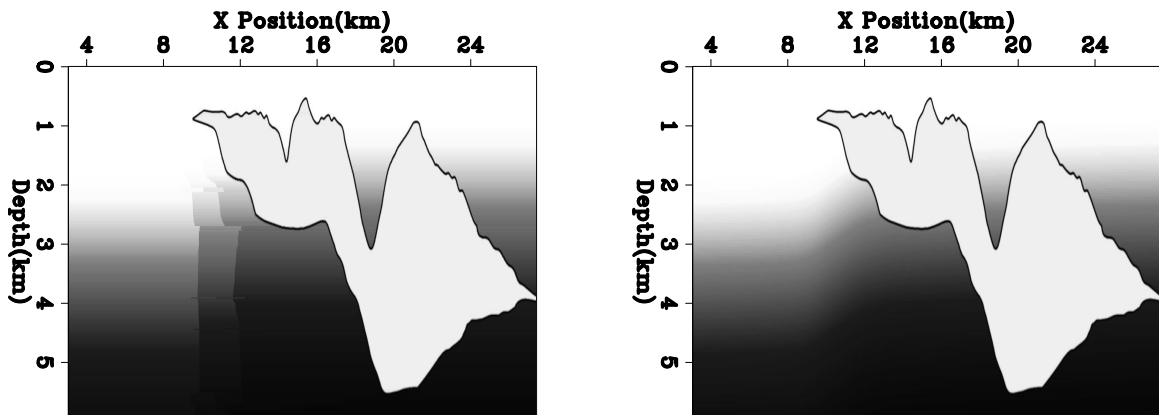


Figure 11: The correct (left) and selected (right) velocity for the sisbee synthetic. Note how the salt velocity was correctly selected and the rest of the model well represented. bob4-zig-chosen [ER,M]

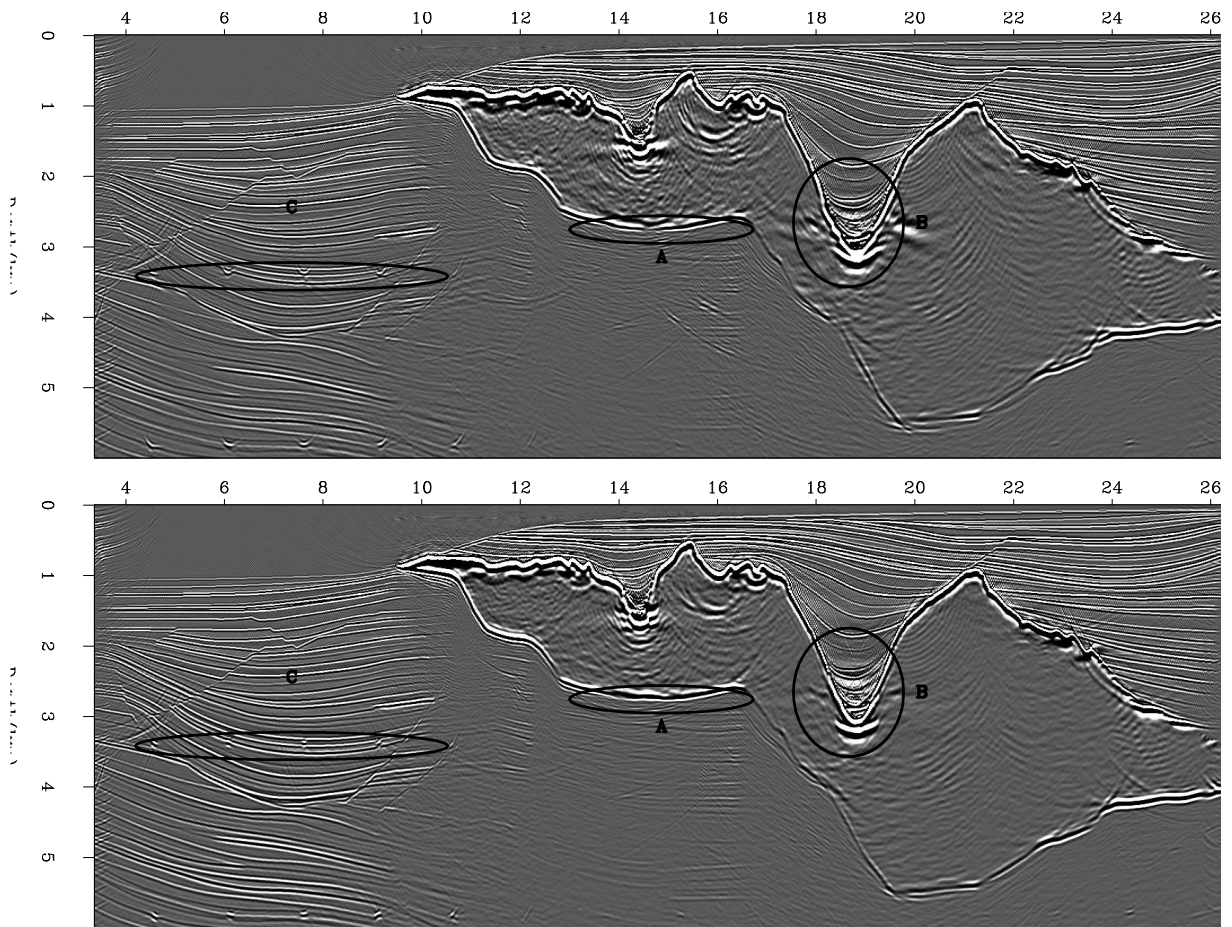


Figure 12: Comparison between regularly sampling the velocity range (top panel) and using Lloyd's algorithm for velocity selection. The bottom of the salt reflection, 'A', is better focused. At 'B' the right canyon is more accurately positioned and better focused. At location 'C' we see how the point diffractors are collapsed using the Lloyd velocity selection method while using the more standard approach they are not collapsed. `bob4-zig-mig` [CR,M]

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