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Non-stationary, multi-scale prediction-error filters and irregularly sampled data

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ABSTRACT

Non-stationary prediction-error filters have previously been used to interpolate sparse, regularly sampled data. I take an existing method used to estimate a stationary prediction-error filter on sparse, irregularly sampled data, and extend it to use non-stationary prediction-error filters. I then apply this method to interpolate a non-stationary test case, with promising results. I also examine a more complex three dimensional test case.

INTRODUCTION

Data interpolation can be cast as in inverse problem, where the known data remains constant, and the empty bins are regularized to constrain the null space. A two-stage linear approach was developed (Claerbout, 1999) where a prediction-error filter (PEF) is estimated on known data, and is then used to constrain the unknown data by minimizing the output of the model after convolution with the PEF. When the data is not stationary, a non-stationary filter has been used to fill the unknown data (Crawley, 2000). This gives better results than a patching approach, where the data is broken up into separate patches that are assumed to be stationary, largely because most data is smoothly non-stationary. In the case of sparsely (but regularly) sampled data, the non-stationary filter can be stretched over various scales to fit the data. Most recently, a PEF was estimated on irregularly sampled data by scaling the data to various grid sizes and simultaneously estimating a single filter on the various scales of data in a multi-scale approach (Curry and Brown, 2001).

Here, I take the multi-scale approach for irregular data, and extend it to estimate a non-stationary PEF. I examine how to choose the parameters needed for this non-stationary PEF estimation, namely micro-patch size, scale choice, regularization, and filter size, and how they are related when using this estimation method. I use this approach to interpolate a poorly sampled 2D test case, where existing methods would fail, with promising results. I then interpolate a suitable 3D test case with very promising results, with the eventual goal of seismic data interpolation.

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BACKGROUND

A PEF is traditionally estimated by minimizing the output of the known data **d**, convolved (**D**) with a filter **f** which is unknown except for the first coefficient, which is constrained to 1. This is expressed below, with **K** representing a mask which is 1 when all filter coefficients lie on known data, and 0 when coefficients lie on missing data.

$$\mathbf{KDf} + \mathbf{d} \approx \mathbf{0} \tag{1}$$

Fitting goal 1 works well for estimating the PEF if there are sufficiently contiguous data. However, if the data are irregularly sampled so that there are an inadequate number of fitting equations, a different method is used (Curry and Brown, 2001), where more fitting equations are generated by regridding the data (**D**) with an operator $\mathbf{S_i}$ (normalized linear interpolation followed by its adjoint), and then simultaneously estimating a single filter (**f**) on all of the versions of the scaled data ($\mathbf{S_i d}$). The mask for the known data ($\mathbf{K_i}$) must also be regridded accordingly for each scale.

$$\begin{bmatrix} \mathbf{K_0 D} \\ \mathbf{K_1 S_1 D} \\ \mathbf{K_2 S_2 D} \\ \dots \\ \mathbf{K_n S_n D} \end{bmatrix} \mathbf{f} = \begin{bmatrix} \mathbf{d} \\ \mathbf{S_1 d} \\ \mathbf{S_2 d} \\ \dots \\ \mathbf{S_n d} \end{bmatrix}. \tag{2}$$

Several user-defined parameters must be set during this procedure, specifically the choice of scales to be used in the estimation as well as the size of the PEF. The only constraint on these parameters is that the aspect ratio of the data remain constant from scale to scale, meaning that the ratio of the number of bins in each dimension remain constant. For example, a $50 * 40$ data set should not be regridded to $26 * 21$, since the aspect ratio is changed by the round-off from 20.8 to 21. A better choice would be to use $25 * 20$ as a scale. The PEF size is only constrained by the size of the coarsest scale of data.

A non-stationary filter varies with position, so instead of only having indices corresponding to the lag of the filter, there are also indices corresponding to the position of the filter. The filter would go from looking like `a(ia)` to `a(ia,id)`, where `ia` is the lag of the filter, and `id` is the position of the filter. An illustration of this concept is shown in figure 1. Only two indices are used for lag and position, thanks to the helical coordinate system (Claerbout, 1998).

Since we have moved from estimating a single filter with `na` filter coefficients to estimating a non-stationary filter with `na*nd` filter coefficients, PEF estimation becomes an underdetermined problem instead of an over-determined problem. As a result, we need to incorporate some type of regularization into the estimation in order to get enough equations. Laplacian or radial rougheners of common filter coefficients (constant `ia`) across the spatial axes (`id`) are both used to ensure a filter bank that varies smoothly spatially (Clapp et al., 1999).

Another method used to constrain the filter coefficients is called micro-patching (Crawley, 2000). Instead of the filter varying at every data point, micro-patching uses the same filter within a small region within the data, reducing the number of filter coefficients that need to be

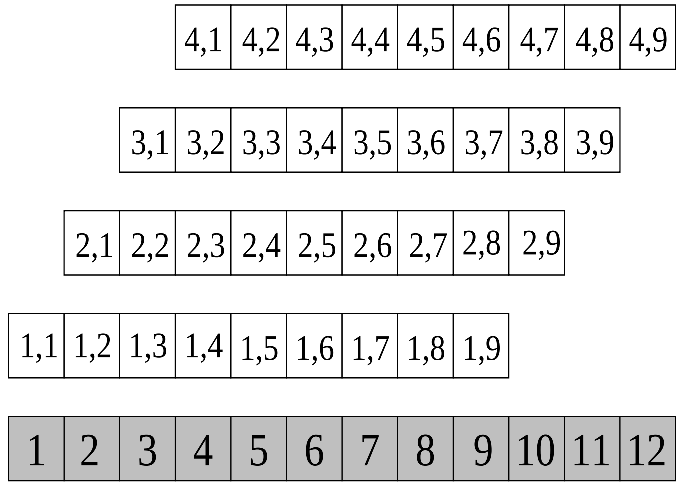

Figure 1: An illustration of non-stationary convolution. The shaded boxes represent the data, and the hollow boxes represent the filter at various positions. The two indices on each filter point correspond to the data position (`id`) and the filter lag (`ia`), respectively. At each point in the convolution, the filter is different. bill1-nstat [NR]

estimated. This has two benefits: the PEF estimation problem becomes less under-determined, and the amount of memory required for the filter, which was `na` times the size of the data `nd`, is now the number of micro-patches, `np` times `na`.

MULTI-SCALE NON-STATIONARY PEFS

The combination of non-stationary filters and estimation on multiple scales of data introduces a new issue, that non-stationary filters are linked to the size of the data they operate on. If the dimensions of the data change, the dimensions of the PEF must change as well. This causes issues regarding the consistency of the PEF across scales, as well as limits on the type of regularization that can be applied to the filter coefficients.

In order to maintain a consistent PEF across scales, the filter must be sub-sampled so that the same spatial coordinates of the PEF correspond to the proper locations within the scaled data. Since our non-stationary PEF has micro-patches where the filter coefficients are constant, we can scale the patches to match the scaling of the data, so that the number of filter coefficients in the non-stationary filter remains constant, but the size of the micro-patches has decreased. The limiting case for this scaling is when a micro-patch reduces to a single point. Beyond this point, the patches could be sub-sampled during the scaling. This avenue has not been explored, since a need for that level of scaling has not yet been shown.

I represent the sub-sampling of the patch table by $\mathbf{P_i}$, which acts upon the non-stationary filter $\mathbf{f}$ in the non-stationary fitting goal shown below:

$$\begin{bmatrix} \mathbf{K_0 D} \\ \mathbf{K_1 S_1 D P_1} \\ \mathbf{K_2 S_2 D P_2} \\ \cdots \\ \mathbf{K_n S_n D P_n} \end{bmatrix} \mathbf{f} \approx \begin{bmatrix} \mathbf{d} \\ \mathbf{S_1 d} \\ \mathbf{S_2 d} \\ \cdots \\ \mathbf{S_n d} \end{bmatrix}. \tag{3}$$

In addition to the above fitting goal, a set of regularization equations must also be solved,

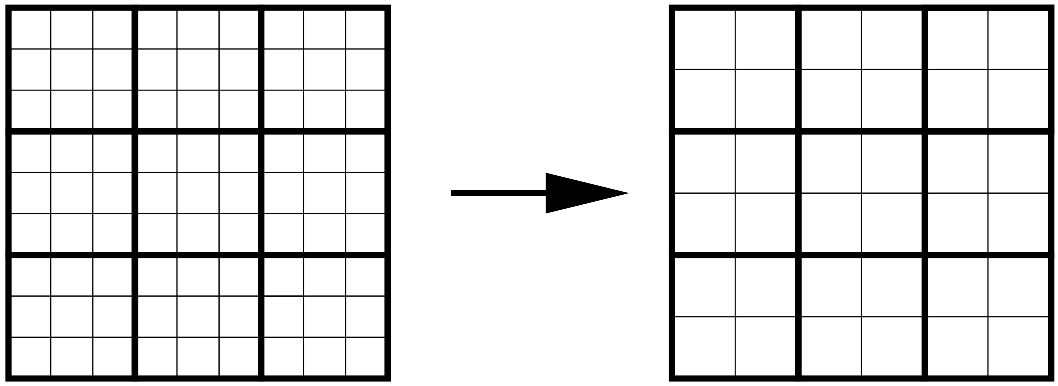

Figure 2: Regridding both the data (fine grid) and the micro-patches (thick grid) simultaneously. In this case a $9*9$ grid with $3*3$ micro-patches is regridded to a $6*6$ grid with $2*2$ micro-patches. The sizes of the micro-patches remain constant. bill1-patchscale [NR]

where **A** is our regularization operator:

$$\mathbf{A}\mathbf{f} \approx \mathbf{0} \tag{4}$$

The scaling of both the filter and the data to some extent limits the choices of regularization available. Specifically, radial micro-patches do not scale well, so applying radial regularization would have to be done over square micro-patches. Laplacian regularization of filter coefficients across rectangular micro-patches is also a reasonable approach in some cases.

TEST CASE

A non-stationary, two-dimensional test case has been created as a proof of concept example. This test is based upon a simple plane wave model (Brown et al., 2000; Curry and Brown, 2001).

In this case, there is one plane wave on each half of the example, and the plane wave on the right varies in amplitude from left to right. There is also Gaussian noise present in the data, which largely obscures the low amplitude portion of the plane wave on the right side of the example.

The data was randomly sub-sampled, keeping only $10-20$ percent of the data. This sparse data, along with a mask describing the position of the known data, was used in the non-stationary interpolation scheme. The results of the interpolation are shown in Figure 4.

The results are on the whole quite encouraging. The data was very heavily sub-sampled, and the interpolation scheme was able to identify the dips in the data and interpolate them properly. The dip from the left side of the example was smoothed over the area on the right side of the figure with the low signal-to-noise ratio, which was expected. As the number of micro-patches drops, the results deteriorate, as the regularization loses its effect and one patch

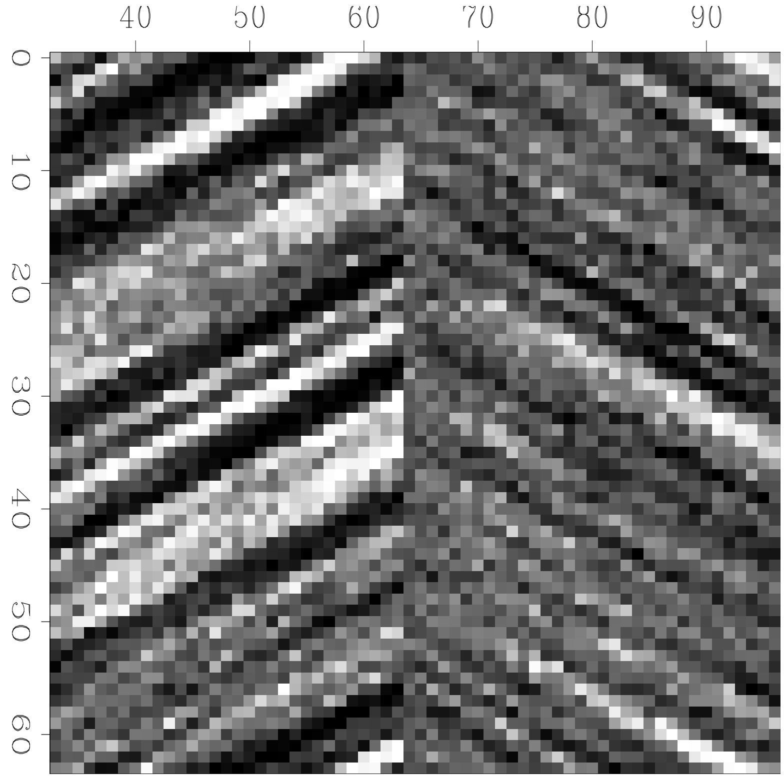

Figure 3: The fully sampled version of the non-stationary test data. bill1-testcase [ER]

becomes unstable. A higher number of micro-patches preserves the dips as well as the low amplitude area.

A more relevant case to interpolating seismic data is the qdome model (Claerbout, 1999), which has been highly sub-sampled along two of three dimensions, with the vertical axis still fully sampled. The qdome model is a collection of folding layers, flat layers and a fault, which acts as an excellent overall test for this interpolation method. I have randomly removed 88 percent of the traces from the data set, and use the non-stationary multi-scale PEF-based interpolation to attempt to recreate the original model.

The results for the qdome model are very promising. The smoothly varying dips were correctly estimated and interpolated almost everywhere, excluding very steep dips. This is due to two things, the size of the PEF might not have been large enough to capture the spatially aliased dips, and that the dips were changing rapidly within a small area, which was only covered by a small number of micro-patches.

The results for this 3D case are much more impressive than in the 2D case, even though more of the data was removed. This is due to several things: The extra dimension of data allows for more constraints to be applied by the regularization, the greater size of all of the dimensions allows for more fitting equations to be found, and most importantly that the well-sampled z-axis gives better coverage of the data space.

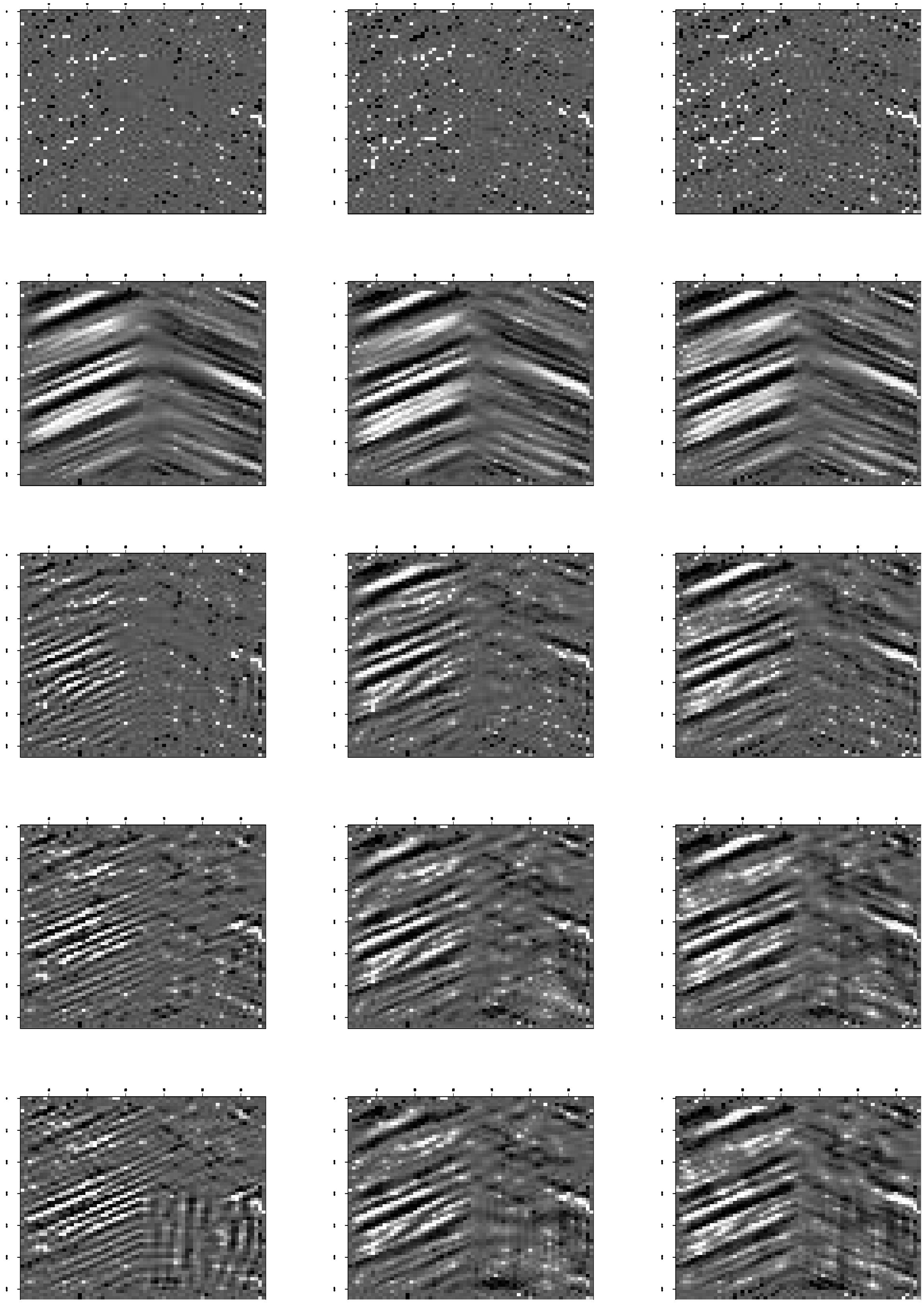

Figure 4: Results for three different sampling levels, from left to right: 10, 15 and 20 percent. From top to bottom: sparse data, sparse data filled with PEF trained on fully sampled original data, sparse data filled with PEF from sparse data with 64, 16 and 8 micro-patches, respectively. bill1-testfill [ER]

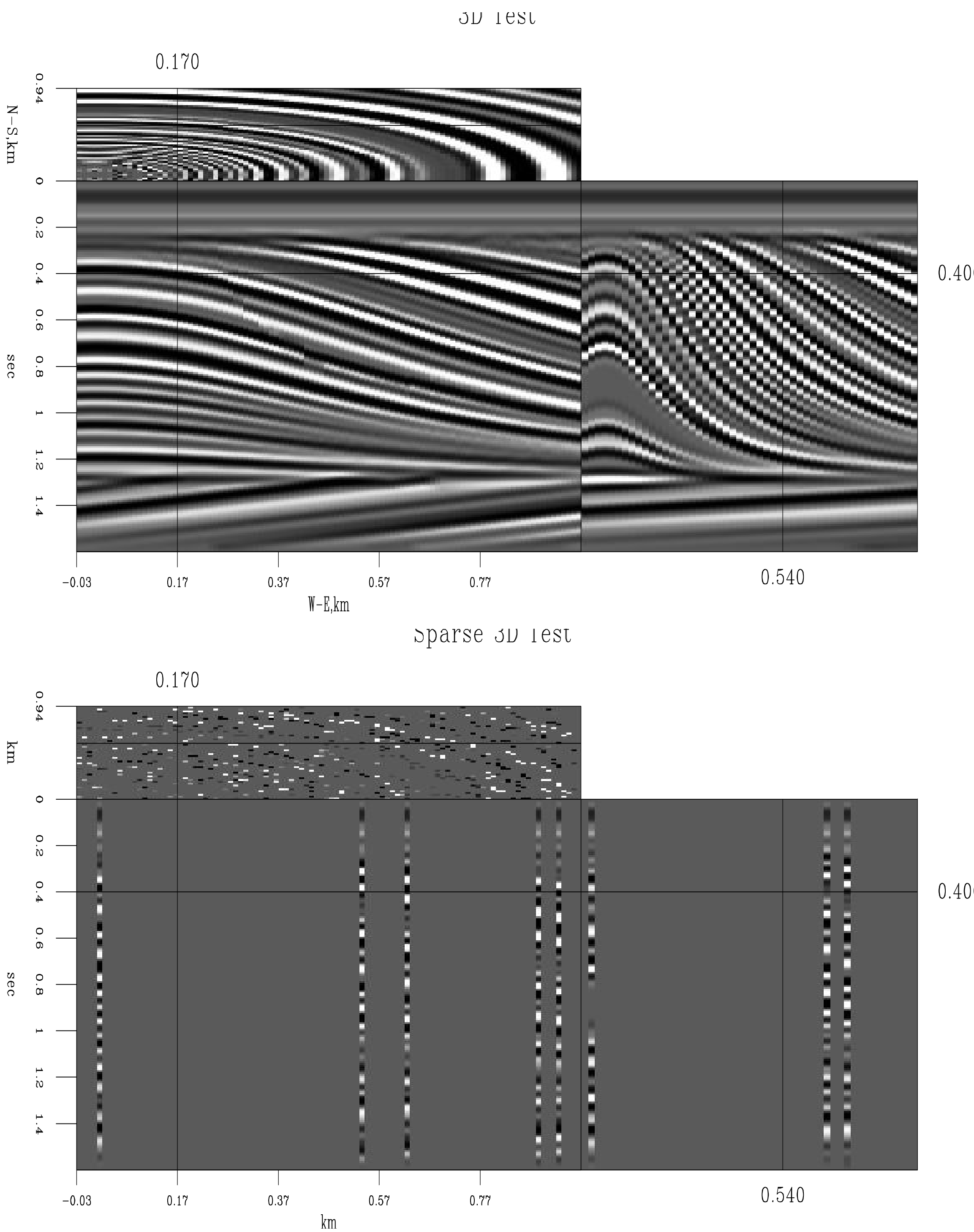

Figure 5: Fully sampled and sub-sampled versions of the qdome model. bill1-qdometest [ER]

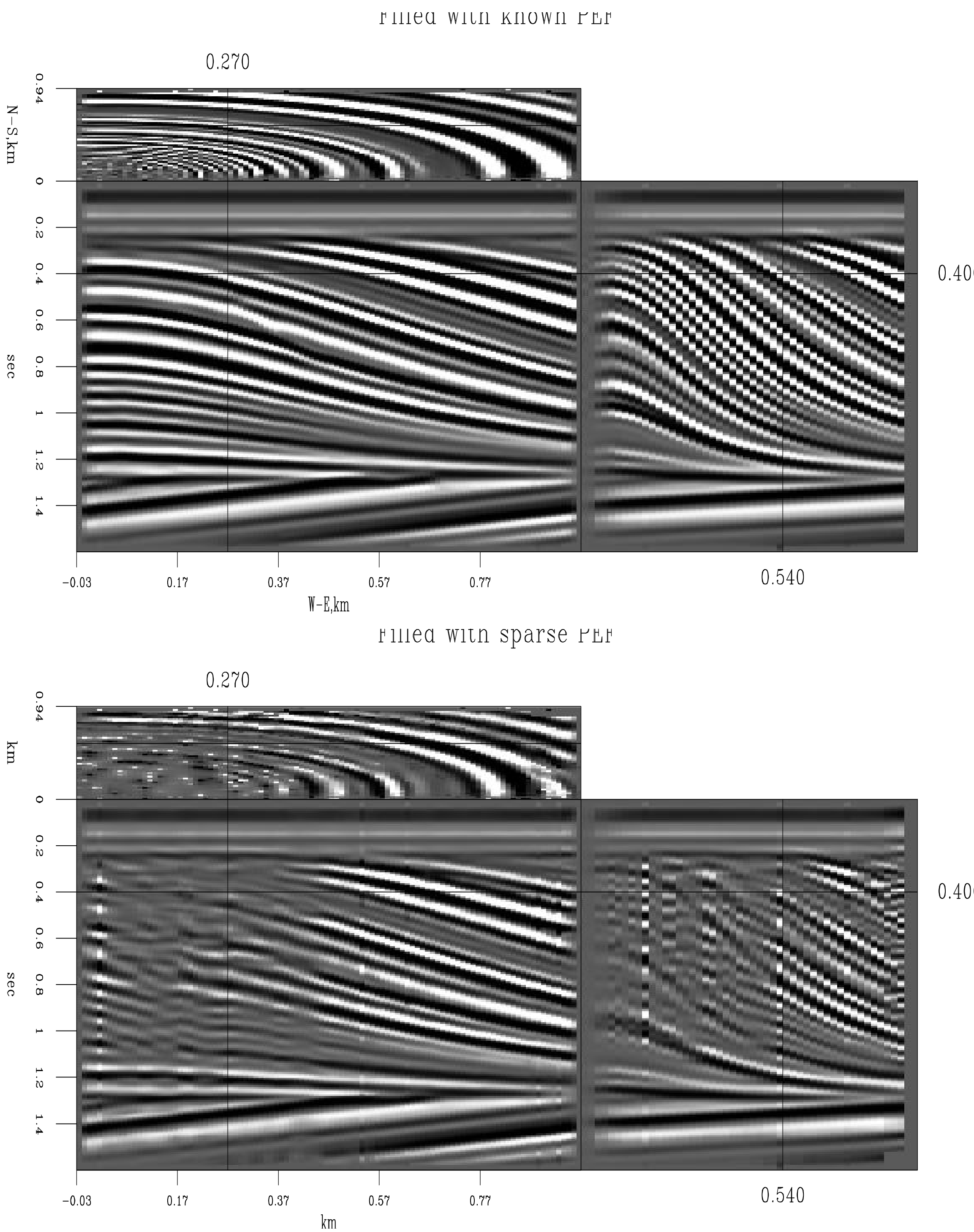

Figure 6: Above: Sparse data filled with a non-stationary PEF trained on all data. Below: Sparse data filled with a non-stationary PEF trained on sparse data. bill1-qdomefill [CR]

CONCLUSIONS AND FUTURE WORK

Overall, estimating a non-stationary prediction-error filter with multiple scales of data appears to be successful. The method interpolates a very heavily decimated 2D test case successfully. Results for a 3D case are even more successful, even though the amount of data removed from the case was greater than in the 2D case.

In the future, this method can be used on real seismic data in two, three, or even five dimensions, so that prestack 3D data can be interpolated in cases where surface topography, structures, or other obstacles cause irregularly sampled data.

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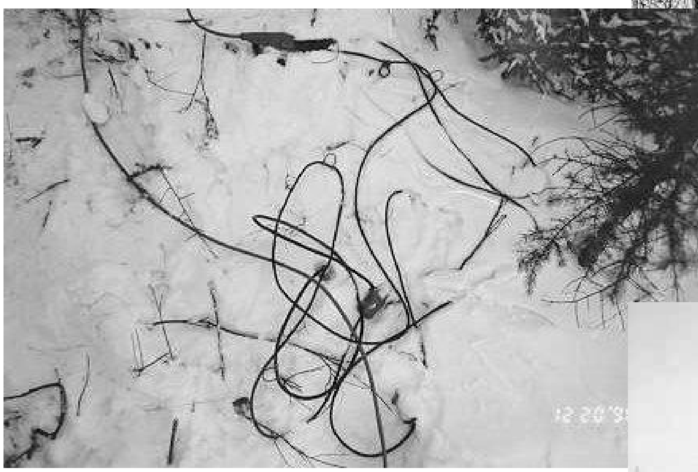


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