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A hybrid adaptive subtraction method

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ABSTRACT

A hybrid adaptive subtraction scheme is proposed. This hybrid scheme uses prediction-error filters as covariance operators within the filter-estimation step. This methods proves to be the most efficient when the noise and signal interfere. Although prediction-error filters are utilized, this technique is not a pattern-recognition technique: it simply tries to remove the correlated signal information to unbias the estimation of the matched-filters. Tests on synthetic and real data for a multiple attenuation problem illustrate the efficiency of the proposed scheme.

INTRODUCTION

Often the subtraction of noise is done via an adaptive subtraction of a noise model from the data. The resulting signal components become, by construction, orthogonal to the noise components (Spitz, 1999). Consequently, adaptive subtraction is not appropriate when noise and signal are correlated.

A very common noise attenuation problem is the subtraction of multiples. It is well known that the subtraction of multiples becomes very challenging when they interfere strongly with primaries (Spitz, 1999). Some solutions have been proposed by various authors to cope with correlated noise and signal. For instance, the so-called pattern-based methods have proved to be particularly efficient at attenuating multiples in the most complex areas (Spitz, 1998; Guitton et al., 2001).

In this paper, I investigate an improved adaptive subtraction scheme that can separate interfering multiples and primaries. With this method, I do not assume that the signal has minimum energy. Building on Guitton (2002), I estimate a signal covariance operator with time domain (t, x) prediction-error filters (pef). This covariance operator is then utilized within an inversion scheme to remove the signal spectrum in the data leading to an unbiased estimation of the matched-filters.

In the first section, I review theoretical issues about adaptive subtraction and I present the new hybrid scheme. In the second section, I illustrate the advantages of the proposed method with synthetic and field data examples.

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IMPROVING ADAPTIVE SUBTRACTION

In this section I review basic notions on adaptive subtraction. Then I present an improved scheme that better separates interfering noise and signal. All my derivations are based upon the assumption that we want to subtract a noise model from the data. In the multiple attenuation problem the noise becomes the multiples and the signal becomes the primaries.

Theory of adaptive subtraction

The goal of adaptive subtraction is as follows: given a time series $\mathbf{b}$ and a desired time series $\mathbf{d}$, we seek a filter $\mathbf{f}$ that minimizes the difference between $\mathbf{f} * \mathbf{b}$ and $\mathbf{d}$ where $*$ is convolution. We can rewrite this definition in the fitting goal

$$\mathbf{0} \approx \mathbf{Bf} - \mathbf{d} \tag{1}$$

where $\mathbf{B}$ represents the convolution with the time series $\mathbf{b}$. We can minimize this fitting goal in a least-squares sense leading to the objective function

$$g(\mathbf{f}) = (\mathbf{Bf} - \mathbf{d})'(\mathbf{Bf} - \mathbf{d}) \tag{2}$$

where ($'$) is the transpose. The minimum energy solution is given by

$$\hat{\mathbf{f}} = (\mathbf{B}'\mathbf{B})^{-\mathbf{1}}\mathbf{B}'\mathbf{d}. \tag{3}$$

where $\hat{\mathbf{f}}$ is the least-squares estimate of $\mathbf{f}$. This approach is very popular but has some intrinsic limitations. In particular $\mathbf{B}\hat{\mathbf{f}}$ is by construction orthogonal to the residual $\mathbf{B}\hat{\mathbf{f}} - \mathbf{d}$. In the multiple attenuation problem $\mathbf{d}$ is the data, $\mathbf{b}$ the multiple model and $\mathbf{B}\hat{\mathbf{f}} - \mathbf{d}$ the estimated primaries. If both signal and noise are correlated, the separation will suffer because of the orthogonality principle.

From now on I will refer to this method as the “standard approach”.

In the next section I propose improving the adaptive subtraction scheme. This improvement leads to an unbiased matched-filter estimation when both signal and noise are correlated.

A hybrid attenuation scheme

In the multiple attenuation case, most of the problems encountered with the adaptive subtraction technique stem from the correlation that might exist between the noise and signal. In addition, the minimum energy assumption forces the residual (the signal) to be white, which is not valid all the time. Fortunately we can derive a fitting goal that can cope with interfering noise and signal and non-white spectrum.

In Guitton (2002), I presented a method that approximates covariance operators with pef. The goal was basically to obtain independent and identically distributed (iid) residual components. I propose using the same approach for the filter estimation in adaptive filtering.

Following this idea, I have the new fitting goal

$$\mathbf{0} \approx \mathbf{A_s}(\mathbf{B}\mathbf{f_h} - \mathbf{d}) \tag{4}$$

where $\mathbf{A_s}$ is a pef that whitens the signal spectrum only. The corresponding least-squares estimate for $\mathbf{f_h}$ becomes

$$\hat{\mathbf{f}}_\mathbf{h} = (\mathbf{B}'\mathbf{A_s}'\mathbf{A_s}\mathbf{B})^{-1}\mathbf{B}'\mathbf{A_s}'\mathbf{A_s}\mathbf{d}. \tag{5}$$

If the signal has a white spectrum, then this new estimate of the filter $\hat{\mathbf{f}}_\mathbf{h}$ is identical to the estimate in equation (3). If the signal is not white, then this new estimate is going to be more accurate than the estimate in equation (3). More specifically, the noise and signal do not have to be orthogonal any more.

I call this scheme hybrid because it puts back together two worlds: the world of adaptive subtraction and the world of pef. Nonetheless, this method is not a pattern-based technique because the multiples and primaries are not separated according to their spatial predictability. I am only proposing to unbias the filter estimation.

Once the filter has been estimated [equation (5)] I compute the noise and signal as follows:

$$\begin{aligned} \mathbf{n} &= \mathbf{B}\hat{\mathbf{f}}_\mathbf{h} \\ \mathbf{s} &= \mathbf{B}\hat{\mathbf{f}}_\mathbf{h} - \mathbf{d}. \end{aligned} \tag{6}$$

One unsolved problem is the pef estimation. I give few guidelines in the next section.

How to estimate the pef $\mathbf{A_s}$ for the signal?

It is always difficult to estimate the pef for the signal $\mathbf{A_s}$ because the signal is what we are looking for! It is a chicken and egg story. Fortunately, I have three simple recipes that seem to work well in practice.

The first one has been used by Brown and Clapp (2000) and Guitton (2001) and consists of deconvolving the data pef by the noise pef. It works pretty well but the deconvolution might be unstable when dealing with non-stationary filters (Rickett, 1999). If $\mathbf{A_d}$ and $\mathbf{A_n}$ are the data and noise pef respectively, the estimated signal pef becomes

$$\mathbf{A_s} = \mathbf{A_d}/\mathbf{A_n}. \tag{7}$$

The second one is a simple technique that requires the noise pef only. I first estimate the pef from the noise and apply it to the data. My hope is to obtain a good model for the signal. Then I estimate the signal pef from this model (Spitz, 2001, personal communication).

The last method consists of estimating the signal with the standard adaptive subtraction scheme. A pef is then estimated from the signal and used inside the hybrid scheme. You might be tempted to repeat this process iteratively in order to improve your signal pef. In my experience, I find that an iterative process might diverge, especially when non-stationary filters are involved.

In the next two sections, I show synthetic and field data examples. They prove that the hybrid adaptive filtering gives a better estimate of the noise when primaries and multiples interfere.

SYNTHETIC DATA EXAMPLE

The synthetic example is inspired by Spitz (1998) and shows clearly that the hybrid method is more accurate than the standard approach.

In Figure 1a, the signal, I show a linear event with a gradient of 1.05 from trace to trace. Figure 1b, the noise, displays another linear event with constant amplitude that perfectly overlaps with Figure 1a. The sum of Figures 1a and 1b gives Figure 1c, the data.

To make matters worse, I applied a phase-shift to the noise in order to compute the noise model (Figure 1d). My goal is to adaptively subtract this noise model from the data in Figure 1c in order to retrieve the signal in Figure 1a.

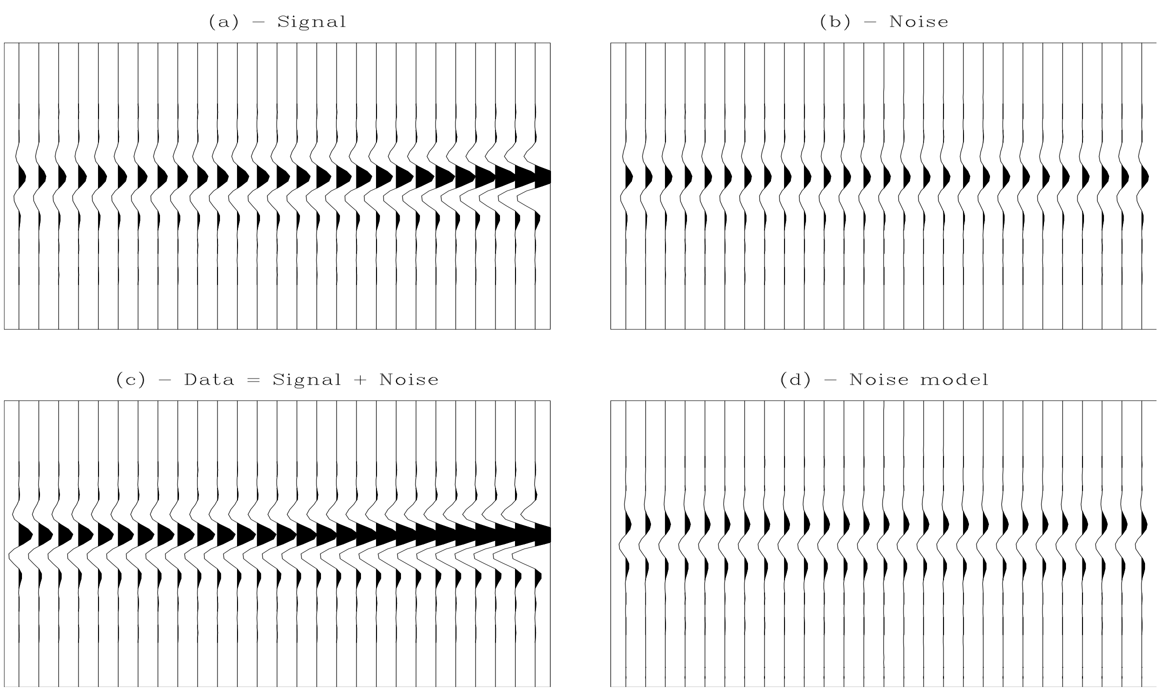


Figure 1: (a) A linear event, the signal, with a gradient of 1.05. (b) A linear event, the noise, with constant amplitude. (c) The sum of (a)+(b), the data. (d) A noise model obtained after applying a phase-shift to (b). antoine2-synth [ER]

Now I estimate the filter with the standard approach and compute the signal in Figure 2b. The estimated signal clearly does not resemble the true signal in Figure 1a.

The hybrid method gives a perfect result. First I estimate a pef with two coefficients for the signal by deconvolving the data pef by the noise pef. I obtain for the signal pef $\mathbf{A_s}' =$

$(\mathbf{1}, -\mathbf{1.05})$. Then I estimate the matched-filter with the hybrid approach and compute the signal in Figure 2c. The separation is perfect as shown in the difference panel in Figure 2d.

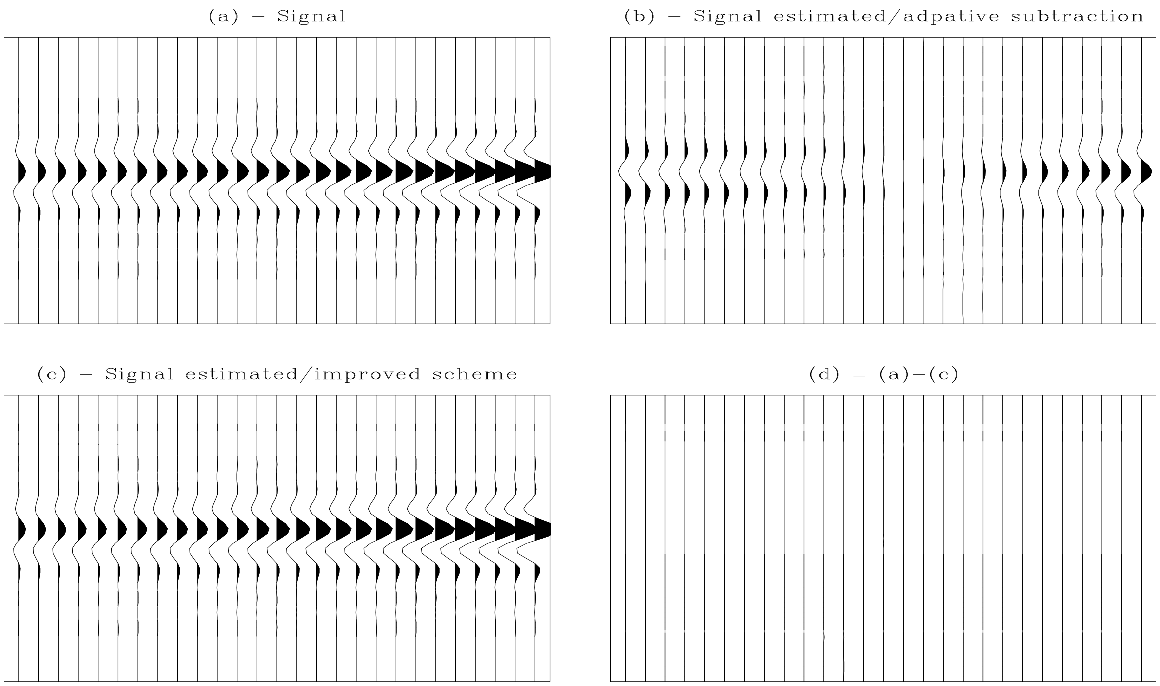


Figure 2: (a) The signal. (b) Estimated signal with the standard approach. (c) Estimated signal with the hybrid approach. (d) Difference between (a) and (c). The noise removal is perfect. antoine2-synth-res [ER]

In the next section I show prestack land data examples.

PRESTACK LAND DATA EXAMPLES

The hybrid adaptive subtraction technique has been tested on few shot records from a land data survey. My goal is to subtract multiple models from shot records. The preprocessing is described in Kelamis et al. (1999) and the goal is to attenuate surface-related multiples only. Note that for these shot records primaries and multiples are strongly correlated.

To better accomodate for the spatial variability of the data, I do not estimate one filter only but many filters. Instead of estimating filters in patches, I estimate a bank of non-stationary time domain (t, x) filters for both $\mathbf{f}$ and $\mathbf{A_s}$ (Crawley, 2000; Rickett et al., 2001).

Figures 3a and 4a show the land data and the multiple model respectively. Figures 3b and 4b display the estimated signal and noise respectively with the standard approach whereas Figures 3c and 4c show the estimated signal and noise with the hybrid approach. The hybrid subtraction improves the noise subtraction at far offsets. It also preserves the signal better as illustrated in Figure 3c around 1.4 seconds.

A different shot record is processed in Figure 5. The corresponding noise model and estimated multiples are displayed in Figure 6. The same conclusions hold true.

A more interesting shot record with its multiple model are shown in Figures 7a. In this example, the multiple attenuation is greatly improved with the hybrid approach.

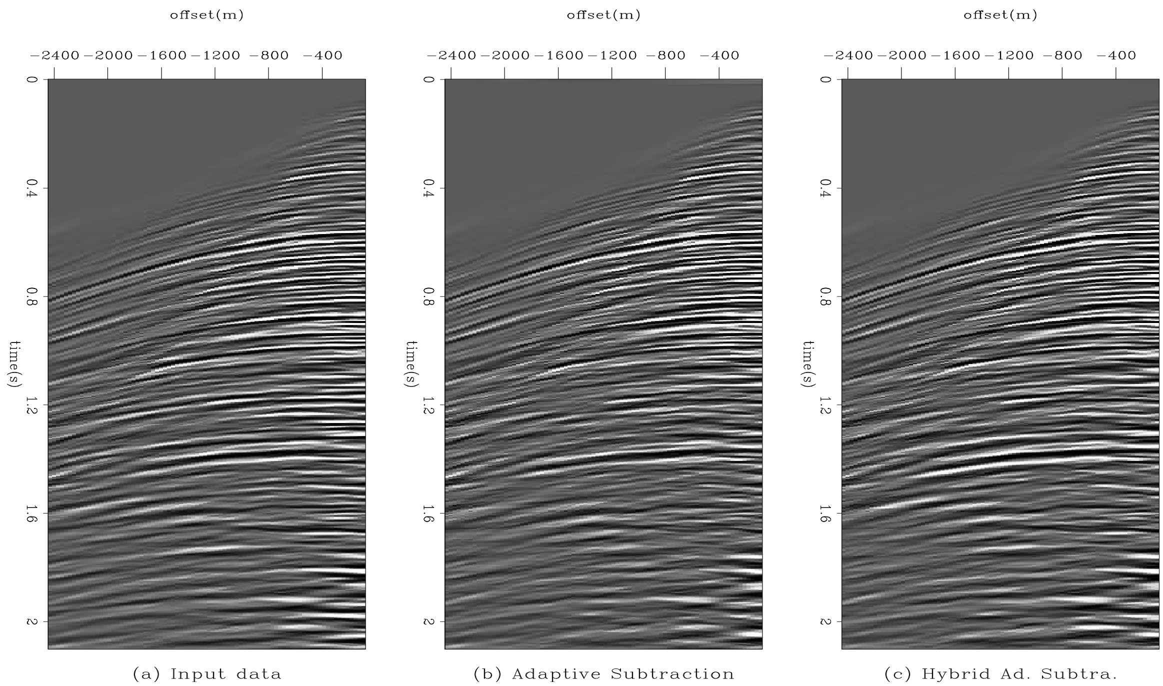


Figure 3: (a) Input data. (b) Estimated signal with the standard approach. (c) Estimated signal with the hybrid adaptive subtraction technique. The strong primary at 1.4 seconds is better preserved with the hybrid adaptive subtraction. antoine2-comp_0 [ER,M]

The land data examples illustrate the efficiency of the hybrid approach when noise and signal are correlated. In the next section I discuss some limitations of the method and illustrate them with a marine data example.

LIMITATIONS OF THE METHOD

It is important to keep in mind that the hybrid adaptive subtraction is not a pattern-based method. Noise and signal are not separated because of their spatial predictability. The pef $\mathbf{A_s}$ intends to unbias the filter $\mathbf{f}$ estimation.

In the following example, I show that when the signal/noise ratio is low, the hybrid subtraction is equivalent to the standard approach. In short if the signal level is well below the noise level, then the hybrid adaptive subtraction is equivalent to the standard approach.

To illustrate this last point I extracted one shot gather (Figure 9a) from a Gulf of Mexico 2-D line and estimated the corresponding multiple model (Figure 9b) with the Delft approach

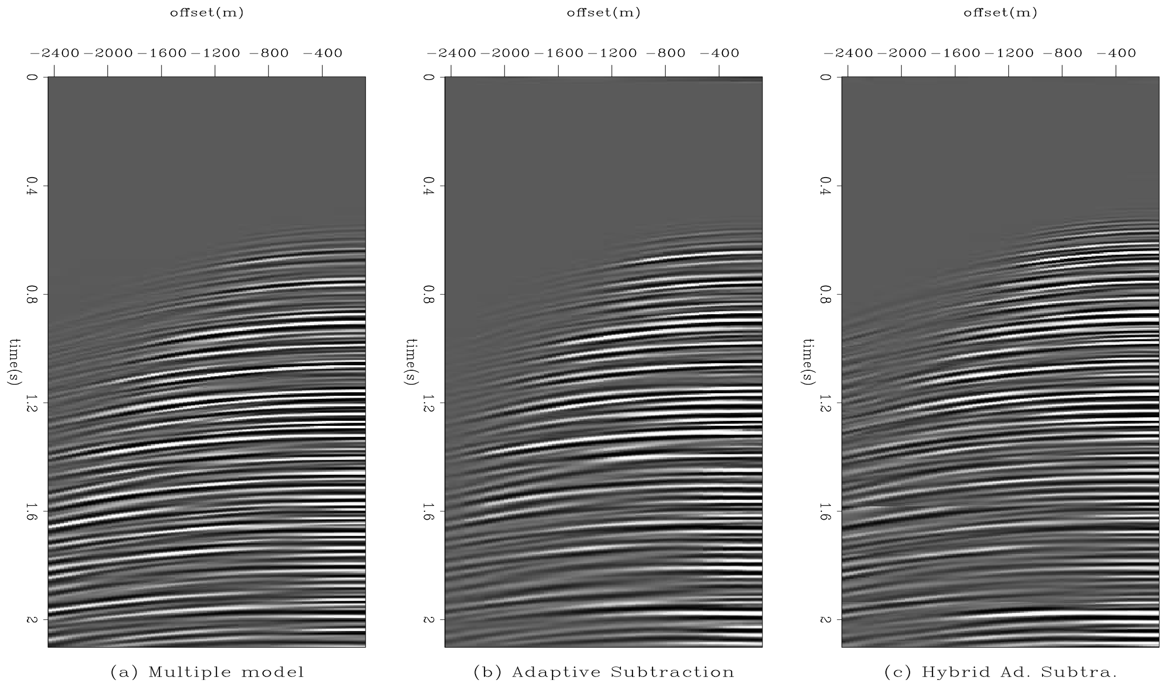


Figure 4: (a) Multiple model for the data in Figure 3. (b) Estimated noise with the standard approach. (c) Estimated noise with the hybrid adaptive subtraction technique. Far offset events are better subtracted. antoine2-comp2_0 [ER,M]

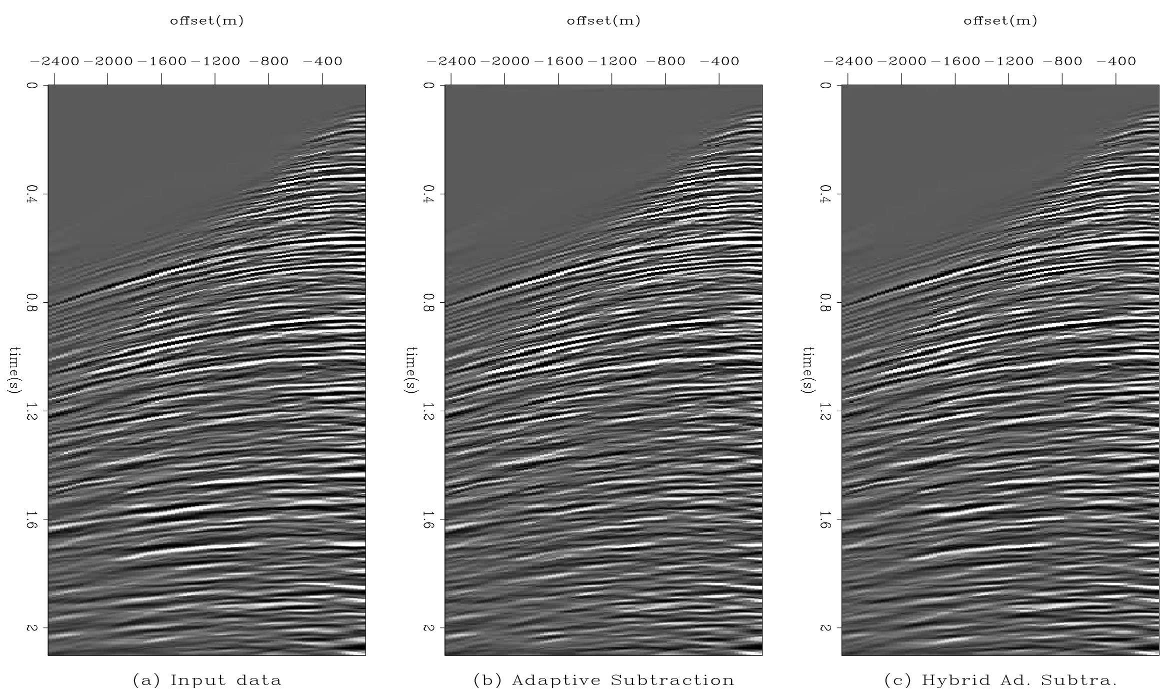
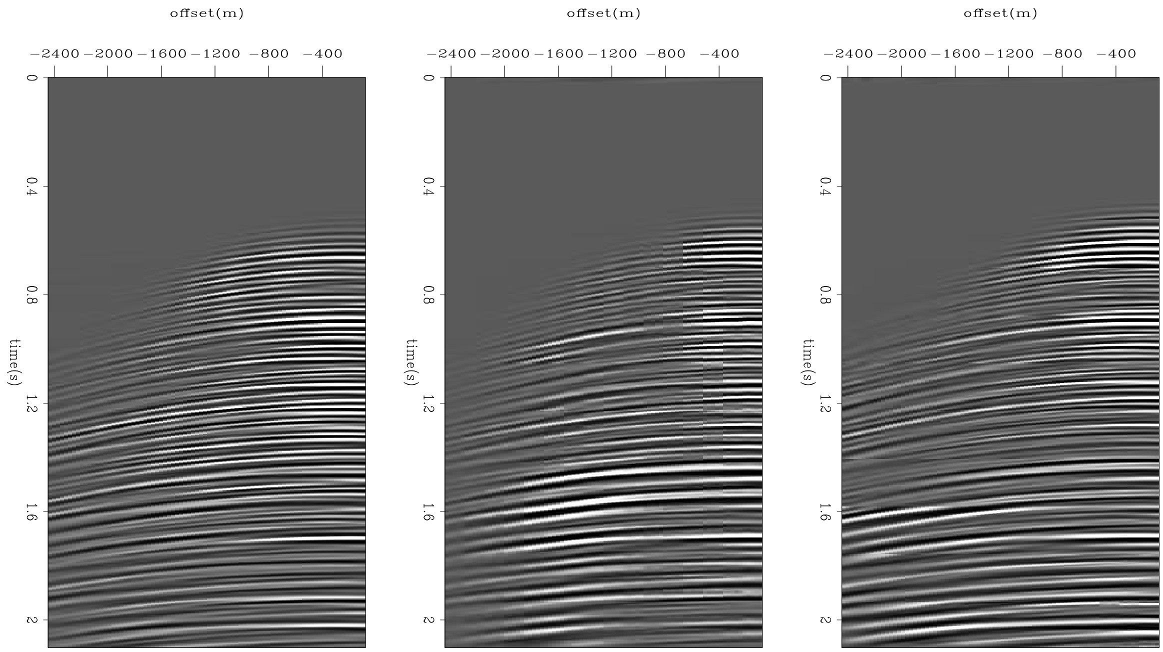
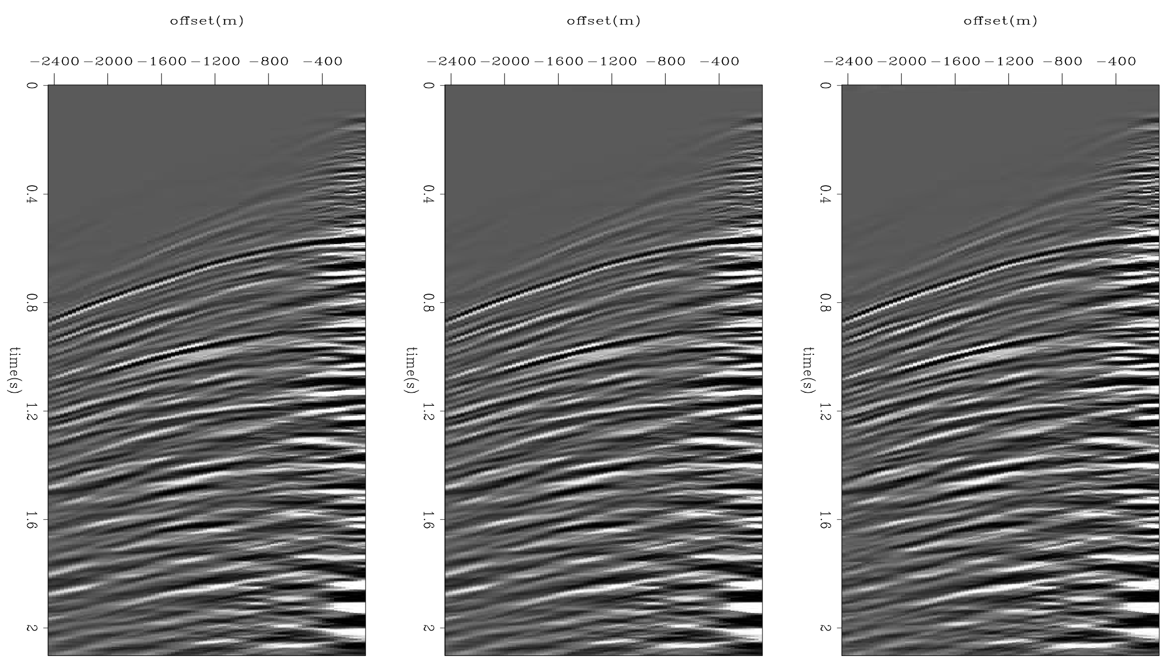


Figure 5: Another shot record. (a) Input data. (b) Estimated signal with the standard approach. (c) Estimated signal with the hybrid adaptive subtraction technique. antoine2-comp_1 [ER,M]



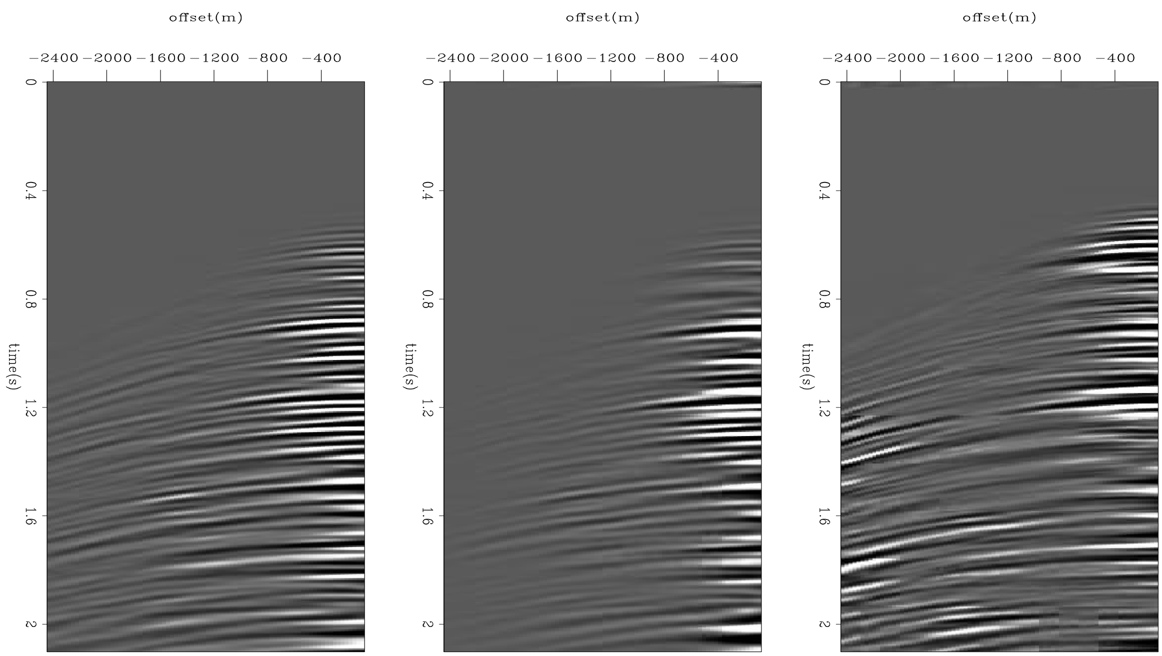
(a) Multiple model (b) Adaptive Subtraction (c) Hybrid Ad. Subtra.

Figure 6: (a) Multiple model for the data in Figure 5a. (b) Estimated noise with the standard approach. (c) Estimated noise with the hybrid adaptive subtraction technique. antoine2-comp2_1 [ER,M]



(a) Input data (b) Adaptive Subtraction (c) Hybrid Ad. Subtra.

Figure 7: Another shot record. (a) Input data. (b) Estimated noise with the standard approach. (c) Estimated noise with the hybrid adaptive subtraction technique. antoine2-comp_4 [ER,M]



(a) Multiple model (b) Adaptive Subtraction (c) Hybrid Ad. Subtra.

Figure 8: (a) Multiple model for the data in Figure 7a. (b) Estimated noise with the standard approach. (c) Estimated noise with the hybrid adaptive subtraction technique. The hybrid adaptive subtraction approach attenuates more multiples. Far offset events are better subtracted too. antoine2-comp2_4 [ER,M]

(Verschuur et al., 1992). The first surface-related multiples appears at 3.5 seconds within an area where the signal is very weak. Consequently, Figures 9c and 9d which have been processed with the standard and hybrid approach respectively, are identical.

We can recognize in Figure 9 some well-known weaknesses of the adaptive subtraction approach at far offset where the multiples are not well attenuated. Guitton et al. (2001) obtain better results with a pattern-based approach.

CONCLUSION

I presented a method that improves the signal/noise separation. This method incorporates a prediction-error filter in the standard formulation of the noise removal by adaptive subtraction. By whitening the signal spectrum in the data, this formulation leads to an unbiased estimation of the matched-filter. As a result the noise and signal can be separated when both are correlated.

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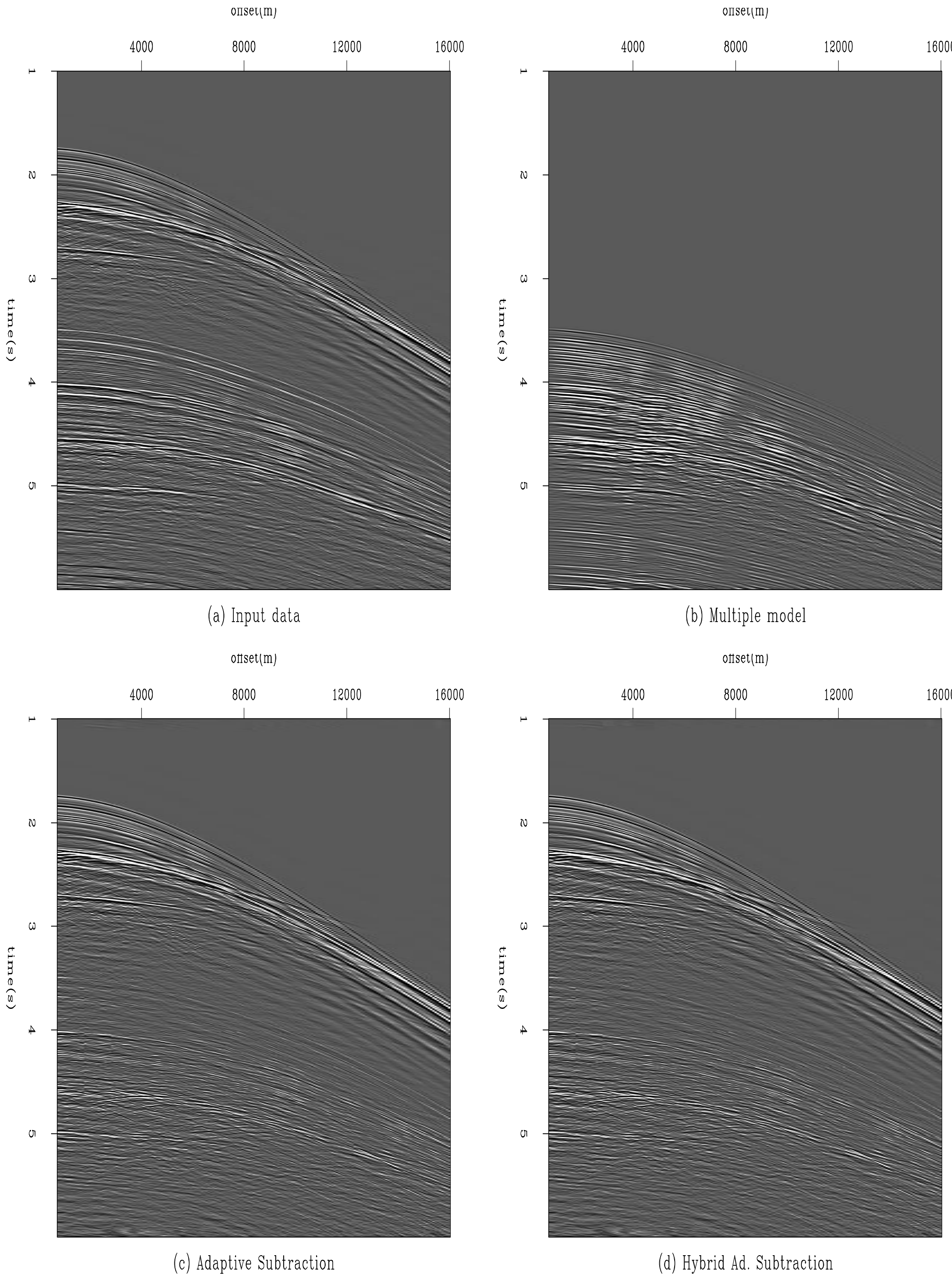

Figure 9: (a) A shot gather from a Gulf of Mexico 2-D line. (b) The corresponding multiple model. (c) Estimated primaries with the standard approach. (d) Estimated primaries with the hybrid approach. Both estimated signal are similar. antoine2-gom_res [ER,M]

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