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Multidimensional imaging condition for shot profile migration

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ABSTRACT

Conventional shot profile migration schemes determine the reflection strength at each subsurface point taking into account only the downgoing and the upgoing wavefields at that location. Since events in the subsurface are not uncorrelated, a better imaging condition could be one that makes use of the downgoing and upgoing wavefields in a neighborhood of the point where the reflection strength is calculated. A generalized multidimensional deconvolution imaging condition could be the solution to integrating information from the neighboring points, but issues related with deconvolution stability still need to be solved. An alternative to deconvolution may be a new regularized least squares imaging condition. This could be a feasible approach since the regularization operator can favor a predetermined distribution of the reflectivity. Improvements can be done in the conventional industry imaging condition adding a spatially variant damping factor, even without including information of the neighboring points.

INTRODUCTION

Shot profile migration includes three different steps: downgoing wavefield propagation, upgoing wavefield backward propagation and imaging. The last step, imaging, is based on Claerbout's imaging principle (Claerbout, 1971). According to this principle, a reflector exists at a point where the upgoing and the downgoing wavefields coincide in time and space (Figure 1).

There are two distinct aspects behind this principle: the kinematic (coincidence in time and space of upgoing and downgoing wavefields) and the dynamic (reflection strength at the coincidence point). Conventional migration schemes (Jacobs, 1982) determine the reflection strength in each subsurface point taking into account only the downgoing and the upgoing wavefields at that location. But these approaches don't consider that the reflectors in the subsurface are spatially correlated. We discuss a multidimensional imaging condition that makes the reflectivity strength dependent on the downgoing and upgoing wavefields in a neighborhood of the subsurface point.

Based on the equivalence of the deconvolution imaging condition with the exact solution of the least squares fitting goal, we propose a regularization scheme for the imaging condition that has the potential of including the previous knowledge of the image in the regularization operator. This regularization approach has been used to steer the final image in least squares

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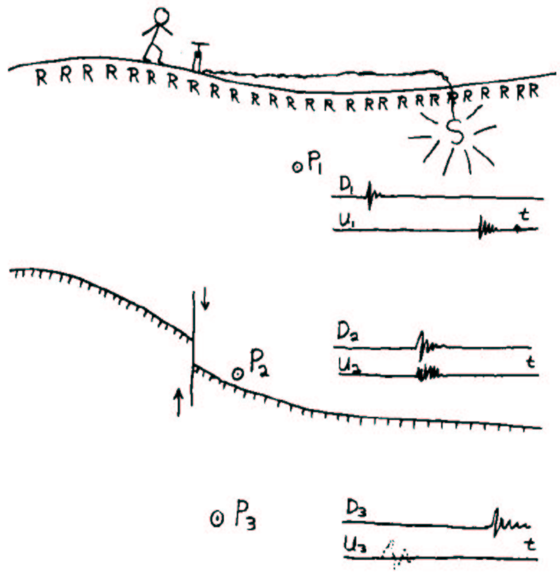

Figure 1: Reflector mapping imaging principle, note that the reflector exists at a point where the upgoing and downgoing wavefields coincide in time and space. Taken from Claerbout (1971) alejandro1-jon [NR]

inversion (Prucha and Biondi, 2002).

First, we explore a multidimensional deconvolution imaging condition. No clear results were obtained using this approach yet, but it allows us to study the difficulties of multidimensional deconvolution. Second, a least squares regularized scheme for the imaging condition is stated.

Finally we test, in a synthetic experiment, a space variable damping factor to improve conventional industry imaging condition. In this case we only consider point to point dependence of the reflection strength.

MULTIDIMENSIONAL DECONVOLUTION IMAGING CONDITION

Claerbout (1971) expresses the reflector mapping principle by the formula

$$\mathbf{r}(x,z) = \frac{\mathbf{u}(x,z,t_d)}{\mathbf{d}(x,z,t_d)}, \tag{1}$$

where x is the horizontal coordinate, z is the depth, t_d is the time at which the downgoing wave $\mathbf{d}(x,z,t_d)$ and the upgoing wave $\mathbf{u}(x,z,t_d)$ coincide in time. This principle states that for time equal t_d the reflectivity strength $\mathbf{r}(x,z)$ depends only on the downgoing wave at (x,z) and on the upgoing wave at (x,z). No particular distribution is assumed for the reflectivity in the horizontal direction or in depth. Neither a dependence of the reflectivity of the future (wavefields anteceding t_d) or of the past (wavefields preceding t_d) is assumed.

Based on the imaging principle described in equation (1) we can propose a more general imaging condition that makes the reflectivity in (x,z) dependent on the downgoing and upgoing wavefields in the neighborhood of (x,y), shown in Figure 2 .

This more general imaging condition can be stated by:

$$\mathbf{r}(x,z) = \sum_t \frac{\mathbf{u}([x-\sigma_x, x+\sigma_x],[z-\sigma_z, z+\sigma_z])}{\mathbf{d}([x-\sigma_x, x+\sigma_x],[z-\sigma_z, z+\sigma_z])}, \tag{2}$$

where the division symbol (−) means 2-D deconvolution of the upgoing wavefield with the downgoing wavefield in the (x,z) plane. The σ_x, σ_z are small numbers that define a rectangular neighborhood (x,y). This 2-D imaging condition states that there will be more than one point in the downgoing wavefield **d** and the upgoing wavefield **u** contributing to the strength at the point (x,y).

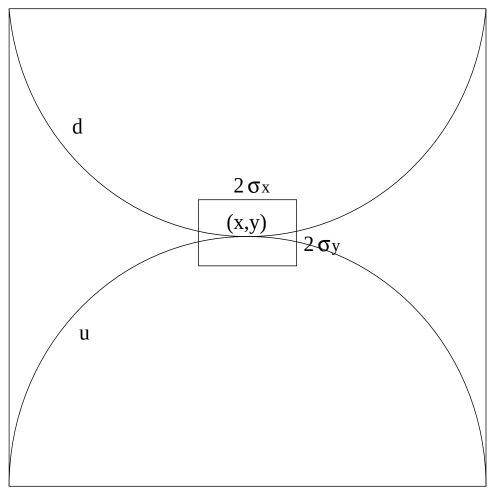

Figure 2: Multidimensional imaging geometry alejandro1-cuboxfig [NR]

To address the multidimensional deconvolution we can make use of the helix concept (Claerbout, 1998). If we put the upgoing and the downgoing wavefields in helical coordinates, we will be able to treat the multidimensional deconvolution as a 1-D deconvolution.

But deconvolution is not an easy task. To have a stable deconvolution we need **d** to be minimum phase, so an approximation of equation (2) could be

$$\mathbf{r} = \sum_{\mathbf{t}} \frac{\mathbf{u}}{\mathbf{d_{mp}}}, \tag{3}$$

where $\mathbf{d_{mp}}$ can be computed in an helix by means of Wilson spectral factorization (Sava et al., 1998) in spatial coordinates (x,y) or by means of Kolmogoroff spectral factorization (Claerbout, 1976) in the Fourier domain.

Now, a new question arises: Does the new imaging condition formulation equation (3) honor Claerbout (1971) imaging principle?

The answer to this question is no, equation (3) gives a shifted version of the image. The minimum phase transformation produces a shift in spatial coordinates (x,y). This shift has to be calculated to obtain a properly placed image.

Some attempts were made to implement the anteceding procedure using Wilson spectral factorization to obtain a minimum phase version of the downgoing wavefield. No convergence of factorization results were obtained. More work needs to be done to understand the causes.

LEAST SQUARES IMAGING CONDITION

Changing deconvolution for convolution, a different imaging condition can be stated for each time in terms of the following fitting goal:

$$\mathbf{D}\mathbf{r} = \mathbf{u}, \tag{4}$$

where **D** is a convolution matrix in which columns are downshifted versions of the downgoing wavefield **d**.

The least squares solution to this problem is

$$\mathbf{r} = (\mathbf{D}^{'}\mathbf{D})^{-\mathbf{1}}\mathbf{D}^{'}\mathbf{u}.$$

A damped solution is usually used to guarantee $\mathbf{D}^{'}\mathbf{D}$ to be invertible as in

$$\mathbf{r} = (\mathbf{D}^{'}\mathbf{D} + \varepsilon^{\mathbf{2}})^{-\mathbf{1}}\mathbf{D}^{'}\mathbf{u}$$

where ε is a small positive number to guarantee no zeroes in $\mathbf{D}^{'}\mathbf{D}$ diagonal. This is equivalent to the fitting goal

$$\begin{aligned} \mathbf{0} &\approx \mathbf{D}\mathbf{r} - \mathbf{u} \\ \mathbf{0} &\approx \varepsilon\mathbf{I}\mathbf{r}, \end{aligned} \tag{5}$$

where **I** is the identity matrix that is used here as the regularization operator. Using this regularization scheme we are adding to the denominator a constant value where it is needed and where it is not.

As it is our intention to use the previous knowledge of how the image should be, we could choose an smarter way to fill the zero values off $\mathbf{D}^{'}\mathbf{D}$ diagonal. We can substitute the regularization operator for one constructed with a priori information, using

$$\begin{aligned} \mathbf{0} &\approx \mathbf{D}\mathbf{r} - \mathbf{u} \\ \mathbf{0} &\approx \varepsilon\mathbf{A}\mathbf{r} \end{aligned} \tag{6}$$

where our regularization operator **A** could be a steering filter (Clapp et al., 1997). Steering filters can efficiently guide the solution toward a more geologically appealing form. This type of filter has been used with success to smooth existing reflectors and fill shadow zones in least squares inversion (Prucha and Biondi, 2002).

SPACE VARIABLE DAMPING IN CONVENTIONAL IMAGING CONDITION

Conventional shot profile migration schemes determine the reflection strength at each subsurface point taking into account only the downgoing and the upgoing wavefields at that location. Jacobs (1982) compares two different imaging conditions

$$\mathbf{r} = \sum_{\mathbf{t}} \mathbf{ud}, \tag{7}$$

and

$$\mathbf{r} = \sum_{\mathbf{t}} \frac{\mathbf{ud}}{\mathbf{d}^2 + \varepsilon^2}. \tag{8}$$

The first is one commonly used by the industry. It has the advantage of being robust, but has the disadvantage of not computing the correct amplitudes. The second computes the correct amplitudes (except for a damping factor ε), but has the disadvantage of being unstable due to zero division. That is why a damping factor ε is needed.

We propose to add a mask function defined as

$$\mathbf{w} = \begin{cases} 0 & \text{if} \quad \mathbf{ud} > \alpha \\ 1 & \text{otherwise} \end{cases} \tag{9}$$

where α can be variable in space.

When **ud** has enough energy to contribute to the image, the damping factor ε is set to zero. When factor **ud** is small, the damping factor is kept to avoid zero division. Thus, the imaging condition can be set as

$$\mathbf{r} = \sum_{\mathbf{t}} \frac{\mathbf{ud}}{\mathbf{d}^2 + \mathbf{w}\varepsilon^2}, \tag{10}$$

where the damping is now variable in space.

A simple synthetic was generated to test the preceding idea using wave equation modeling. Figure 3a shows the downgoing wave, and Figure 3b the upgoing wave, at a fixed time. Figure 4 shows the mask **w** used in this example.

Figure 5a shows the reflection strength calculated using the imaging condition stated in equation (7). Figure 5b shows the reflection strength calculated using division of the upgoing wavefield **u** by the downgoing wavefield **d**. Figure 5c shows the reflection strength calculated using the imaging condition stated in equation (8), and Figure 5d shows the reflection strength calculated using the imaging condition stated in equation (10). The advantage of Figure 5d's result over the others is that it has the correct reflection strength value inside the masked area and doesn't diverge outside it because of the damping factor.

In Figure 6 we compare the two imaging conditions stated in equations (8) and (10) inside the masked area for two different ε. We can see for the imaging condition stated in equation

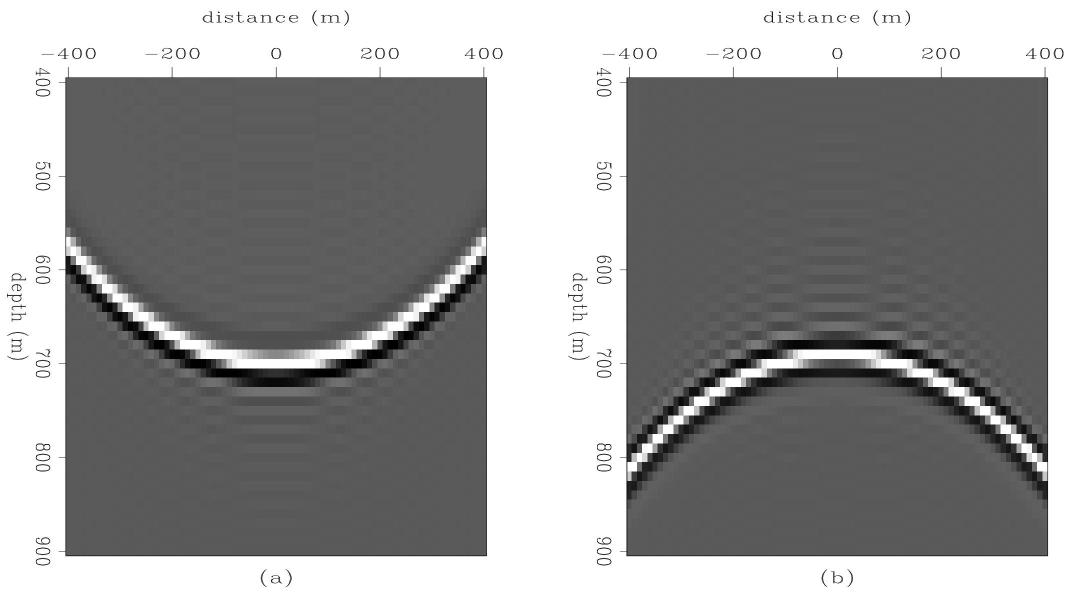

Figure 3: Wavefields at a fixed time. a) Downgoing wave, b) Upgoing wave. alejandro1-DU [ER]

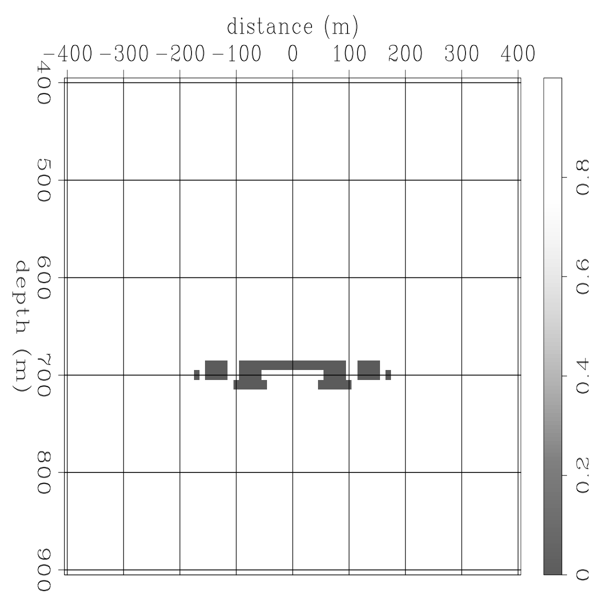

Figure 4: Mask used in equation (10). Zero at masked area and one out of the masked area. alejandro1-ma [ER]

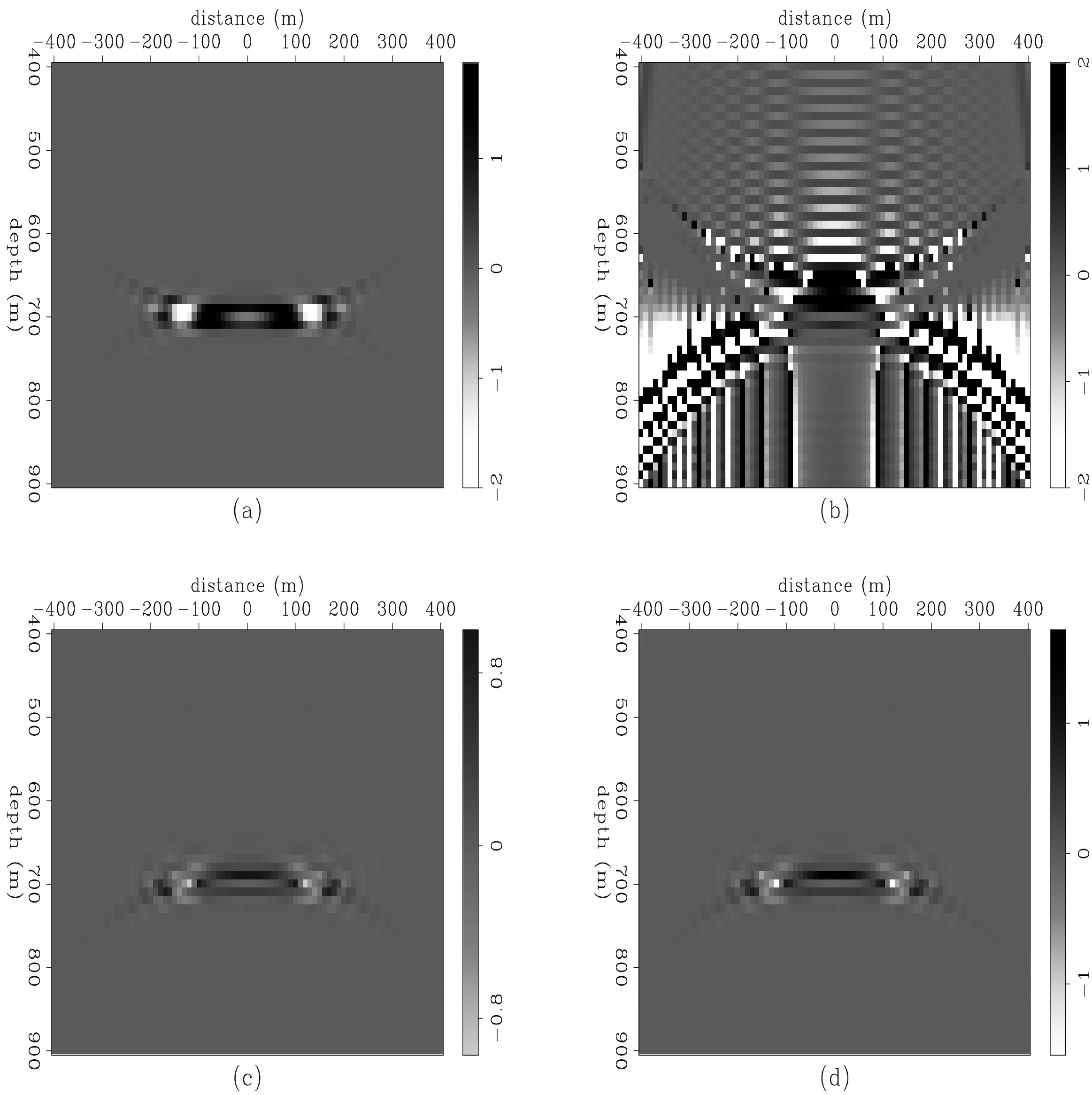

Figure 5: Comparison between four different imaging conditions a) Calculated by wavefield multiplication equation (7), b) Calculated by wavefield division ($\mathbf{u}/\mathbf{d}$), c) Calculated using constant damping equation (8), and d) Calculated using space variable damping equation (10). alejandro1-comp_1shot [ER]

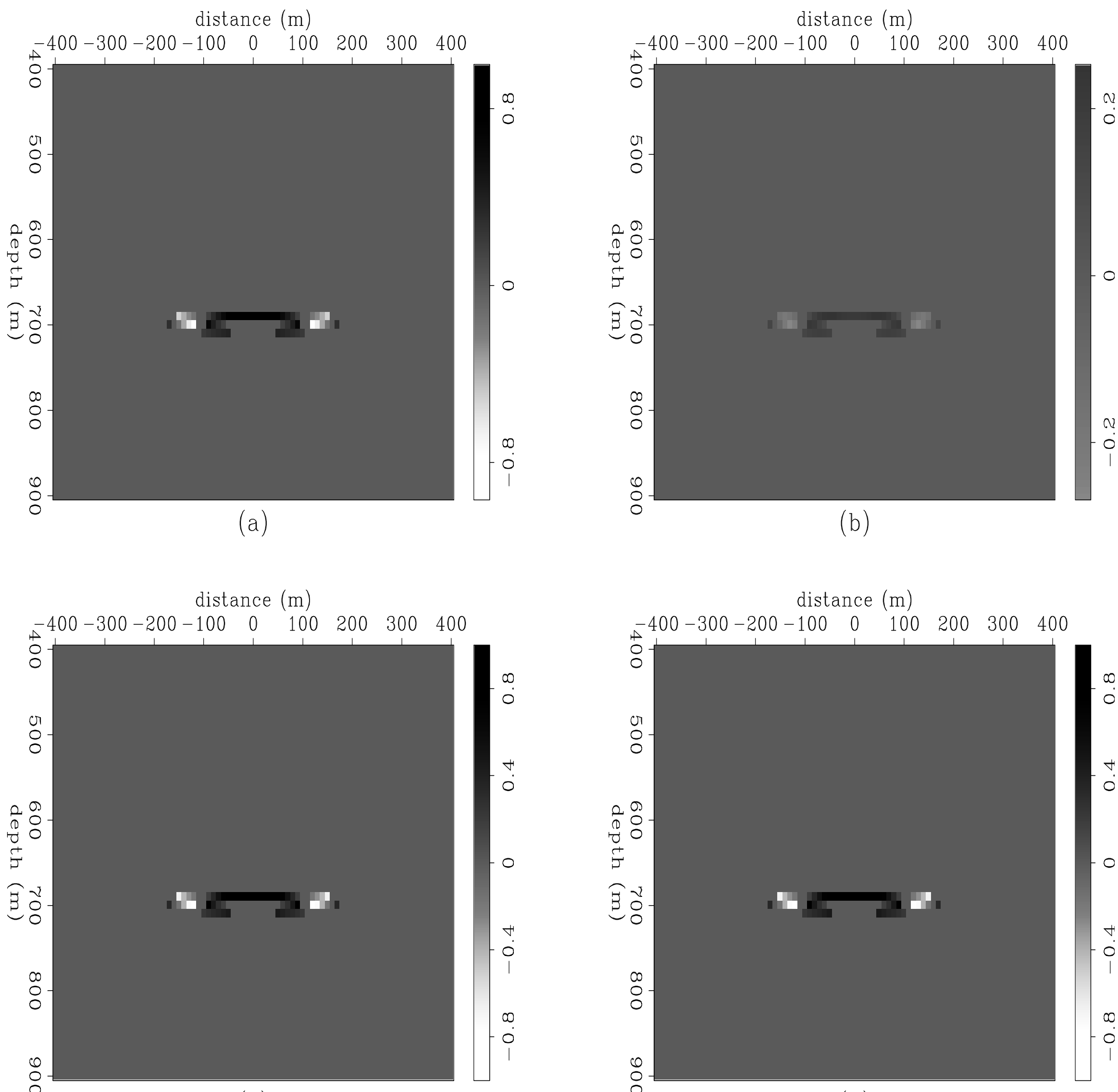

Figure 6: Comparison between imaging condition stated in equations (8) and (10) inside the masked area. a) $\varepsilon = 0.5$, b) $\varepsilon = 5$, c) $\varepsilon = 0.5$, d) $\varepsilon = 5$ alejandro1-comp_im [ER]

(10) that the reflection strength inside the masked area doesn't change. This is an important advantage of space variable damping imaging principle, because it let us to build an adaptive mask dependent of the subsurface illumination.

We stack the reflection strength from 11 shots to see how the change observed in one shot affects the final image. The result is shown in Figure 7. We can see imaging condition from equation (10) gives the best resolution.

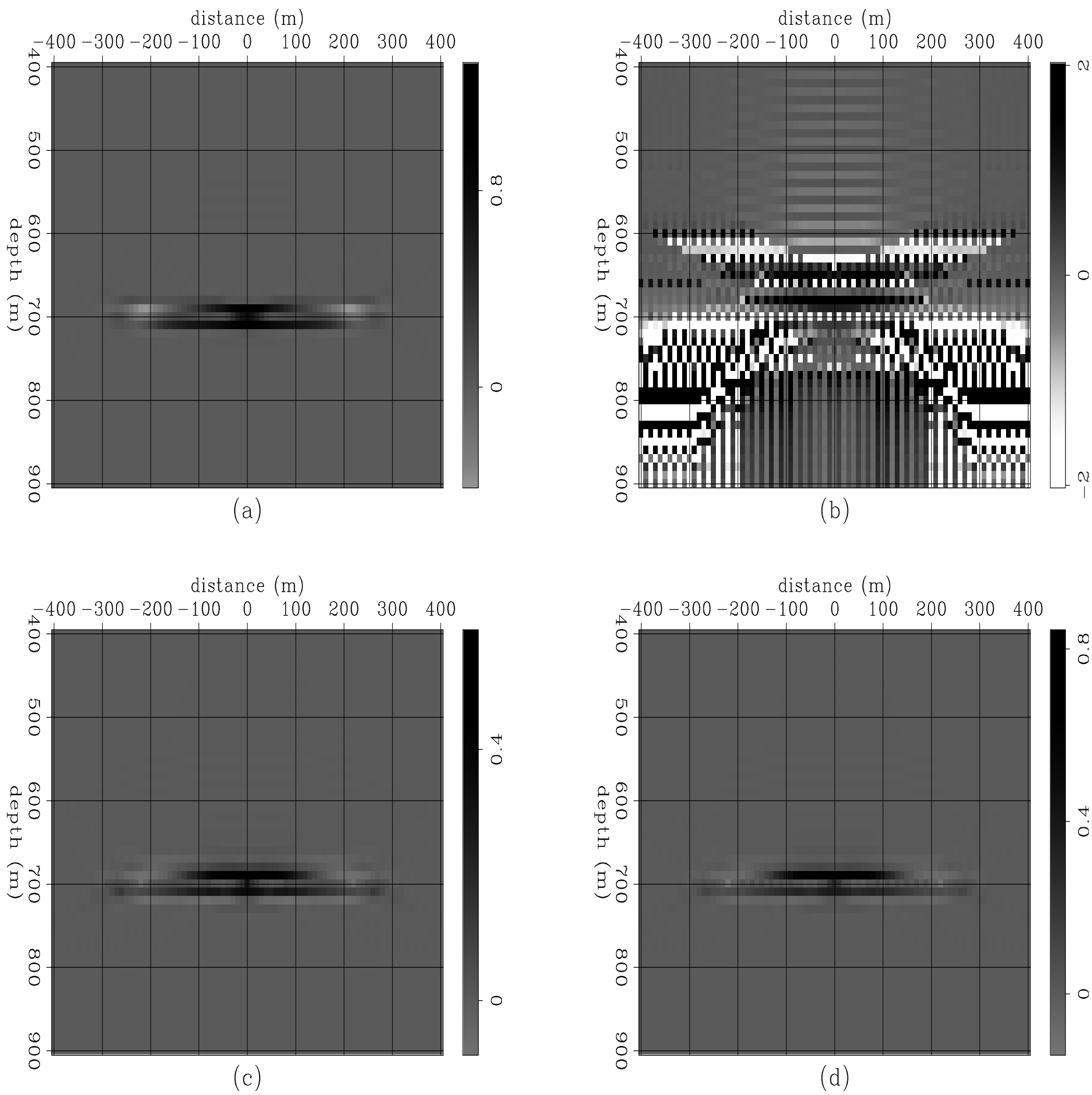

Figure 7: Comparison between 11 shot stacks using three different imaging conditions, a) equation (7), b) wavefield division ($\mathbf{u}/\mathbf{d}$), c) equation (8), and d) equation (10). alejandro1-comp_stack [ER]

CONCLUSION

A generalized multidimensional deconvolution imaging condition could be the solution to integrating information from the neighboring points in the computation of reflection strength for shot profile migration. However, issues related to deconvolution stability and proper placement of the image still need to be solved. An alternative to deconvolution, stating a new regularized least squares imaging condition, could be a feasible approach as the regularization operator can be set to favor a predetermined spatial distribution.

We showed, in a synthetic experiment, that a spatial variant damping factor can improve the resolution and amplitude preservation of the conventional industry imaging condition. The damping factor can be related to reflector illumination, adding the damping factor where it is really needed.

REFERENCES

Claerbout, J. F., 1971, Toward a unified theory of reflector mapping: Geophysics, **36**, no. 3, 467–481.

Claerbout, J. F., 1976, Fundamentals of geophysical data processing: Blackwell.

Claerbout, J. F., 1998, Multidimensional recursive filters via a helix: Geophysics, **63**, no. 5, 1532–1541.

Clapp, R. G., Fomel, S., and Claerbout, J., 1997, Solution steering with space-variant filters: SEP–**95**, 27–42.

Jacobs, B., 1982, The prestack migration of profiles: SEP–**34**.

Prucha, M. L., and Biondi, B. L., 2002, Subsalt event regularization with steering filters: SEP–**111**, 1–17.

Sava, P., Rickett, J., Fomel, S., and Claerbout, J., 1998, Wilson-Burg spectral factorization with application to helix filtering: SEP–**97**, 343–351.