e equation $P_z = a(-\omega^2 \Delta t^2/2 + i\omega \Delta t)P$, unlike the equation of Exercise causal time-domain filter. (Do the extrapolation in z by the Crank- nod, i.e., the bilinear transform method.)

data $P(x, t)$ is believed to satisfy the equation $P_{zt} = P_{xx}$. It is observed ned data $Q(x, t)$, where $Q(x, t) = P(x, t)e^{\alpha t}$, fits into a reasonably small ge so that Q may be represented using integer arithmetic. What differ- n does Q satisfy?

11

SEISMIC DATA PROCESSING WITH THE WAVE EQUATION

The coordinate frames used by theoreticians to describe wave propagation do not include frames in common use by geophysical prospectors to describe observations. Whereas the theoretician generally considers a single source (or shot) location at a time, the experimentalist deals simultaneously with waves which have been generated separately by many shots. Our task in this section is to put the wave equation into some prospectors' coordinate frames.

11-1 DOWNWARD CONTINUATION OF GATHERS AND SECTIONS

Suboceanic prospecting is generally carried out by a ship which carries a repetitive energy source and which trails a cable that is 2 to 3 kilometers long and packed with sonic receivers. Ideally, the ship's course is a straight line which we can take to be the x axis. Ideally, all the seismic waves of interest propagate in a vertical plane through the line of the ship's course. This plane is called the plane of the seismic section. Despite the fact that it is no great problem to describe waves in

ns once difference techniques have been mastered for two
will restrict the theory to two dimensions in order to keep it
the bulk of present-day surveying practice and the capability of
y computing machines. An impulsive wave from a point source
n three dimensions will decay in amplitude in inverse proportion
e. (The area of a spherical wavefront increases as t^2, so the energy
ecreases as t^2, and so the wave amplitude is proportional to t^{-1}.)
ave from a line source (the line would be on the ocean's surface
the ship's course) has an amplitude decay proportional to $t^{-1/2}$.
pt to compress three-dimensional reality into a one- or two-dimen-
tical form often begins (and almost always ends) with a $t^{1/2}$ or a
Of far more practical importance than this scaling is the attempt
seismic rays which emanate from and return to the ship's traverse
a single plane. In other words, we hope to avoid recording side
ide echoes can be reduced or eliminated by careful choice of the
ut, once the data have been recorded, you have to live with what-
are there. One way to think about these side echoes is to imagine
se line as the axis of a cylindrical coordinate system. Instead of
the time delay of an echo is a measure of the depth to a reflector,
es that the travel time is a measure of the radius in the cylindrical
em. Interpretation is easy if the plane of the seismic section is
at tipped away from the vertical. Interpretation problems arise
is so three-dimensionally complex that several wobbly planes are
e observed data have become a superposition of many of them.
the earth gets three-dimensionally inhomogeneous you cannot get
with two-dimensional experimental and calculational techniques.
-1 shows a most important relationship between two coordinate
ordinates of the shot sound source s and geophone sound receiver
g the ship's course, which is the x axis. Also along the x axis are
hone distance offset coordinate f and the midpoint y between the
ophone.
ng to describe waves of pressure $P(s, g)$ where the shot and geophone
taken to be independent variables. In reality, the shots and geo-
distributed in a continuum along the x axis, but they are usually
gether that it is merely a matter of interpolation to find P for any
ata cannot be interpolated, they will not be satisfactory for use in
tions.
dependent variable is time t. The origin point on the time axis is
ime t equals 0 when the shot goes off. After the echoes from the
lied out completely (ordinarily about 6 seconds), the time axis is
or the next shot at $s + \Delta s$. In Fig. 11-1 the t axis may be taken to be
of the paper. Both the (s, g) coordinates and (y, f) coordinate
onal. Nonorthogonal coordinates such as (s, f) are used in
ording, but they are rarely used in data analysis and we will ignore

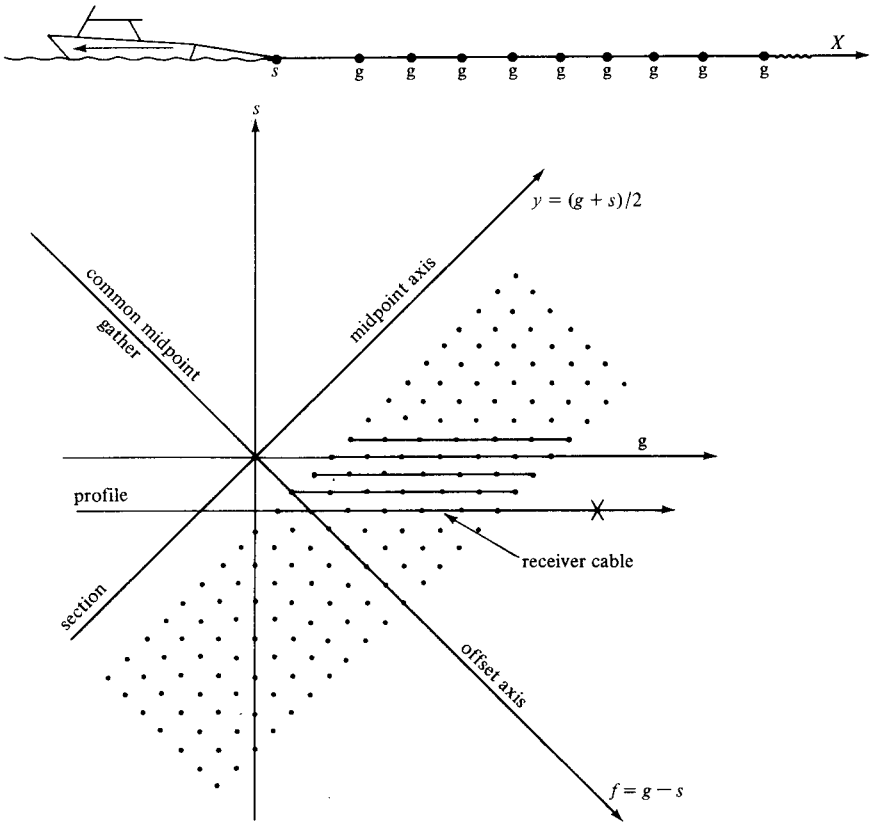


FIGURE 11-1
The relationships among sound source coordinate s, geophone sound receiver coordinate g, offset coordinate $f = g - s$, and midpoint coordinate $y = (g + s)/2$. Theoreticians generally use s and g as coordinates of the wave-pressure field, but interpreters generally use f and y.

Theoreticians usually work in the (g, t) plane for a fixed s. In exploration seismology, data in the (g, t) plane are called a *profile*. Seismic data interpreters usually work with wave amplitude in both the (y, t) plane and the (f, t) plane. A display in the (y, t) plane is called a *seismic section*. A display in the (f, t) plane is called a *common midpoint gather* or a *common reflection point gather*, which, unfortunately, in industry is often called a *common depth point gather*. This terminology originated in the days when analytical methods usually modeled the earth as a stratified medium in which the reflection point, called the *depth point*, lay directly beneath the midpoint. It is unfortunate because of the considerable confusion we now have with the depth z axis. In this book we will avoid the term *depth point*. A data display over midpoint y at fixed offset f, that is, the (y, t) plane, is called a *seismic section* and it is the only one of the orthogonal planes

nue for hundreds or thousands of kilometers. *Profiles* and *gathers* y a few kilometers, first because of the limited length of the re- more fundamentally because of the limited distance over which ard.

d gathers today typically contain about 48 seismic traces. Because lie beyond the 3-kilometer receiver cable, another form of data re- mes used. This is the *sonobuoy*. A sonobuoy is a buoy with a single alled a *hydrophone*) and a radio transmitter. The buoy is cast over- ip sails away, firing at about a six-second repetition rate until the ange of either the radio or the seismic signals. Such data form a r point gather and comprise about 200 to 2000 seismic traces. re conceptually one of the easiest kinds of data to be imagined as ary conditions for wave-extrapolation equations. The principle of hat we could imagine that the ship had carried the sonic receiver y had carried the repetitive sound source. (It is not done this way ic receiver is an inexpensive, lightweight, throwaway item, well oy.) Thus, the principle of reciprocity which says $P(g, s, t) =$ us to imagine a single shot with many hundreds of receivers, just r downward continuation of the upcoming and downgoing wave

s us to the concept of how we learn about the interior of the earth wnward continuing waves. As indicated in Fig. 11-2, an obvious, dea is that *reflectors exist in the earth at places where the onset of ave is time coincident with an upcoming wave.*

strate this idea, monochromatic waves will be used. This enables us think in (x, z) for fixed ω rather than to have to work in the three- e of (x, z, t). A penalty we pay by going to a single frequency is time coincidence of two time-dependent waveforms becomes for waves the idea of both waves being coherent with a fixed phase o or 180 degrees) at points in space where reflection can occur. phase equivalence at a single frequency at a single point in space lf provide time coincidence. The same must be true of other nerally, the more frequencies are involved in phase coherency, the spatial resolution.

will recall that a number of assumptions in Sec. 10-5 led us to the d downgoing monochromatic waves can be calculated with equa-

$$D'_z = \frac{iv}{2\omega} D'_{xx} \qquad (11\text{-}1\text{-}1)$$

$$U'_z = -\frac{iv}{2\omega} U'_{xx} - c(x, z) D' e^{2imz} \qquad (11\text{-}1\text{-}2)$$

rd problem, synthetic data are calculated from a model. In an a model is calculated from the data. To do a forward problem we

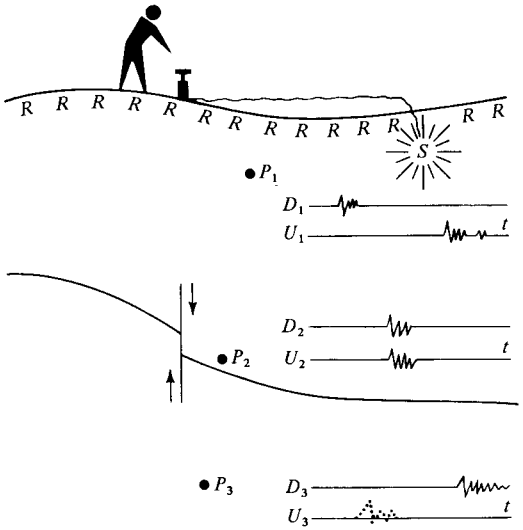
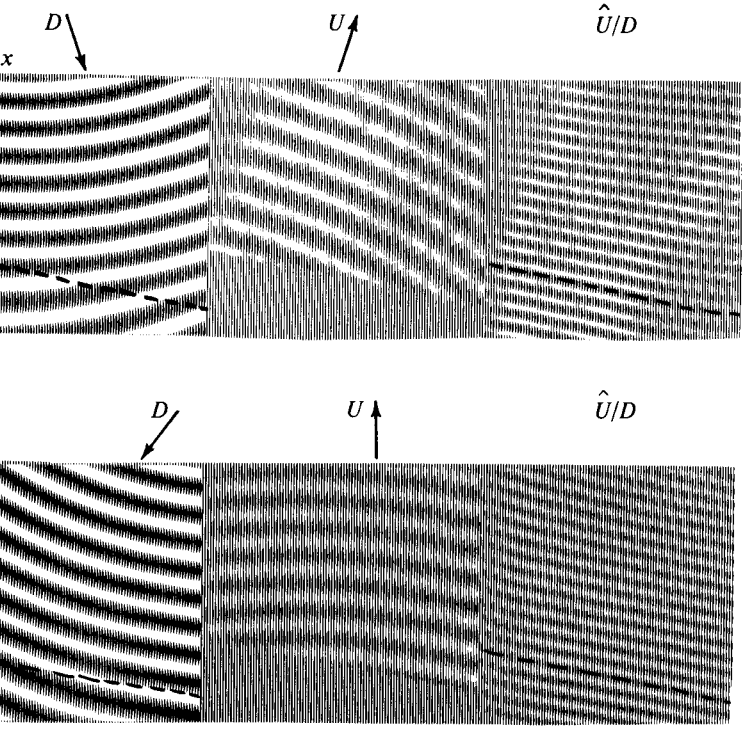


FIGURE 11-2
Illustration of the basic principle of reflector mapping. There are a near-surface source S and many surface receivers R. At a shallow depth above the reflector at the typical point P_1 the downgoing wave D_1 occurs much earlier than the upcoming wave U_1. The upgoing wave U_1 represents energy which has traveled from the source to one or more places on the reflector and then back up to the point P_1. At a point P_2, which is at or near a reflecting interface of arbitrary shape, there will be overlap in time of the down- and upgoing waves D_2 and U_2. The time overlap may be used in the construction of a map of reflector positions. Below the reflector at the point P_3, there is, in principle, no upcoming wave. However, practical schemes for estimating the upcoming waves U at various depths in large part amount to shifting the upcoming waves seen at R to earlier and earlier times corresponding to greater and greater depths. Since the practical schemes will have no knowledge of the interface, they will predict an erroneous upcoming wave U_3 at P_3, indicated by dots before the arrival of the downgoing wave. This error has no bad effect on the reflector mapping formulas which utilize time coincidence of up- and downgoing waves. These ideas are valid in situations where there are many reflectors at many depths. (From Ref. [5], p. 468.)

first assume a shot location at, say $z = 0$ and $x = 0$. This provides a delta function initial condition for the downgoing wave. Assumption of a velocity model then allows use of an equation like, say (11-1-1) to continue the downgoing wave downward to arbitrary depth. At a sufficiently great depth, the upcoming wave U is taken to be 0. It is integrated upward with (11-1-2) where the product of the reflection coefficient c and the downgoing wave D act as a source for the upgoing wave.

Now we approach the *inverse* problem where we are trying to determine the reflection coefficient $c(x, z)$, but we are given the observation of the upgoing wave U at the earth's surface ($z = 0$, all x). We calculate D as before. Since $c(x, z)$ is



GURE 11-3
cation of a dipping bed with a single source emitting a single frequency. Top bottom represent two possible source locations. The left-hand panel shows real part of downgoing wave D. The dashed line represents a transition from to high velocity as can be seen by the greater wavelength below. The center el represents the upcoming reflected wave U, which originates at the velocity p. On the right-hand panel is plotted the real part of the ratio $\hat{U}/D$. The right-d panel tells where downgoing and upcoming waves are in phase. It gives the rect dip of the bed. With only a single frequency the depth cannot be deter-ded except to within multiples of a half-wavelength. The estimated upcoming e $\hat{U}$, is computed from the true upcoming wave U, observed at the surface the velocity v of the medium; Y_z/Y is not used. (From Ref. [3], p. 417.)

-2) we can compute $\hat{U}$, an approximation to U, by marching the

$$\hat{U}'_z = -\frac{iv}{2\omega}\hat{U}'_{xx} - 0 \qquad (11\text{-}1\text{-}3)$$

urface $z = 0$ where $\hat{U}$ is given.
estion is, how will $\hat{U}$ depart from U? Between the earth's surface st reflector (shallowest nonzero c) there will be no difference be-d (11-1-3). In that region, both U and $\hat{U}$ will be waves of identical s, rates of divergence, and all other properties. As (11-1-2) is

projected downward (opposite to the direction of propagation), the source terms in (11-1-2) serve to "turn off" the upcoming waves until U, unlike $\hat{U}$, totally vanishes beneath the deepest reflector. Now the question is whether there is practical significance to the fact that $\hat{U}$ does not vanish beneath reflectors. Our purpose in computing $\hat{U}$ is to determine the location of reflectors by the time-coincidence idea in Fig. 11-2. For this time-coincidence idea, it does not matter that the upcoming wave is not turned off beneath an interface. Figure 11-3 illustrates these concepts for a monochromatic wave. To display the earth model, the reflection coefficient $c(x, z)$ is estimated by displaying $\hat{c}$ where

$$\hat{c}(x, z) = \frac{\hat{U}(x, z)}{D(x, z)} \qquad (11\text{-}1\text{-}4)$$

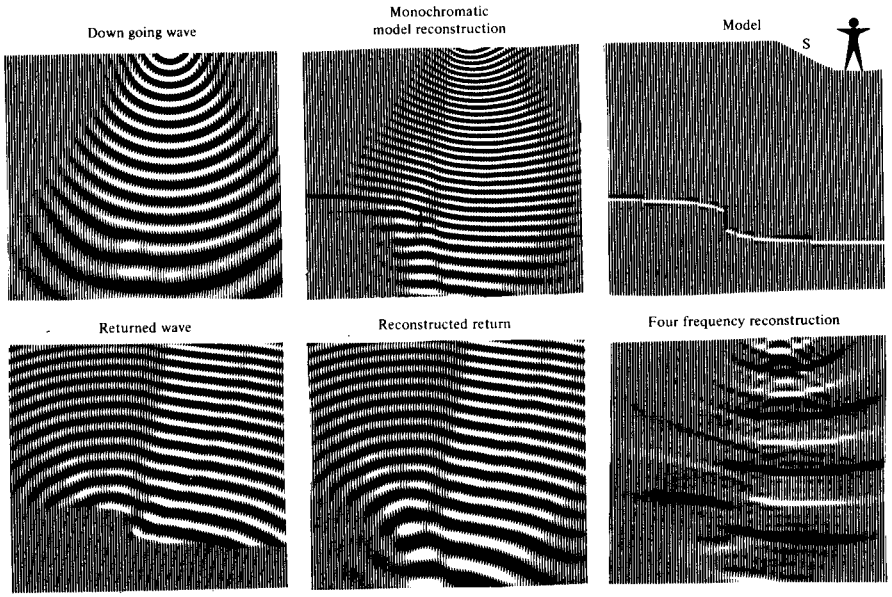


FIGURE 11-4
Synthesis of waves from a reflector which is warped and offset by a fault and the reconstruction of an image of the fault. Top left is the wave traveling away from a point source. Bottom left shows the upgoing reflected wave. It vanishes below the interface. At the bottom center, we see the reconstruction from surface observations of the upcoming wave. In doing the reconstruction, one does not assume knowledge of the reflectors, since they are what we are looking for. Hence, in the reconstructed return, one has an upgoing wave below the reflector. At the top center panel we show the product of the upgoing wave with the complex conjugate of the downgoing wave. The reflector exists along some line of zero phase, but with a single frequency, one cannot tell which line represents the reflector. The top center panel was summed over four frequencies to get the lower right panel, which gives a better indication of the reflector position. More frequencies would define it more clearly. (From Ref. [5], p. 479.)

or a monochromatic wave there will be phase coincidence of
at the reflector but also at half-wavelength intervals both above
flector. (If we had U instead of $\hat{U}$, there would be 0 below.)
at we can repeat the whole calculation for monochromatic waves
ies and sum the results. The summation will always be in phase
t of variable phase away from the reflector.
shows a calculation like that in Fig. 11-3, but more terms were in-
tions (in order to better represent waves traveling at larger angles
and more frequencies were used (in order to illustrate construc-
t the reflector). Another difference between Figs. 11-3 and 11-4 is
estimate is $\hat{U}/D$ in (11-1-3), but in (11-1-4) it is $\hat{U}D^*$ (where D^*
njugate of D). Both $\hat{U}/D$ and $\hat{U}D^*$ have the same phase, but they
mplitude. The advantage of $\hat{U}/D$ is that it has the magnitude of
fficient. The disadvantage of $\hat{U}/D$ is that nodes in D, which
ld imply nodes in U, may cause us to have practical problems
small numbers. Another advantage of $\hat{U}D^*$ is that if it is to be
over many frequencies, but also over many shot locations, it has
racteristic that it is small where illumination is poor and large
n is good.
ack the important practical matter of how to continue sections
bvious approach is to solve a separate problem in the (g, t) plane
t. Alternatively, we could use the reciprocity principle and use
t as a separate problem in the (s, t) plane. For reasons we will
, there are considerable practical advantages in leaving the theory
and continuing downward directly in the interpreter's (y, f)
mpelling practical reason is that many data are collected with a
ingle receiver which move together across the surface of the earth.
oint there is only one receiver point, so there are clearly insuf-
ialize a downward continuation in the (g, t) plane. Nonetheless,
nts are considered, it turns out that with good accuracy we can
tant offset section downward.
lop two separate equations, one for downgoing waves and one for
The conversion from field coordinates (s, g, e, t) to interpreta-
y, f, z, t') is accomplished with the definitions

$$y = \frac{s+g}{2} \qquad (11\text{-}1\text{-}5a)$$

$$f = g - s \qquad (11\text{-}1\text{-}5b)$$

$$z = e \qquad (11\text{-}1\text{-}5c)$$

$$t' = t - \left(\pm \frac{e}{\bar{v}}\right) \qquad (11\text{-}1\text{-}5d)$$

itions (11-1-5a, 11-1-5b) are merely the transformation from shot
coordinates to midpoint y and offset f coordinates described

earlier. Equation (11-1-5c) indicates that the receiver elevation e is a point on the vertical z axis. In (11-1-5d) we have a definition of receiver-elevation-dependent time t'. Comparing to Sec. 10-4, it is clear that the plus sign is for waves propagating in the $+z$ direction (down) and the minus sign giving $t + z/\bar{v}$ is for waves going up. It is important to distinguish the constant velocity $\bar{v}$ in the coordinate transform (11-1-5) from the spatially variable velocity $\tilde{v}(x, z)$ which will be used in the wave equation. Although the coordinate transformation is based on a constant-velocity medium, the transformed wave equation can still describe waves in a variable-velocity medium.

Now we state that we are trying to describe the same disturbance in the new coordinate system as that in the old one.

$$P(g, s, t, e) = Q(y, f, t', z) \qquad (11\text{-}1\text{-}6)$$

Next, we compute the partial derivative of P with respect to its independent variables.

$$\begin{aligned} P_g &= Q_y y_g + Q_f f_g + Q_{t'} t'_g + Q_z z_g \\ P_g &= (.5\partial_y + \partial_f)Q \end{aligned} \qquad (11\text{-}1\text{-}7a)$$

$$\begin{aligned} P_e &= Q_y y_e + Q_f f_e + Q_{t'} t'_e + Q_z z_e \\ P_e &= (t'_e \partial_{t'} + \partial_z)Q \end{aligned} \qquad (11\text{-}1\text{-}7b)$$

$$P_t = (t'_t \partial_{t'})Q \qquad (11\text{-}1\text{-}7c)$$

Now we need the second partial derivatives of P with respect to its independent variables. Since all the coefficients of ∂_y, ∂_f, $\partial_{t'}$, and ∂_z in (11-1-7) are constants, then the second derivatives can be found by squaring the operators in parentheses in (11-1-7).

We are familiar with the wave equation in the form

$$P_{xx} + P_{zz} = \tilde{v}^{-2} P_{tt} + \text{source} \qquad (11\text{-}1\text{-}8)$$

We can imagine that the geophones used to observe P could be placed anywhere in the (x, z) space. A quantity like P_{xx} on the surface of the earth could be measured by setting out geophones at $g = x$ and measuring P_{gg}. We regard the coordinates of the geophone location $(x, z) = (g, e)$ as independent variables. Thus, we may write the wave equation as

$$P_{gg} + P_{ee} = \tilde{v}^{-2} P_{tt} + \delta(g - s, e, t) \qquad (11\text{-}1\text{-}9)$$

where we have used a source term defined as a delta function at $(x, z, t) = (s, 0, 0)$. Since we do not intend to use this equation in the vicinity of sources, we can drop the delta function at the source. Now we take the operators in (11-1-7), square them and insert them into (11-1-9). This gives

$$\tfrac{1}{4}Q_{yy} + Q_{ff} + Q_{yf} + (t'_e)^2 Q_{t't'} + 2t'_e Q_{t'z} + Q_{zz} = \frac{(t'_t)^2 Q_{t't'}}{\tilde{v}^2} \qquad (11\text{-}1\text{-}10)$$

glect Q_{zz} in a Fresnel-like approximation. (Higher accuracy can Sec. 10-3 if Q_{zz} is estimated.) Specializing to homogeneous media, $t'_e = -(\pm 1/\bar{v})$ and $t'_t = 1$ our equation has reduced to

$$\pm \frac{2}{\bar{v}} Q_{t'z} = \frac{1}{4} Q_{yy} + Q_{ff} + Q_{yf} \quad (11\text{-}1\text{-}11)$$

11-1-11) is a partial differential equation in four dimensions starting ditions which are data in three dimensions. Frequently a problem e will not be computationally feasible, so we now consider how to t dimension. Let us integrate (11-1-11) over offset. We obtain

$$0 = \left(\frac{\mp 2}{\bar{v}} \partial_{zt'} + \frac{1}{4} \partial_{yy}\right) \int Q \, df + \partial_y \int \frac{\partial Q}{\partial f} \, df + \int \frac{\partial^2 Q}{\partial f^2} \, df \quad (11\text{-}1\text{-}12)$$

and Q_f should vanish at the great offsets which we take to be the egrals, then the two terms farthest right vanish. Let us define a *l section* by

$$S = \int Q \, df \quad (11\text{-}1\text{-}13)$$

done without time shifting; hence, it is similar to but not precisely s the familiar common reflection-point stack. Thus, for vertically we have

$$S_{zt'} = \pm \frac{\bar{v}}{8} S_{yy} \quad (11\text{-}1\text{-}14)$$

ave obtained (11-1-14) from (11-1-11) by merely asserting that for he offset derivatives may be neglected and that (11-1-14) applies tions.

11-1-14) is no stranger to us. Earlier we learned how the equation ntrols propagation of a wave field (like a common shot-point ans that (11-1-14) should convert hyperbolas to other hyperbolas; use of the Fresnel-like approximation (which is improvable) it as to other parabolas. In other words, (11-1-14) does just the kind s of use in seismic migration. Further details are in succeeding

EQUATION MIGRATION

of a cross section of reflectivity within the earth from a seismic *migration*. At many locations on the earth the subsurface consists ayered sedimentary rocks. At such locations the migration of be extremely simple because waves propagate vertically to the y will have a round-trip travel time which is in direct proportion to n is, then, just applying the proportionality factor to the time axis.

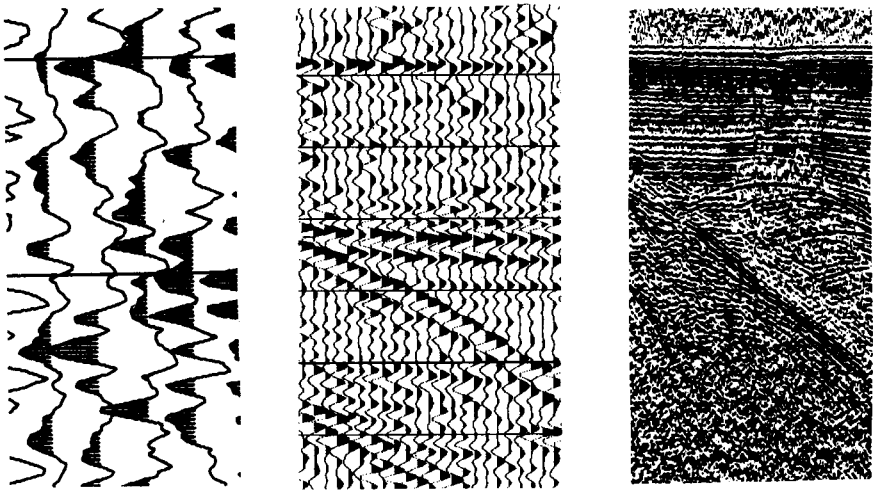
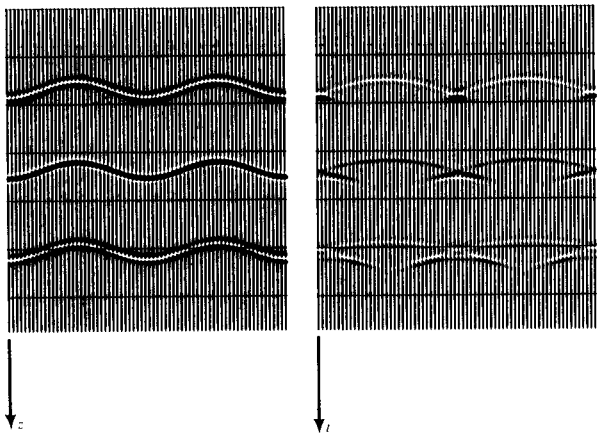


FIGURE 11-5
Display of wave fields. Side-by-side display of a collection of shaded seismograms with information density increasing from left to right becomes a picture of a wave field. The picture usually represents acoustic pressure $P(x, t)$ as a function of the horizontal space coordinate x and the vertical time coordinate t. When the time axis t is taken to be the vertical travel time of echoes, then the picture shows a cross section through the earth.

Thus, a picture of the waves, as in Fig. 11-5, which may actually show $P(x, t)$ at $z = 0$ can be regarded as a cross section through the earth, say $P(x, z/2v)$. Ideally the contours between light and dark delineate boundaries between different types of sedimentary rocks. Unfortunately, the appearance of alternating strata (between black rocks and white rocks?) is usually deceptive. It is unavoidably caused by filtering effects in the sources, receiving equipment, or even the earth. The most obvious, and perhaps the most important, information carried in the seismic section is in the departure of the earth from horizontally stratified models. Seismic sections are usually displayed with some vertical exaggeration. Such vertical exaggerations commonly range from a factor of 1 to a factor of 20. The right-hand frame of Fig. 11-5 happens to have a vertical exaggeration of 5 so that the obvious strong reflector which appears to have a 45° slope actually in the earth has approximately a 9° slope. From such pictures we may attempt to deduce the present state of the earth's subsurface and perhaps some of its history.

The purpose of migration calculations is to account for the fact that waves will not go just straight up and down where the strata within the earth are not just horizontally layered. Such calculations can incorporate a seemingly endless list of complicating factors, but luckily a straightforward application of our equation $U_{tz} = U_{xx}$ can in a practical way accommodate the important departures from horizontal layering which are found under many regions of the earth. Figure 11-6 is an example of the migration techniques to be described in this section. It illustrates that the difference in appearance between a seismic time section and a depth



GURE 11-6
ree sinusoidal reflectors at increasing depths (left) and the calculated zero-
et reflection seismic section (right). There is no vertical exaggeration. Since
h reflector has the same shape, it must have the same maximum dip (about
) as the others. The departure of the time section from the depth section
iously increases as one looks down the frames. A rule of thumb is that
ificant departure begins to occur when the depth becomes comparable to
smallest radius of curvature of the reflector (buried focus). The radius about
als the depth at the shallowest reflector where strong focusing is apparent.
om Ref. [8], p. 758.)

arise solely from the dip of the strata; in fact, the curvature is
factor. Hence, migration can be important even in relatively flat

e incident on an interface gives an impulsive reflection; however,
me filtering effect in the source, receiver, or earth which converts
pulsive waveform to a little wavelet. Ideally, the cross section
h should display an impulse at the interface, but the migration
ries the wavelet from the time section into the depth section. In
ving power limitation on the time axis is converted to a resolving
on the depth axis. An interesting question is that of the resolving
zontal x axis. Waves propagating at an angle convert the resolving
velet to an angular axis rather than the vertical axis. Clearly, the
resolving power you could get would be from waves traveling
r 45° waves, horizontal resolving distance would be 1/sin 45° times
ertical resolving distance. This is illustrated in Fig. 11-7 and in

peration of migration can be understood in simple terms without
wave equation. Figures 11-8 and 11-9 illustrate the transforma-
ne and depth for the two special cases where one domain or the
ly an impulse function. Real data comprise a continuum in (x, t),

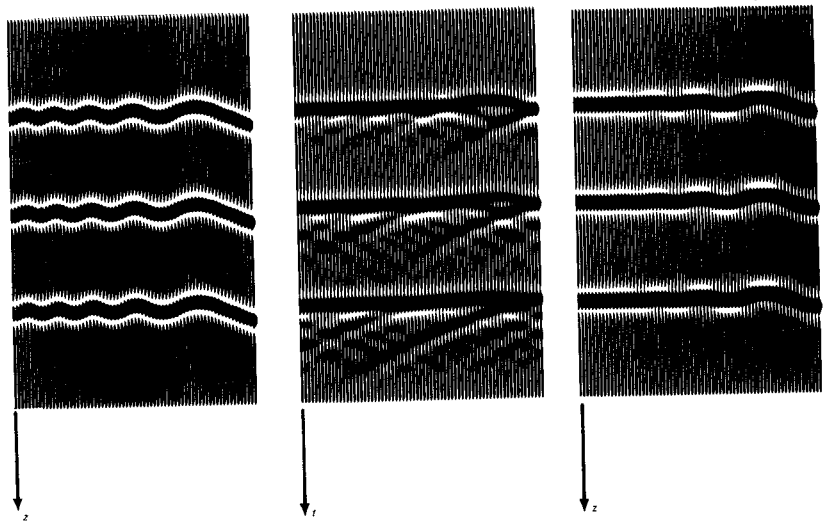


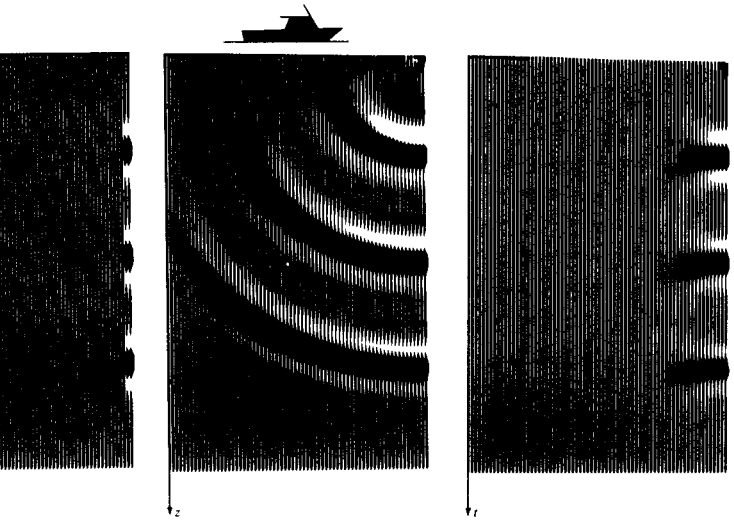
FIGURE 11-7
Reflections from oscillatory interfaces illustrating lateral resolving-power limitations. On the left is a starting model of some oscillatory interfaces. Center is the construction of a zero-offset section by the method of this section extended to the 45° approximation and modified by numerical viscosity to reject energy with dips beyond about 30°. The right-hand frame is the attempted reconstruction of starting model. It is no longer possible to resolve short-wavelength oscillations on the left-hand side of the interface. (From Ref. [8], p. 759.)

but Fig. 11-8 suggests a migration technique based on the linear superposition principle; namely, take every data point in the (x, t) domain and throw it out to a circular arc in the (z, t) domain. The migrated section is the superposition of all these arcs. A natural question one might ask is: Why bother to use the wave equation for migration when it can be done with the circular arcs? The answer to such a question depends upon a multitude of practical factors, some of which are data-dependent. One consideration favoring the wave equation approach is that velocity inhomogeneity is more easily and accurately described by wave equations than by ray tracing. Both methods are presently commercially available.

Figure 11-10 is an example which illustrates the origin of the term *migration.* It shows that a dipping interface which terminates at a point will appear in the time data to have the termination point *migrated* down-dip from its true location. In an example of this type, the departure of the depth section from the time data is really rather modest and it becomes even more slight as the dip is decreased.

This does not imply that data with slight dips need not be migrated because, as we have seen, of the dominating importance of curvature. An interesting practical example of this is shown in Fig. 11-11.

Field data examples of many of the synthetic data analyses are shown in the data of Fig. 11-12.



GURE 11-8
e depth response to time-domain impulses and reconstruction of the impulses. e fact that the left frame is mostly blank depicts a situation in which no echo eceived when a source and receiver move together in the horizontal direction il they reach the right-hand edge of the frame where the three blips indicate t there are three echoes at successively increasing times. With these as ob- ved data, the logical conclusion is that the reflection structure of the earth is e concentric circles with centers on the right-hand margin. The center frame ws the circles. (For economy, the right-hand edge of the frame is a plane of metry.) It will be noticed that the bottom of the circles is darker than the This is indicative of the 45° phase shift of bringing two-dimensional waves n a focus away from the focus. Waves with dips greater than about 45° have n filtered away by numerical viscosity. The loss of this energy plus the loss he energy of waves which propagate at complex angles results in a recon- ction (right-hand frame) in which the impulses are somewhat spread out in horizontal direction. (From Ref. [8], p. 750.)

ne explanation of how to migrate sections by recalling the basic (and replacing t' by t).

$$S(y, z, t) = \int Q(y, f, z, t)\, df \quad (11\text{-}1\text{-}13)$$

$$S_{zt} = \pm \frac{v}{8} S_{yy} \quad (11\text{-}1\text{-}14)$$

3) defines a sum over offset of all the waves. Equation (11-1-14) im S can be extrapolated down into the earth from the surface We recall that the plus sign or the minus sign is chosen according trapolation is done on the downgoing wave or the upgoing wave. rm $\delta(g - s, e, t)$ in (11-1-9). In midpoint-offset coordinates this mes $\delta(f)$ const$(y)\delta(z)\delta(t)$. Since the source term is a constant

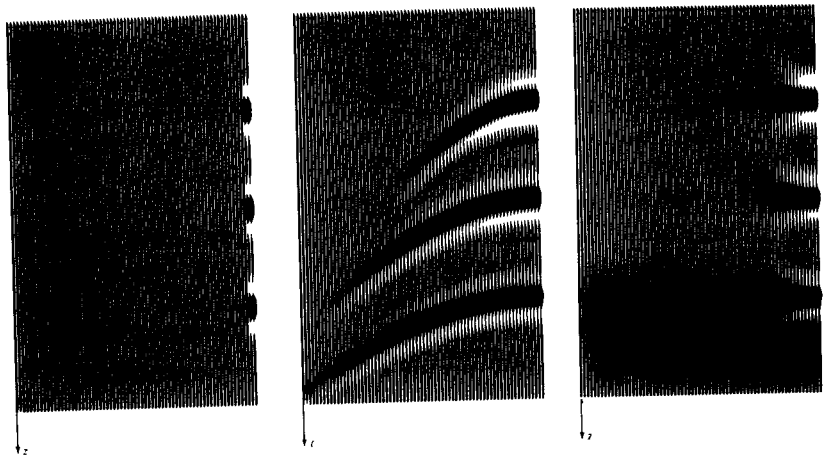


FIGURE 11-9
The time response to depth-domain impulses and reconstruction of the impulses. The left-hand frame depicts a model of the earth which consists of three point scatterers beneath one another along the right-hand edge. The second frame shows the synthetic time data created from the model. Basically one observes the hyperbolic travel-time curves to the reflecting points. The third frame represents migration of the synthetic data back to the point scatterers. As in Fig. 11-8 there is a reduced resolution because, in principle, horizontal resolution cannot be better than vertical resolution (which is controlled by the frequency content of the waves) and in practice we have included only rays up to angles of about 40°. (From Ref. [8], p. 751.)

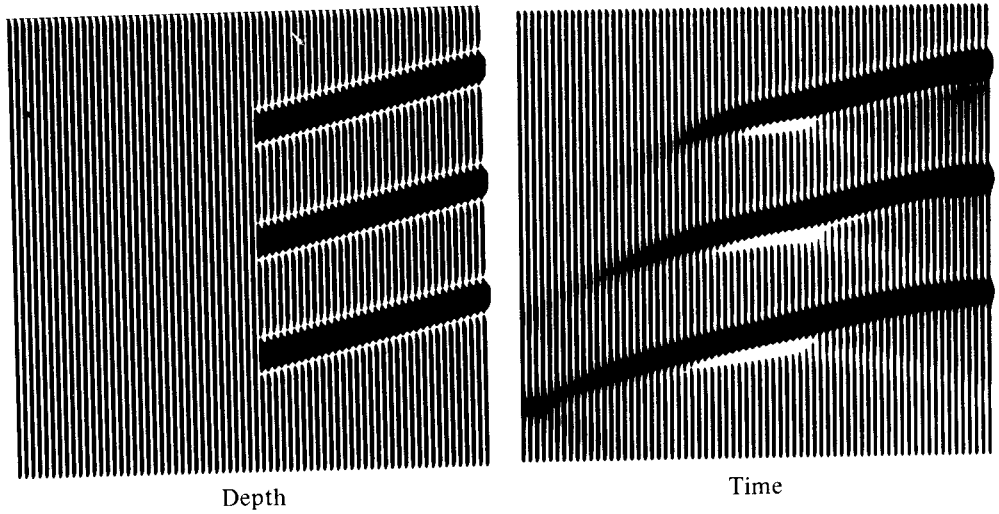


FIGURE 11-10
An illustration of the origin of the term *seismic migration*. Left is a depth section consisting of three terminating interfaces. On the right is the synthetic time data. Note that besides the broadening of the termination point (by spreading it into a hyperbola) there is a general *migration* of the termination "point" in the down dip direction. (From Ref. [8], p. 756.)

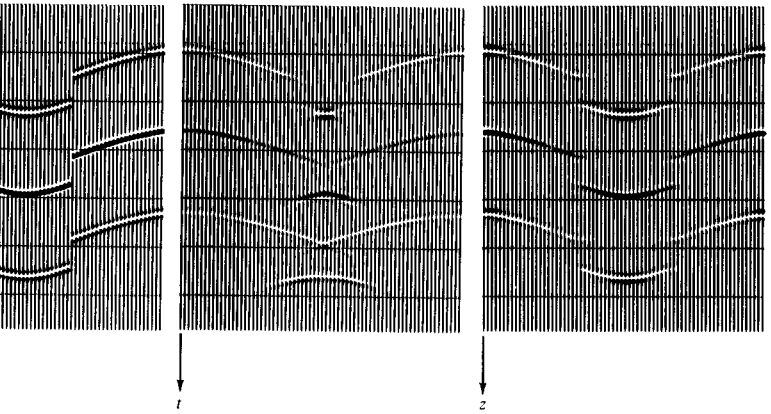
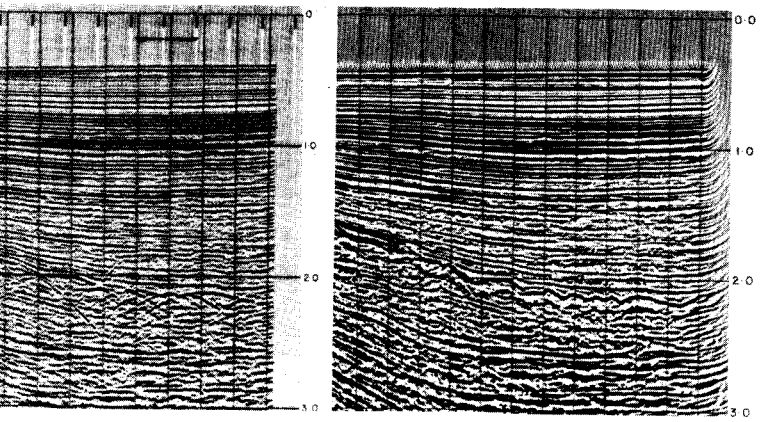


FIGURE 11-11
The classic graben model. Of practical significance is the fact that a concave structure appears convex in the time section. Thus, a geologic syncline may be confused with an anticline and the conditions for petroleum accumulation could be erroneously inferred. (From Ref. [8], p. 757.)

dpoint y, we can expect (11-1-14) for the downgoing wave to reduce e general solution to $S_{zt} = 0$ is an arbitrary function of z added to an on of t. Because we are interested in wave oscillations, not potential t out the arbitrary function of z. The remaining time-dependent is independent of z must be equal to the time dependence of the



IGURE 11-12
xample of migration of marine reflection seismic data. Left-hand frame data ourtesy of Royal Dutch Shell) were migrated with the wave equation (by igicon, Inc.) and the result is shown in the right-hand frame. A prominent regular reflector just below two seconds in the migrated section (right) is ghly diffracted on the surface data (left).

source function; that is, a delta function of time. Since we now have an analytic function for the downgoing S at all depths, we can next turn to the numerical downward extrapolation of the upcoming S. Equation (11-1-14) with the minus sign enables us to do the downward extrapolation of the upcoming wave. Because the downgoing wave is a plane wave, the upcoming wave, if seen down at the reflectors, will take on the shape of the reflectors. Thus, the migrated section comprises the surface observations extrapolated downward to the proper depth.

As a practical matter, it turns out that investigators do not presently use the vertical stack defined by (11-1-13). They inject a time shift as a function of offset before doing the summation. The main effect of this is to make all offset traces more closely resemble the zero offset trace. The time delay of waves recorded on a zero offset section is equally divided between the downgoing path and the upgoing path. The two paths have unequal delays when the downgoing wave is a plane wave and the upcoming wave represents complicated scattering. From these considerations, we can now guess a result from (11-3-18) which is that the migration equation for a zero offset section, or an NMO stack resembles (11-1-14) but contains an extra factor of 2. Thus, we begin with

$$Q_{zt} = -\frac{\bar{v}}{4} Q_{yy} \qquad (11\text{-}2\text{-}1)$$

There may be some practical justification in allowing $\bar{v}$ to be variable in some of the three coordinates (y, z, t); however, by keeping the velocity constant in y we can simplify our first encounter with the details of migration. Fourier transforming the y variation with e^{iky} we reduce (11-2-1) to

$$0 = \left(\frac{v}{4} k_y^2 - \partial_{tz}\right) Q \qquad (11\text{-}2\text{-}2)$$

Now we will discretize the z and t coordinates. Then the function Q in (11-2-2) may be tabulated in the (z, t) plane and the differential operator in (11-2-2) becomes a 2×2 convolution operator in this two-dimensional plane. The operator is

$$\frac{vk_y^2}{16}\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 1 & 1 \\ \hline \end{array} - \frac{1}{\Delta z\, \Delta t}\begin{array}{|c|c|}\hline 1 & -1 \\ \hline -1 & 1 \\ \hline \end{array} \qquad (11\text{-}2\text{-}3)$$

Defining a scale factor

$$a = \frac{\Delta z\, \Delta t\, vk_y^2}{16} \qquad (11\text{-}2\text{-}4)$$

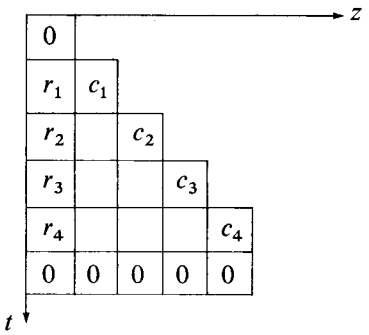
the operator multiplied by $\Delta z\, \Delta t$ is

$$\begin{array}{|c|c|}\hline (a-1) & (a+1) \\ \hline (a+1) & (a-1) \\ \hline \end{array} \qquad (11\text{-}2\text{-}5)$$

When the operator (11-2-5) is laid upon an arbitrary place in the Q table and the four numbers in the operator are multiplied onto the four Q numbers beneath the operator, the meaning of (11-2-2) is that the sum of these four products should vanish. If we find a place in the Q table where only three of the four numbers of Q

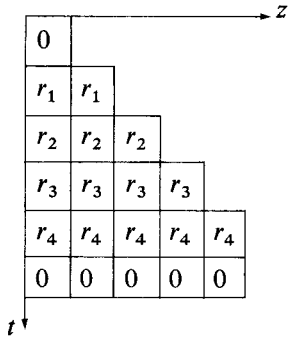
wn, then we can calculate the fourth, unknown number. In fact, only along a few boundaries in the (z, t) plane and we may be rest of the plane.

and its inverse may be thought of on the following grid.



(11-2-6)

$r_1, \ldots, r_4$ represents the observed surface seismogram and c_0, ents the migrated section. The zeros in the bottom row represent smograms vanish at a sufficiently late time. Notice that in filling e is a lot more work (diffraction) in going from r_4 to c_4 than in c_2. This is because for fixed dip, deep events migrate farther than Letting $k_y^2 = 0$, we have $a = 0$ and we are describing a stratified with surface data r_1, r_2, r_3, and r_4 and the bottom row of zeros -5) to fill in the table (11-2-6) and it becomes

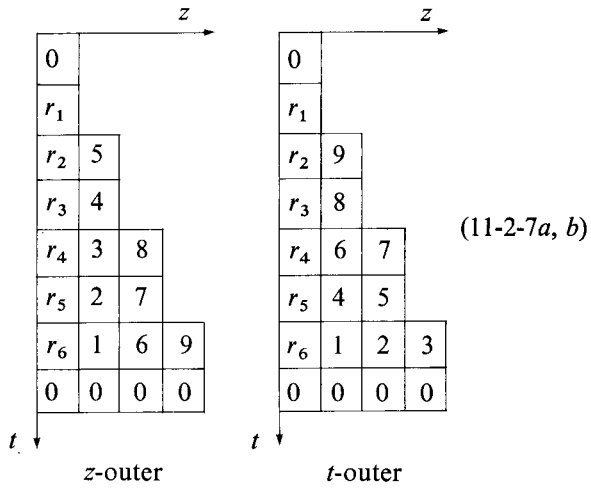


(11-2-7)

the table (11-2-7) we see that numbers remain absolutely constant e z direction. If we had not taken $k_y^2 = 0$ but had instead taken we would see the numbers in the table changing gradually in the en k_y^2 is not zero some caution must be exercised in the order in lane is filled up. The number a is always positive. Obviously, if ppens to equal $+1$, then it will be impossible to find unknown Q table if they should lie under the multiplier $1 - a$. It turns out of what numerical (positive) value a takes, the process of recur- mbers in Q which underlie $1 - a$ will be unstable as is polynomial

division by a nonminimum-phase filter. One reason is that such a process actually is polynomial division but the polynomials are two-dimensional polynomials. These complications are all the mathematical manifestations of the physical idea of causality.

From the point of view of solving hyperbolic differential equations it is most economical if you can get roughly the same number of points per wavelength (typically eight) on each coordinate axis. This means that in the development till now we have drastically over-sampled the z axis. The 15° limitation of the Fresnel approximation implies that Δz could always be taken five times or more coarser than $v\,\Delta t$. The subject of optimal selection of grid spacings is somewhat involved. Suffice it to say here that some field data have been migrated satisfactorily at a $\Delta z/v\,\Delta t$ ratio as great as 100 with a corresponding reduction in cost. Anyway, for the purpose of illustrating the point we will now redraw the section table with twice as coarse a sampling on the z axis. Numbers in the two tables below indicate two different possible orderings, both causal, of the use of (11-2-5) for the migration calculation. In either case the resulting migrated section is interpolated (perhaps very crudely) off the diagonal.



(11-2-7*a*, *b*)

The reader should check in each of the above tables that the number computed at any stage is based on three already known table entries and that the unknown being computed multiplies $(1 + a)$ and not $(1 - a)$. To synthesize data from a hypothetical depth section the calculation proceeds in reverse numerical order.

Study of the two optional procedures (11-2-7*a*) and (11-2-7*b*) reveals that the one labeled "t-outer" has an extraordinary advantage over the one labeled "z-outer" in the much smaller computer memory requirement.

Another practical reality is the need to be able to handle depth-variable velocities. This can be achieved by taking the migrated data $c_1, c_2, \ldots$, not from the diagonal in the (z, t) plane, but from another curve through the (z, t) plane as illustrated in Fig. 11-13. A nice feature of wave-equation migration is that velocity

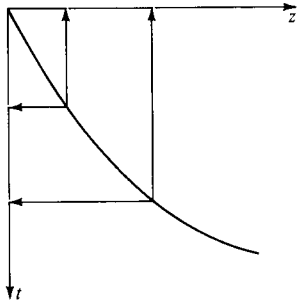


FIGURE 11-13
Plane for migration with depth-variable velocity. The curve in this plane is chosen to be the curve of two-way vertical travel time. The midpoint axis y comes out of the page. Section data along the curve may be projected to either the z axis or the t axis creating a migrated depth section or a migrated time section.

y be easily made without having to redo the calculation. Once the een filled, it is just a question of deciding which curve to display. e (z, t) plane tend to be small. This means that changes in the .e., the curve in Fig. 11-13, result in more exaggerated changes to ɔth section than to the migrated time section. For such reasons e section is often preferred by data interpreters over the migrated

or to consider is the possibility of lateral y variations in the velocity detailed analysis has not been given, it seems clear that simply iable velocity into (11-2-1) should give a valid technique if the change too rapidly in the y direction. How much change is too on which is undoubtedly data-dependent and beyond the scope forts. It should be noted, however, that if a y-dependent velocity n Fourier transformation over the y axis does not reduce (11-2-1) result of this, it is then necessary to regard every element in the s a scalar but as a vector whose different components arise from ns along the y axis. We then regard $1 + a$ not as a scalar divisor ɔnal matrix of the form $(I - \delta_{yy})$ which must be inverted. This cal difficulty at all and is, in fact, the way the figures in this section

TY ESTIMATION

s focused on the task of delineating earth structure. Mathemati- eant that we have taken the material velocity as known (and, for nstant) but the impedance as having unknown discontinuities at nown shape between geologic structures. Now we seek to find the . Traditionally this has been done by assuming the earth structure horizontal layers. Then the material velocity is deduced from the time shift (called the *normal moveout correction* or NMO) which events on the common midpoint gathers. In this section, it will be assumption of flat layers may be eliminated. We will see how

velocity can be estimated even in an earth consisting of random point scatterers. This can be expected to be useful in fractured zones or even perhaps in "no record" areas. An NR or no record area is one where the best-processed section shows no coherence along the midpoint y coordinate. An area may be NR not only because of poor data quality but also because the geologic structure itself has no continuity. But, as we will see, there is no theoretical reason why material velocity cannot be determined in such an NR area.

Basically, the procedure is to downward-continue both the theoretical downgoing wave and the observed upgoing wave. They are projected back down to the reflectors where their nearly constant ratio should represent the reflection coefficient as a function of offset. If they are projected downwards with an incorrect velocity, the ratio will be an oscillatory function of offset. The task, then, is to find the velocity which gives the best fit of the two waves. It does not matter whether the reflectors have any lateral continuity or not because the fitting is done for variable offset at a fixed midpoint at the reflector depth. When reflectors have no lateral continuity they may be called scatterers. An earth model with randomly located scatterers would produce migrated seismic data which was a random function of (moveout-corrected) time and midpoint but which was a constant function of offset.

It is easy to think of a good means to downward-continue the downgoing waves. From the shot point these waves expand spherically. For a homogeneous medium, we can just write down an analytic solution. For a moderately inhomogeneous medium, we can use the methods of earlier chapters. One problem is that the approximation $Q_{zz} \approx 0$ restricts validity to angles of about 15° from the vertical. This is easily improved by transforming from cartesian (x, z) coordinates to polar (r, θ) coordinates. The approximation $Q_{rr} \approx 0$ requires rays to stay within 15° of a radius line. Obviously a "stratified media coordinate frame" could be designed to handle even stronger velocity inhomogeneity of that type.

The problem which is more difficult is to find a good coordinate system for the upcoming waves. It took me two years to come up with a practical solution. A hint is provided by observing why, for the downgoing wave, the polar system is preferable to the cartesian system. For a quasi-spherical wave $Q_{\theta\theta}$ will be nearly 0, whereas Q_{xx} gets large quickly unless you are directly under the source. Because we deal with equations like $Q_{zt} = Q_{xx}$ or $Q_{rt} = Q_{\theta\theta}/r^2$, this means that Q_r will generally be small; whereas Q_z is small only on the z axis directly under the source. Consequently, the approximation $Q_{rr} \approx 0$ is much better than $Q_{zz} \approx 0$. Our observation is that the advantage of the (r, θ) coordinates is that the downgoing wave $D(r, \theta)$ is nearly independent of the lateral θ coordinate. What we need is a coordinate frame in which the upcoming wave U is nearly independent of the lateral coordinate. Experienced geophysicists will immediately recognize that normal moveout-corrected data fill this requirement. Normal moveout (NMO) correction is a compression of the time axis on far offset seismograms intended to make the far offset waves arrive at the same (NMO corrected) time as the vertically incident waves. Thus, the partial derivative of the wave field Q with respect to shot-geophone offset at a fixed NMO corrected time should be small.

The closer our data come to those from flat horizontal reflectors in an earth

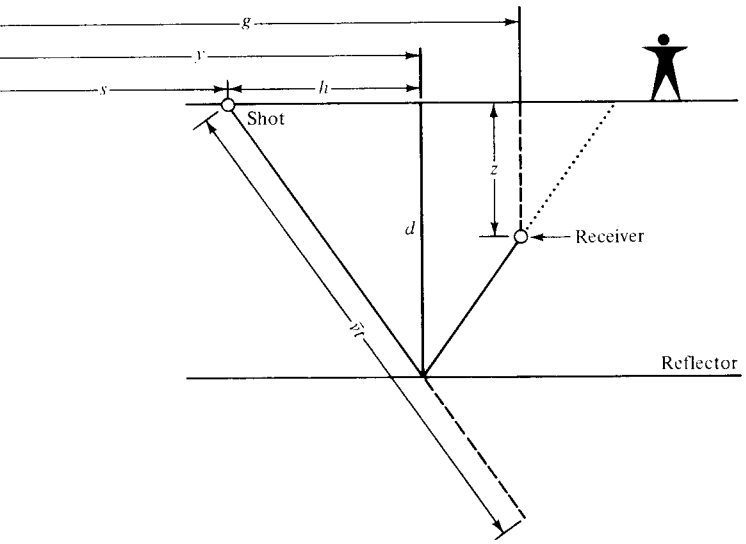


FIGURE 11-14
Geometry for normal-moveout correction of downward-continued data.

ty the smaller the offset derivative will be. The purpose of a wave andle the departure from such an idealized situation.

pression of the time axes of the far offset seismograms is really a ge. The usual definition of NMO correction does not anticipate oject our geophones deeply into the earth. As we project our geo- rd along a ray path we will retain the *surface midpoint* y and the $et\ h = f/2$ as lateral coordinates of the wave field. Lateral deriva- l data should vanish.

14 shows the geometry for normal moveout correction of downward- in homogeneous media of velocity $\bar{v}$. The transformation from ariables to observation variables is

$$s(h, y, d, z) = y - h \tag{11-3-1a}$$

$$g(h, y, d, z) = y + \frac{(d - z)h}{d} \tag{11-3-1b}$$

$$t(h, y, d, z) = \frac{(d^2 + h^2)^{1/2}(2d - z)}{d\bar{v}} \tag{11-3-1c}$$

or geometric means yield the inverse transformation

$$d(s, g, t, z) = \tfrac{1}{2}\{[\bar{v}^2t^2 - (g - s)^2]^{1/2} + z\} \tag{11-3-2a}$$

$$y(s, g, t, z) = \frac{1}{2}\left\{(g + s) + \frac{z(g - s)}{[\bar{v}^2t^2 - (g - s)^2]^{1/2}}\right\} \tag{11-3-2b}$$

$$h(s, g, t, z) = \frac{1}{2}\left\{(g - s) + \frac{z(g - s)}{[\bar{v}^2t^2 - (g - s)^2]^{1/2}}\right\} \tag{11-3-2c}$$

That (11-3-2) is indeed inverse to (11-3-1) is readily checked by substituting (11-3-1) into (11-3-2).

In a homogeneous medium of velocity $\tilde{v}$ we may write the solution for the downgoing wave as a delta function on an expanding circle

$$D(g, s, t, z) = \delta[(g - s)^2 + z^2 - \tilde{v}^2t^2] \tag{11-3-3}$$

The upcoming wave U will be computed in the (h, y, d, z) variables and we want to compare it to the downgoing wave D, expressed by (11-3-3) in (g, s, t, z) variables. We can convert D to (h, y, d, z) variables by substitution of (11-3-1) into (11-3-3); a meaningful simplification arises if we assume the medium velocity $\tilde{v}$ equals the moveout coordinate frame velocity $\bar{v}$. We get

$$D(h, y, d, z) = \delta[4d(z - d)] \tag{11-3-4}$$

In this case, the downgoing wave turns out to be independent of the lateral coordinates h and y.

Now let us consider an earth model which contains only a single point scatterer located at (x_0, z_0). This scatterer is illuminated by a delta function source located at $(s, 0)$. Excluding horizontally propagating waves, we have for the upcoming wave $U(s, g, t, z)$ an infinitesimal distance above the scatterer

$$U(s, g, t, z_0 - 0) = \delta(g - x_0)\delta[\bar{v}^2t^2 - (s - x_0)^2 - {z_0}^2] \tag{11-3-5}$$

Substituting the transformation (11-3-1) at $z = d$ into (11-3-5), we obtain

$$U(h, y, d, z = d) = \delta(y - x_0)\delta[d^2 + h^2 - (y - h - x_0)^2 - {z_0}^2]$$

The existence of $\delta(y - x_0)$ allows us to set $y = x_0$ in the other delta function, getting

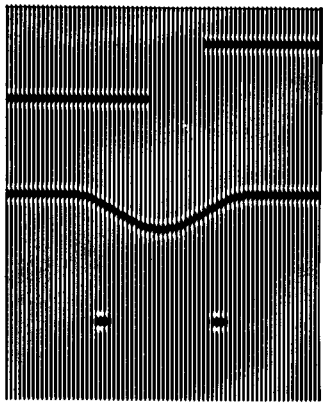
$$U(h, y, d, z = d) = \delta(y - x_0)\delta(d^2 - {z_0}^2)$$

We now see the central concept that the wave at the reflector in moveout corrected coordinates is indeed independent of the half-offset h. Obviously, the superposition of a random collection of point scatterers will create a migrated wave field which is random in y and d but still constant in the offset h. Indeed, the concept would also seem to be valid even if the scatterers were randomly distributed out of the plane of the section. In three-dimensional space it is only necessary to regard z as the radial distance from the traverse line.

The purpose of all this is to estimate velocity; but velocity is needed for the first step, namely the migration. Use of an erroneous velocity in the migration prevents total collapse to a delta function on the midpoint axis. This causes some destructive interference between adjoining midpoints representing some information loss for a random scatterer model but it is of no consequence in a layered earth model (where even the migration itself is unnecessary).

Stephen M. Doherty [Ref. 37] made a calculation to illustrate these concepts. Figure 11-15 shows an earth model. Figure 11-16 shows surface data and downward-continued data for the model.

From the point of view of velocity determination, it is immaterial what coordinate frame is used to downward-continue the observed waveforms. However,



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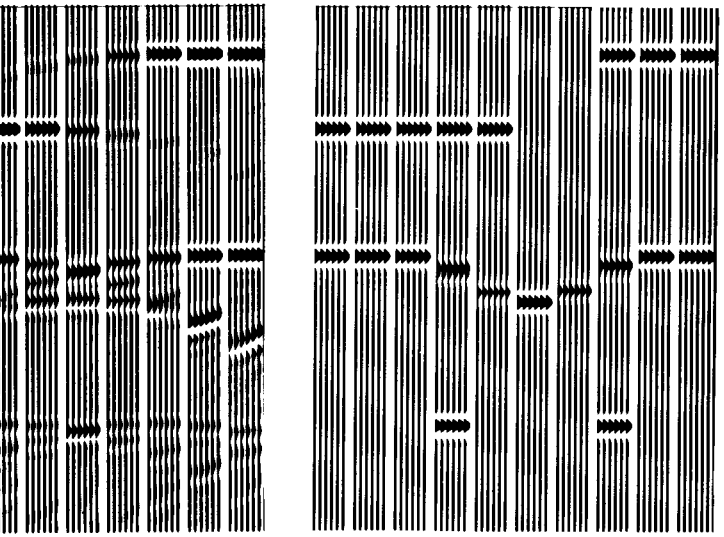


URE 11-16
ace data and downward-continued data for the model of Fig. 11-15. The
dinates are designed to display three-dimensional data (y, h, d) on a two-
nsional page. The vertical axis, as usual, is the d coordinate. For the hori-
al axis the h coordinate has been sampled at six values of h which are displayed
her in groups (common midpoint gathers). There are ten of these gathers
ed along the y axis. Within each group $h = 0$ is on the left and h_{max}, cor-
onding to about 45° rays, is on the right. The left-hand frame shows the
ce data and the right-hand frame shows the data down at the reflectors.
he reflectors we see horizontal alignment of waveforms indicating that the
are independent of offset h.

it is convenient to downward-continue these waves in the NMO coordinate frame. This proceeds in a fashion similar to that in our earlier work. To simplify the algebra, first note that (11-3-2*b*) and (11-3-2*c*) imply that

$$\frac{\partial y}{\partial(g, t, z)} = \frac{\partial h}{\partial(g, t, z)} \tag{11-3-6}$$

The wave equation

$$\left(\partial_{gg} + \partial_{zz} - \frac{\partial_{tt}}{\tilde{v}^2}\right)P = 0 \tag{11-3-7}$$

in NMO coordinates will take the form

$$\left[(d_g\partial_d + y_g\partial_y + h_g\partial_h)^2 + (\partial_z + d_z\partial_d + y_z\partial_y + h_z\partial_h)^2 - \frac{(d_t\partial_d + y_t\partial_y + h_t\partial_h)^2}{\tilde{v}^2}\right]Q = 0 \tag{11-3-8}$$

As before, when we square these partial differential operators we will take the coefficients to be constant. This is the high frequency assumption that the wave field changes more rapidly than the coordinate frame. Before we compute all the required derivatives we define a simplifying combination b where

$$b = [\bar{v}^2t^2 - (g - s)^2]^{1/2} \tag{11-3-9}$$

The required derivatives are computed, recalling (11-3-6) to be

$$\begin{bmatrix} d_g & d_z & d_t \\ y_g & y_z & y_t \\ h_g & h_z & h_t \end{bmatrix} = \tfrac{1}{2}\begin{bmatrix} \dfrac{-(g-s)}{b} & 1 & \dfrac{\bar{v}^2t}{b} \\ 1 + \dfrac{z\bar{v}^2t^2}{b^3} & \dfrac{g-s}{b} & \dfrac{-(g-s)\bar{v}^2tz}{b^3} \\ 1 + \dfrac{z\bar{v}^2t^2}{b^3} & \dfrac{g-s}{b} & \dfrac{-(g-s)\bar{v}^2tz}{b^3} \end{bmatrix} \tag{11-3-10}$$

First we quickly discover that if moveout-correction velocity $\bar{v}$ equals media velocity $\tilde{v}$, say $\tilde{v} = \bar{v} = v$, then three of the terms in (11-3-8) vanish identically. By direct substitution, the reader may verify that

$$\left(d_g^2 + d_z^2 - \frac{d_t^2}{v^2}\right)Q_{dd} = 0 \tag{11-3-11a}$$

$$2\left(d_gy_g + d_zy_z - \frac{d_ty_t}{v^2}\right)Q_{dy} = 0 \tag{11-3-11b}$$

$$2\left(d_gh_g + d_zh_z - \frac{d_th_t}{v^2}\right)Q_{dh} = 0 \tag{11-3-11c}$$

n the three cross terms with ∂_z.

$$2y_z\, Q_{yz} = \frac{g-s}{bQ_{yz}} = \frac{h}{dQ_{yz}} \quad (11\text{-}3\text{-}12a)$$

$$2h_z\, Q_{hz} = \frac{g-s}{bQ_{hz}} = \frac{h}{dQ_{hz}} \quad (11\text{-}3\text{-}12b)$$

$$2d_z\, Q_{dz} = Q_{dz} \quad (11\text{-}3\text{-}12c)$$

we realize that the coefficients of Q_{yy}, Q_{hh}, and $2Q_{yh}$ are identical. siderable amount of algebraic reduction we obtain

$$^2 - \frac{y_t^2}{v^2}\Big)(\partial_y + \partial_h)^2 Q = -\left(\frac{d}{2d-z}\right)^2\left(1+\frac{h^2}{d^2}\right)(\partial_y + \partial_h)^2 Q \quad (11\text{-}3\text{-}13)$$

nake the Fresnel-like approximation by dropping the Q_{zz} term. In etry, this limits accurate treatment of rays to within a cone of about ical. In the NMO geometry, this would seem to be more like a 15° tructural dips. Of course, the higher-accuracy techniques can always required. Gathering (11-3-11) to (11-3-13) together, we obtain the

$$\left[\partial_d + \frac{h}{d}(\partial_y + \partial_h)\right]\partial_z Q = \left(\frac{d}{2d-z}\right)^2\left(1+\frac{h^2}{d^2}\right)(\partial_y + \partial_h)^2 Q \quad (11\text{-}3\text{-}14)$$

3-14) may be used for downward continuation of moveout-corrected ions for velocity determination.

worthwhile to inspect (11-3-14) in some special cases. At the surface Q_h vanishes by symmetry. At a point scatterer we saw that Q was f h. For idealized data from layered reflectors Q is a function of d e variety of practical situations it turns out to be reasonable to simplify $Q_d \gg Q_y \gg Q_h$. This leaves us with

$$Q_{dz} = -\left(\frac{d}{2d-z}\right)^2\left(1+\frac{h^2}{d^2}\right)Q_{yy} \quad (11\text{-}3\text{-}15)$$

al to wonder about the variable coefficient $d/(2d-z)$ in comparison equations with constant coefficients. We can now show that with ration that there is no practical difference. Define a new variable

$$z' = \frac{zd}{2d-z} \quad (11\text{-}3\text{-}16)$$

he surface $z = 0$ we have z' equal to zero and at the reflectors $z = d$ d. Thinking of $Q(z, d) = Q'(z', d)$ we find

$$Q_z = z'_z\,\partial_{z'}\, Q'$$
$$Q_d = (\partial_d + z'_d\,\partial_{z'})Q'$$

With these, the left-hand side of (11-3-15) becomes

$$Q_{dz} = (\partial_d - z'_d\,\partial_{z'})z'_z\,\partial_{z'}\, Q$$

In a Fresnel-like approximation, we drop $\partial_{z'z'}$ obtaining

$$Q_{dz} = z'_z\, Q_{dz'} = \frac{2d^2}{(2d-z)^2}\, Q_{dz'}$$

which reduces (11-3-15) to

$$Q_{dz'} = -\frac{1}{2}\left(1+\frac{h^2}{d^2}\right)Q_{yy} \quad (11\text{-}3\text{-}17)$$

To justify the factor of 2 which was asserted in Sec. 11-2, we may make another transformation from d to a two-way travel-time coordinate t' given by

$$t' = \frac{2d}{v}$$

which gives

$$Q_{t'z'} = -\frac{v}{4}\left[1+\left(\frac{2h}{vt'}\right)^2\right]Q_{yy} \quad (11\text{-}3\text{-}18)$$

Of course (11-3-18) must be integrated from $z' = 0$ to $z' = t'v'/2$. A convenient rescaling of the depth axis is in terms of two-way travel time t'' where

$$t'' = \frac{2z'}{v}$$

This leads to the equation

$$Q_{t't''} = -\frac{v^2}{8}\left[1+\left(\frac{2h}{vt'}\right)^2\right]Q_{yy} \quad (11\text{-}3\text{-}19)$$

in which t' is the two-way travel time and t'' is the two-way travel-time depth axis which is integrated from the surface $t'' = 0$ to the reflectors at $t'' = t'$.

Strictly speaking, (11-3-19) should be applied separately to data of each offset h before the data are summed over offset (stacked). For reasons of economy, the data are often stacked before migration with (11-3-19). In such a compromise, h in (11-3-19) is taken as zero or some average value of $2h/vt'$ is used.

So far, we have shown that downward-continued, moveout-corrected seismograms will be independent of offset if downward continued with the correct velocity. What we have not seen is how to estimate the velocity error from the downward-continued data. For this we must recognize another important term which has been omitted from the entire analysis. We saw this term in earlier studies of propagation in inhomogeneous media. We must carry through the distinction between media velocity $\tilde{v}(x, z)$ and NMO velocity $\bar{v}$ [generalizable to $\bar{v}(z)$] which was abandoned for the sake of the simplifications beginning at (11-3-11). Recalling that for small departures from layered models, $Q_d \gg Q_y \gg Q_h$, we see

he three terms in (11-3-11) will be the most important. Making the een the two velocities, (11-3-11*a*) now introduces the significant

$$\left(d_g^2 + d_z^2 - \frac{d_t^2}{\tilde{v}^2}\right) Q_{dd} \neq 0 \quad (11\text{-}3\text{-}20)$$

$$\frac{\bar{v}^2 t^2}{\bar{v}^2 t^2 - (g-s)^2}\left(1 - \frac{\tilde{v}^2}{\bar{v}^2}\right) Q_{dd} \neq 0$$

$$\left(1 + \frac{h^2}{d^2}\right)\left(1 - \frac{\tilde{v}^2}{\bar{v}^2}\right) Q_{dd} \neq 0 \quad (11\text{-}3\text{-}21)$$

new term but the other approximations, (11-3-15) becomes

$$_{dz} = -\left(\frac{d}{2d-z}\right)^2 \left(1 + \frac{h^2}{d^2}\right) Q_{yy} + \left(1 + \frac{h^2}{d^2}\right)\left(1 + \frac{\tilde{v}^2}{\bar{v}^2}\right) Q_{dd} \quad (11\text{-}3\text{-}22)$$

can consider solving (11-3-22) by a splitting method where the cted downward by alternate use of the two equations

$$Q_{dz} = -\left(\frac{d}{2d-z}\right)^2 \left(1 + \frac{h^2}{d^2}\right) Q_{yy} \quad (11\text{-}3\text{-}23a)$$

$$Q_z = \left(1 + \frac{h^2}{d^2}\right)\left(1 - \frac{\tilde{v}^2}{\bar{v}^2}\right) Q_d \quad (11\text{-}3\text{-}23b)$$

11-3-23*a*) may be called the "diffraction" part and (11-3-23*b*) may in-lens" part. The effect of (11-3-23*b*) is that as Q is projected in each seismogram [a seismogram is a function of (moveout-cor- a fixed half-offset h and midpoint y] undergoes a steady time shift mount of the shift increases with the velocity error according to increases with offset according to $1 + h^2/d^2$. Thus, the effect of change the curvature of the data with half-offset h. However, ins Q_{yy} but it does not contain Q_h or Q_{hh}. This means that the -3-23*a*) and (11-3-23*b*) commute. Thus, we can project all the way lectors with (11-3-23*a*) and then use (11-3-23*b*). It is significant rt of the job, namely (11-3-23*a*), depends on the frame velocity $\bar{v}$, l velocity $\tilde{v}$. This means that we can rather economically test elocities $\tilde{v}$.

can consider the task of selecting our best estimate of the media nust consider matching the upcoming wave U to some reflection tiplied by some downgoing wave D. The matching of these waves he field-recording coordinates, but we prefer to do the matching in

the NMO coordinate system. First, let us get an expression for the downgoing wave in NMO coordinates. Insert (11-3-1) into (11-3-3) to obtain

$$D(h, y, d, z) = \delta\left[4d(z-d) + \left(1 - \frac{\tilde{v}^2}{\bar{v}^2}\right)\left(1 + \frac{h^2}{d^2}\right)(2d-z)^2\right] \quad (11\text{-}3\text{-}24)$$

At present, we are not trying to preserve slow magnitude variations [spherical spreading was omitted from (11-3-3)], so we can divide through the argument of the delta function by $-4d$. Since we are interested in small amounts of variation of $\tilde{v}$ from $\bar{v}$ the delta function will vanish very near to $z = d$. Thus, to a good approximation we can substitute z for d in the coefficient of $(1 - \tilde{v}^2/\bar{v}^2)$, obtaining

$$D(h, y, d, z) = \delta\left[d - z - \left(1 - \frac{\tilde{v}^2}{\bar{v}^2}\right)\left(1 + \frac{h^2}{z^2}\right)\frac{z}{4}\right]$$

$$= \delta(d - z - s) \quad (11\text{-}3\text{-}25)$$

where we have defined a time (d) shift function

$$s(h^2, z, \tilde{v}/\bar{v}) = \left(1 - \frac{\tilde{v}^2}{\bar{v}^2}\right)\left(1 + \frac{h^2}{z^2}\right)\frac{z}{4} \quad (11\text{-}3\text{-}26)$$

Now let us return to the task of matching the up- and downgoing wave. We might hope to determine a reflection coefficient, along with some angular dependence, in the form of a power series, for example:

$$c = c_0 + \frac{c_1 h}{z} + \frac{c_2 h^2}{z^2} + \cdots \quad (11\text{-}3\text{-}27)$$

To simplify the sequel we will estimate only the constant term c_0 by the minimization

$$\min_c \sum_h \sum_d [U(y, z, h, d) - c(y, z)D(y, z, h, d)]^2 \quad (11\text{-}3\text{-}28)$$

The solution is obviously

$$c(y, z) = \frac{\sum_h \sum_d UD}{\sum_h \sum_d D^2} \quad (11\text{-}3\text{-}29)$$

Because D vanishes almost everywhere, we can gain insight by replacing the double sum by a single sum, specifically for the numerator

$$\text{Numerator} = \sum_h \sum_d U(y, z, h, d)\,\delta(d - z - s)$$

$$= \sum_h U[y, z, h, d = z + s(h^2, z, \tilde{v}/\bar{v})] \quad (11\text{-}3\text{-}30)$$

Letting N denote the number of terms in the offset sum, we get for (11-3-29)

$$c(y, z, \tilde{v}) = \frac{1}{N}\sum_h U \quad (11\text{-}3\text{-}31)$$

come to the part of determining the velocity $\tilde{v}$ which provides the f $U - cD$. For this, a computer scan over $\tilde{v}$ may be used to find

$$\min_{\tilde{v}} \sum_h \sum_d (U - cD)^2 = \min_{\tilde{v}} \sum_h (U - c)^2$$
$$= \min_{\tilde{v}} \sum_h^N \left(U - \frac{1}{N}\sum_h^N U\right)^2$$
$$= \min_{\tilde{v}} \left(\sum_h U^2\right) - \frac{1}{N}(\sum U)^2 \geq 0 \quad (11\text{-}3\text{-}32)$$

ound that rather than minimize the sum squared minus the squared ble to maximize the negative logarithm or the semblance ratio

$$\text{Semblance}_{\tilde{v}} = \frac{(\sum U)^2}{N \sum U^2} \leq 1 \quad (11\text{-}3\text{-}33)$$

e advantage of being insensitive to the magnitude of the wave U vell to displays over a wide range of conditions.

PLE REFLECTIONS

ng of multiple reflections makes it possible to subtract theoretical field data, thereby uncovering the more informative, primary will first review a simplified layered model for multiple reflections it for application to a two-dimensionally inhomogeneous earth. e both forward and inverse problems. The forward problem is mensional spatial distribution of reflection coefficients to find the ling diffracted multiples with peglegs. The inverse problem is to dimensional spatial distribution of reflection coefficients from the

dea that we use for the inverse problem is that "reflectors exist at th where the first arrival of a downgoing wave is time-coincident ng wave." As a practical matter, we try to choose reflection co- ensure that the upcoming wave vanishes before the onset of the . In Chap. 8 we learned that, for a layered medium, the Z trans- for the downgoing wave $D(Z)$ is minimum-phase. This means f $D(Z)$, namely $1/D(Z)$, can be expanded into positive powers of Z fficient will be $1/d_0$. Consequently, in the nth layer the reflection bottom of the layer is the coefficient of the lowest power of Z in $U^n(Z)/D^n(Z)$. Our plan is to show that $U^n(Z)/D^n(Z)$ is observable = 0, and then show how U^{n+1}/D^{n+1} can be easily computed from all, pressure P and vertical velocity W must be taken as known at l time.

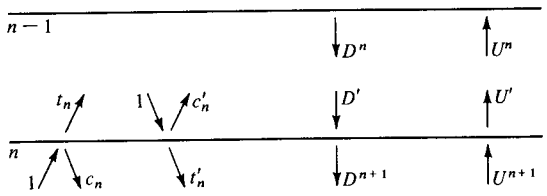


FIGURE 11-17
The defining conventions for waves in layers being scattered at the nth interface.

From Sec. 9-3 and equations (9-3-12a) and (9-3-12b), we have

$$\begin{bmatrix} U \\ D \end{bmatrix} = \tfrac{1}{2}\begin{bmatrix} 1 & -\dfrac{1}{Y} \\ 1 & \dfrac{1}{Y} \end{bmatrix}\begin{bmatrix} P \\ W \end{bmatrix} \quad (11\text{-}4\text{-}1)$$

and

$$\begin{bmatrix} P \\ W \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -Y & Y \end{bmatrix}\begin{bmatrix} U \\ D \end{bmatrix} \quad (11\text{-}4\text{-}2)$$

In (11-4-1) and (11-4-2) we think of the surface $n = 0$ pressure $P^0(Z) = p_0$ as a delta function at time $t = 0$, which is an instantaneous violation of the surface boundary condition $P^0 = 0$ in order to introduce energy into the medium. This causes a delta function behavior in W^0 at $t = 0$; and, later on, the reflectors in the earth cause W^0 to be nonvanishing while P^0 is vanishing. To apply our present method, we will require $U^0 = \frac{1}{2}(P - YW)$ which vanishes until the first echo and then becomes the negative of the observed returning waveform W and $D^0 = \frac{1}{2}(P + YW)$ which is a delta function at $t = 0$ followed by the observed waveform.

Figure 11-17 will recall the conventions of Chap. 8 for scattering at an interface. For propagation across the layer, we have the obvious delays which become multiplication by the half-unit delay operator $Z^{1/2} = \exp(i\omega\,\Delta t/2) = \exp(i\omega\,\Delta z/2v)$.

$$D' = Z^{1/2} D^n \quad (11\text{-}4\text{-}3)$$
$$U^n = Z^{1/2} U' \quad (11\text{-}4\text{-}4)$$

We have the scattering of downgoing waves into upgoing waves at the interface

$$U' = t_n U^{n+1} + c'_n D' \quad (11\text{-}4\text{-}5)$$

Now for the theory to be perfectly general, as in Chap. 8, we would write an equation converse to (11-4-5) which states that downgoing waves are augmented by scattering from upgoing waves. However, to achieve simplicity in our first exposure to diffracted multiple reflections, we will assume that downgoing waves are generated from upcoming waves only at the surface $n = 0$. Everywhere else in the earth we will suppose that downgoing waves go along their merry way without any contributions from upcoming waves. With this presumption, we write

$$D^{n+1} = t'_n D' \quad (11\text{-}4\text{-}6)$$

cal matter, the assumption we have built into (11-4-6) is not a bad
ata where the free-surface reflector is by far the strongest reflector.
e range of reflection coefficients is found in practice, a common
or a sea-floor coefficient of .1 and subsurface coefficients of .01.
it is often important to account for a lot of absorption and scat-
r-surface layers.
liminating the D' and U' variables from (11-4-3) to (11-4-6), we

$$D^{n+1} = t' Z^{1/2} D^n \tag{11-4-7}$$

$$U^{n+1} = \frac{U' - c_n' D'}{t_n} = \frac{Z^{-1/2} U^n + c_n Z^{1/2} D^n}{t_n} \tag{11-4-8}$$

) by (11-4-7), we achieve a simple relationship for the downward
U/D. It is

$$\frac{U^{n+1}}{D^{n+1}} = \frac{Z^{-1} U^n / D^n + c_n}{t_n' t_n} \tag{11-4-9}$$

his simple recurrence relationship we must recall that the coeffi-
D^n vanishes (see Fig. 11-17) and that the coefficient of Z in U^n/D^n
us, the Z^{-1} multiplier in (11-4-9) shifts U^n/D^n to have $-c_n$ as the
and then the c_n in (11-4-9) extinguishes it. Clearly, the downward
o simple that the results can almost be seen as the coefficients of
$U^0(Z)/D^0(Z)$. The only complicating factor is the divisor $t_n' t_n$
nonotonically increasing scale factor of $1/\prod_{k=1}^{n} (1 - c_k^2)$ to the
in $U^0(Z)/D^0(Z)$.
see how we can generalize these layered-model ideas to a two-
nomogeneous earth. The big difference is that upward and down-
n of waveforms can no longer be accomplished simply by the delay
could use (10-5-16) for upward and downward continuation;
s more natural and certainly more economical to use (10-5-17*a*)
ich are

$$D'_{z't'} = \frac{\bar{v}}{2} D'_{x'x'} \tag{11-4-10}$$

$$U''_{z''t''} = -\frac{\bar{v}}{2} U''_{x''x''} - c'(z'')\partial_{t''} D'\left(x'', z'', t'' - \frac{2z''}{v''}\right) \tag{11-4-11}$$

10) and (11-4-11) may be compared with (11-4-7) and (11-4-8).
t transmission coefficients are not accounted for in (11-4-10) and
s the sort of thing which is usually unimportant in the analysis of
ich can be recovered by a more exhaustive derivation if necessary.

First, we will consider the forward problem. It is easier to understand in a finite difference notation than in the differential equation system (11-4-10) and (11-4-11). Let us define a matrix T which is a tridiagonal matrix because of the second-difference operator δ_{xx}

$$T = -\left(\frac{\bar{v}\,\Delta z\,\Delta t}{8\,\Delta x^2}\right)\delta_{xx} \tag{11-4-12}$$

Then (11-4-10) with the boundary condition that D vanishes before $t' = 0$ can be written in tabular form

$T+I$	$T-I$
$T-I$	$(T+I)$

$*$

y \ z					
	0	0	0	0	0
	d_0^0	d_0^1	d_0^2	d_0^3	d_0^4
	d_1^0	d_1^1	d_1^2	d_1^3	
	d_2^0	d_2^1	d_2^2		
	d_3^0	d_3^1			
t'	d_4^0				

$= 0$ (11-4-13)

where $*$ denotes convolution in the (z, t') plane. This table can be filled in by the knowledge of the zeros in the top row and the surface observations in the left column. We will do the forward problem by illustrating a typical step. Suppose both U and D are known for all z before $t = 4$ but not at or after $t = 4$. The typical step we will show is how to get U and D at $t = 4$. That means we know everything entered in the D table (11-4-13) except for d_4^0. The first step is to take D from the t' coordinate in (11-4-13) to the t'' coordinate. If $\Delta z = \bar{v}\,\Delta t/2$, this is achieved by downshifting successive columns in (11-4-13) to

z					
	0	0	0	0	0
	d_0^0	0	0	0	0
	d_1^0	d_0^1	0	0	0
	d_2^0	d_1^1	d_0^2	0	0
	d_3^0	d_2^1	d_1^2	d_0^3	0
t''	d_4^0	d_3^1	d_2^2	d_1^3	d_0^4

(11-4-14)

e t'' coordinate is that if reflection occurs, all elements on a given
" can contribute to the upcoming wave received at a surface
uation (11-4-11) may be rearranged to

$$\left(\frac{\bar{v}}{2}\,\partial_{xx} + \partial_{zt}\right)(-U) = c'(x, z)\partial_t D \quad (11\text{-}4\text{-}15)$$

essed in tabular form as

0	0	0	0	0
$d_1{}^0$	$-u_1{}^1$	0	0	0
$d_2{}^0$	$-u_2{}^1$	$-u_2{}^2$	0	0
$d_3{}^0$	$-u_3{}^1$	$-u_3{}^2$	$-u_3{}^3$	0

$*$

$$= \frac{\Delta z}{2}$$

-1	-1
1	1

$*$

$d_0{}^0$	0	0	0	0
$d_1{}^0$	$d_0{}^1$	0	0	0
$d_2{}^0$	$d_1{}^1$	$d_0{}^2$	0	0
$d_3{}^0$	$d_2{}^1$	$d_1{}^2$	$d_0{}^3$	0
	$d_3{}^1$	$d_2{}^2$	$d_1{}^3$	$d_0{}^4$

$*$

0	c_1'	c_2'	c_3'	c_4'

(11-4-16)

ation (11-4-16) the unknown elements have been left blank. On
e the reflection coefficients are to be convolved upward into the
o create the sources for the upgoing wave. The c_0 entry in the
nt row is set at zero because we will prescribe the boundary con-
separately. Thus, we see the right-hand side of (11-4-16) is
, and the unknowns in the left-hand side may be filled in from
by computing the upcoming wave table at $t'' = 4$ for all z. This
us to go back and fill in another row in the downgoing wave
s us to fill another row in the upgoing wave table, ad infinitum.
calculation proceeds in a similar fashion. Suppose $d_t{}^0$ is known
sh to calculate c_1', c_2', etc., in a recursive fashion. It is sufficient

to show how to compute c_1'. Suppose we skim off the two left-hand columns of (11-4-16). We get

Cross multiply ↓

0	c_1'

$*$

g	e_0
g	e_1
g	e_2
g	e_3
$\vdots$	$\vdots$

$*$

$T-I$	$T+I$
$T+I$	$T-I$

↑ Convolve up

$=$

g	0
g	g
g	g
g	g
g	g

$*$

$$\frac{\Delta z}{2}$$

-1	-1
1	1

(11-4-17)

In (11-4-17) all the boxes filled by g on the left are given. The boxes with g on the right are readily computable as before. If c_1' were known, it would be a straightforward task to compute successively $\ldots, e_3, e_2, e_1, e_0$. We would be compelled to initialize the computation with an approximation such as $e_N = 0$ for some large N. If the correct value of c_1' had been used, then we should find that e_0 vanishes. Since we do not know what value of c_1' to use, we try $c_1' = +1$ obtaining e_0^+ and we try $c_1' = -1$ obtaining e_0^-. The correct value of c_1' is the appropriately weighted linear combination

$$0 = \alpha e_0^+ + \beta e_0^- \quad (11\text{-}4\text{-}18)$$

where

$$1 = \alpha + \beta \quad (11\text{-}4\text{-}19)$$

$$c_1' = \alpha - \beta \quad (11\text{-}4\text{-}20)$$

which inverts to

$$2\alpha = 1 + c_1' \quad (11\text{-}4\text{-}21)$$

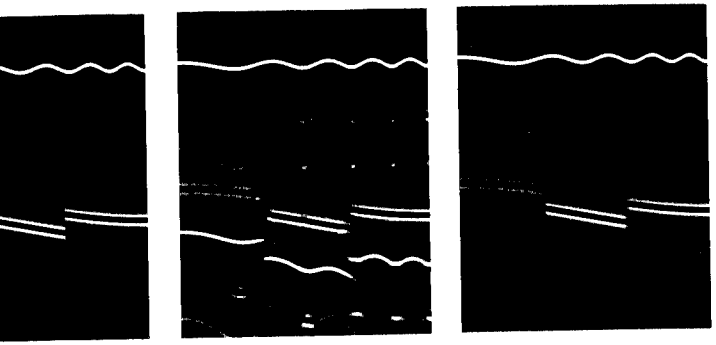
$$2\beta = 1 - c_1' \quad (11\text{-}4\text{-}22)$$

reducing (11-4-18) to

$$0 = (1 + c_1')e_0^+ + (1 - c_1')e_0^-$$

or

$$c_1' = \frac{e_0^+ + e_0^-}{e_0^+ - e_0^-} \quad (11\text{-}4\text{-}23)$$



URE 11-18
left-hand frame is the reflection-coefficient model of a two-dimensional . It consists of an undulating sea floor underlain by a faulted, dipping ture. The horizontal line near the bottom of the frames is the one-second g line. A uniform exponential gain has been applied to all three frames isplay purposes. The vertical exaggeration is 5 on all frames. The center e is the synthesized time section. Below the sea floor are seen the sea-floor iples. Below the faulted, dipping structure are the pegleg multiples. The -hand frame is the attempted reconstruction of the original model.

ich we have used to compute c_1' is the principle that the upgoing (which is e_0) must vanish before the onset of a downgoing wave

f practical difficulties must be overcome before realistic behavior f either the one-dimensional- or the two-dimensional algorithms nted. First of all is the problem of the source spreading out in This calls for a spherical spreading correction. Luckily, since the s a *ratio* between up- and downgoing waves, the true amplitude be very important. In fact, the calculation may be shown to be xponential gain readjustment. One real problem which does arise ude of the initial shot delta function cannot be properly measured for the calculation. In fact, there is a shot *waveform* to be ac- h may also have a slight timing error due to residual moveout. waveform can be calibrated in deep water where the sea floor is a place the returned wave is scaled to zero everywhere except or primary reflection; call this waveform P. Also, another wave- d by weighting the returned wave to 0 everywhere except around . Ideally, P convolved onto itself should give M; the departure of shot amplitude, timing, and waveform.

alize is that strong reflective interfaces may not fall precisely on a nt. This means that (4, 0), (3, 1), (2, 2), (1, 3), and (0, 4) must all uccessive delays of the same impulsive waveform. The problem differ at high frequencies is the same type of problem as dif- ns departing from difference equations for wavelengths shorter

than about ten sample points. There is no need to model the high frequencies accurately; just be sure to sample the data densely enough. The real problem with high frequencies is keeping them from causing instability. For example, in the calculation of $U^0/D^0 = -\hat{W}(Z)/[1 + \hat{W}(Z)]$ it is first necessary to estimate the $\hat{W}$ which would be recorded if the shot P/I had been an ideal delta function. Naturally, the high frequencies in $\hat{W}$ are rather meaningless. The only reason you care about them is that they should not be such as to make $1 + \hat{W}$ nonminimum-phase which would prevent the division. In this case, the high frequencies can be filtered out from $\hat{W}$ and the divisor will tend toward a positive real function. Don C. Riley [Ref. 38] has established that these various practical difficulties can be overcome and Fig. 11-18 shows an example of one of his calculations.