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Fundamentals of Geophysical Data Processing

WITH APPLICATIONS TO PETROLEUM PROSPECTING

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PREFACE

This book is based at the level of a bachelor's degree in physical science. Experience at Stanford indicates that a one-semester class in engineering systems theory provides helpful additional background. It will be readable to a general science and engineering audience and should be useful to anyone interested in computer modeling and data analysis in physical sciences.

Inevitably, the book is strongly flavored by my own research interests which are presently mainly in reflection seismology. However, I have taken an interest in a good many of the data processing problems in general geophysics that have arisen in eight years of teaching graduate students and supervising research. This book is intended to be a textbook rather than a research monograph. The exercises are of a reasonable degree of difficulty for first-year graduate students, and most of them have been thoroughly tested. Since its first publication in 1976, and subsequent translation to Russian and Chinese, this book has become the most widely-referenced textbook in journal articles on exploration geophysics. No other book gives so complete an account of the fundamentals of geophysical data processing. Recently Blackwell Scientific Publications offered to reprint the book, so I took advantage of the opportunity to rewrite the preface and introduction, and to update the references.

The subject of discrete-time filters is easier than calculus and provides the needed platform, usually provided by calculus, to explore many concepts from physics and engineering. Concepts expressed in discrete time are already prepared for computers, thus engineering and science professionals are no longer diverted by analytic solutions, but by computer methods. Now with computers in every room, a curriculum that leaves every thought expressed in the continuum is simply out of date. The real "new math" should not be set theory but elementary calculus reexpressed in discrete form.

Fundamentals of Geophysical Data Processing takes the mathematical part of the undergraduate physics and engineering curricula and translates it into a form digestible by computers. So this is not only a book on *geophysical* analysis but also a book for *any* analysis in modern science and engineering. The book is required in our geophysics *graduate* curriculum because the present science and engineering *undergraduate* curriculum has hardly yet entered the computer age.

a processing draws from mathematical physics, numerical analysis,
damentals of Geophysical Data Processing develops its theme from
eral undergraduate science curriculum and deductions tend to be
ufficient. This book was originally directed to the Bachelor's degree
ny students at that level find the book is too terse for self-study, so I
w study guide, *Imaging the Earth's Interior* (IEI), which supersedes
ers of *Fundamentals of Geophysical Data Processing* (FGDP). For
ew book was taught after FGDP. I taught the material in that order
ial was developed in that way and it seemed natural to teach one-
sis before teaching multidimensional analysis. Subsequently, the
es was reversed and now I teach migration and velocity from IEI
lters and deconvolution from FGDP. The reasons for this are first,
analysis is more geometrical and less algebraic, so it is naturally
ppealing to most students and second, the theory in IEI provides a
nigration and velocity analysis in practice, whereas no book, neither
ks by Ziolkowski or Treitel provides an equally satisfactory practical
lution.
about things that work most of the time, like migration and velocity
ience is about things that are basic, things that should always be true
ficult to show convincingly, things that may require judgment in
ke deconvolution and inversion, the major practical applications of

o a great many friends, associates, and former teachers for much of
ned. I have had many fruitful conversations with Steve Simpson,
, and John Burg about time series analysis. Ted Madden taught me
written in this book on stratified media, but most importantly he
the idea that the time had come to go beyond stratified media. John
ancis Muir introduced me to reflection seismic prospecting and some
of thinking about it. Several generations of students were a great
any of the "bugs" out of the text and the exercises. Phil Schultz, Don
ve Doherty prepared many of the figures in the final chapters. Mrs.
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JON F. CLAERBOUT

January 5, 1985

INTRODUCTION

Geophysical data processing is the use of computers for the analysis of geophysical data. A major task in geophysics is to determine as much as possible about the constitution of the interior of the earth. Where direct penetration is impractical or impossible, seismological, electromagnetic, and gravity measurements are made and the task of making inferences from these measurements is begun. Through systematic application of the laws of physics and the principles of statistics, some of these interpretation tasks can be computerized. When the number of observations is small, it may be satisfactory to match them to the adjustable parameters in known analytic solutions to the equations of classical physics. Today, however, it is common to have massive numbers of observations which contain far more information about the earth than can be modeled by analytic solutions. A typical reflection seismic marine survey ship can collect about a trillion (10^{12}) bits of information per month.

Such massive amounts of data require both statistical reduction and the ability to compute theoretical solutions in many-parameter earth models. Use of digital computers to statistically analyze geophysical data began with the Geophysical Analysis Group (GAG), an industry-supported project at the Massachusetts Institute of Technology which ran from 1953 to 1957 [Ref. 1]. Theoretical geophys-

s made a great step forward in 1954 when Norman Haskell
ed a famous paper in which he showed how seismic surface waves
ted for an earth modeled by an arbitrary number of plane parallel
arbitrarily prescribed physical properties. This enabled utilization
smic waveform in fitting an arbitrarily stratified earth model. (By
s meant that material properties are a function of one coordinate
depth or radius). Haskell's method has been intensively developed
enty years to the point where we can now readily compute seismic
netic responses to arbitrary source distributions in any desired
of the earth. Indeed, it seems that the stratified medium has
the homogeneous medium as the most popular framework for
nathematical geophysics.
often consist of hundreds of oscillations, most of which may be
laborate methodologies have evolved for fitting seismograms
dia models with random variations on layer parameters and data.
g, however, to observe that explosion seismograms with all their
explicable details are completely reproducible. Even earthquake
ill be reproducible when the source region is small. Thus, the intro-
lom variables into data analysis often serves mainly to force fit the
d models.
o our well developed stratified media tools, most of the questions
asked about the earth are really questions about its *departure* from
odel. Foremost are the matters of verifying the mechanics of con-
understanding earthquakes, and seeking to locate petroleum and
today, the frontiers in geophysical data processing lie in the recon-
data with two- and three-dimensionally inhomogeneous models of
efore we start we need a good foundation in the traditional material.
data processing begins with the study of the sampled time form of
nd spectral analysis. The mathematical restraints imposed by the
usality are very important. Even arbitrarily complex models of the
ct to this principle. Computational stability often hinges on per-
herence to it. Then basic concepts of resolving power, statistics,
re reviewed preparing the reader for the general theory of least
vith lots of examples. Least squares has been, of course, the principal
reconciliation of data with theoretical models. While it remains in
role, high-resolution techniques (maximum entropy) and robust
L_1 norm and linear programming) are challenging it.
is development of fundamental data processing ideas, the rest of the
ned with treating earth models of successively increasing complexity.
multiple reflected plane waves in layered media from a base of only
sality, and energy conservation (no more physics than that). Waves
ted from knowledge of the media and the media can be calculated
s. Then, the more general theory of mathematical-physical computa-
ed media is introduced and the essential features of finite-difference
partial differential equations are surveyed.

The final chapters are devoted to wave extrapolation and data processing with partial differential equations. I developed this material with my graduate students over the past six years at Stanford University (see Refs. [3] to [8] and [36] and [37]). The basic objective is like that in holography. A wave field is observed on a plane (the surface of the earth) and the goal is to create a two- or three-dimensional model of the scattering objects to one side of (beneath) the plane. The main problems and the techniques used are quite different from holography. Velocity inhomogeneity, diffraction, interference, and multiple reflection are ubiquitous features of seismic propagation though they are rare in common visual experience. The eye is easily deceived in a house of mirrors or when looking into an aquarium.

As I predicted in the original preface to this book, the material on wave-equation data processing developed rapidly. In 1985 it became the subject of a new book, IMAGING THE EARTH'S INTERIOR also published by Blackwell Scientific Publications.