Amplitude preserving prestack imaging of irregularly sampled 3-D data

Biondo Biondi and Ioan Vlad, Stanford University*

SUMMARY

We introduce a computationally efficient and robust method to regularize acquisition geometries of 3-D prestack seismic data before prestack migration. The proposed method is based on a formulation of the geometry regularization problem as a regularized least-squares problem. The model space of this least-squares problem is composed of uniformly sampled common offset-azimuth cubes. The regularization term fills the acquisition gaps by minimizing inconsistencies between cubes with similar offset and azimuth. To preserve the resolution of dipping events in the final image, the regularization term includes a transformation by Azimuth Moveout (AMO) of the common offset-azimuth cubes. The method is computationally efficient because we applied the AMO operator in the Fourier-domain and we precondition the least-squares problem. Therefore, no iterative solution is needed and excellent results are obtained by applying the adjoint operator followed by a diagonal weighting in the model domain. We tested the method on a 3-D land data set. Subtle reflectivity features are better preserved after migration when the proposed method is employed as compared to more standard geometry regularization methods.

INTRODUCTION

When the seismic data are irregularly sampled, images are often affected by amplitude artifacts and phase distortions, even if the imaging algorithm is designed to preserve amplitudes. The application of a simple imaging sequence that relies on standard 'adjoint' imaging operators can generate misleading amplitude results. Inverse theory is useful in producing a footprint-free image. There are two distinct ways of applying inverse theory to the problem: regularizing the data before migration (Duijndam et al., 2000), or during migration (Albertin et al., 1999; Bloor et al., 1999; Audebert, 2000; Rousseau et al., 2000; Duquet et al., 1998; Nemeth et al., 1999). The latter approach exploits the intrinsic physical correlation between seismic traces recorded at different locations, but it depends on accurate knowledge of the interval velocity. We propose a method that has the advantages of both approaches. We regularize the data geometry before migration by using a partial migration operator – Azimuth Moveout [AMO, Biondi et al. (1998)] – that exploits the intrinsic correlation between prestack seismic traces, and depends not on the interval velocity, but on the RMS velocity, which can be estimated much more robustly from the data.

Ronen (1987) was the first to use a partial migration operator (DMO) to improve the estimate of a regularized data set. Chemingui and Biondi (1997; 1999) have previously inverted AMO to create regularly sampled common offset-azimuth cubes. The main advantage of our method over the previous methods is computational efficiency due to: a)using a Fourier-domain implementation of AMO as opposed to a Kirchhoff one; b)preconditioning of the regularization term in the inversion, made possible by the use of AMO in the regularization term instead of in the modeling one. c)not using an iterative approach, but instead approximating the solution of the preconditioned least-squares problem by applying normalization weights to the model vector after the application of the adjoint operator.

Our formulation of the geometry regularization problem as a regularized least-squares problem is similar to the formulation that Fomel presented in his Ph.D. thesis (2001). He uses a finite difference implementation of offset continuation where we use a Fourier implementation of AMO. These two operators are kinematically equivalent and their computational efficiency is similar. However, the methods are different with respect to items b) and c) listed above. Our method should be more efficient because it explicitly preconditions the regularization term by inverting it. The inversion is fast because the regularization matrix is factored into the product of a block lower-diagonal matrix with a block upper-diagonal matrix, which are easily invertible by recursion. The preconditioning substantially improves the conditioning of the problem; therefore, a simple diagonal normalization of the model vector yields a good and fast solution.

NORMALIZED PARTIAL STACKING AND INVERSE THEORY

We will use partial stacking to create uniformly sampled common offset/azimuth cubes that can be migrated with an amplitude-preserving algorithm. 3-D prestack data traces do not share the same exact midpoint location. Stacking them involves spatial interpolation followed by averaging. Let us define a simple linear operator linking the recorded traces (at arbitrary midpoint locations) to the stacked volume (defined on a regular grid). Each data trace is obtained by interpolation (weighted sum of the neighboring stacked traces):

$$d_i = \Sigma_j \, a_{ij} \, m_j; \text{ subject to the constraint } \Sigma_j \, a_{ij} = 1. \quad (1)$$

The weights are independent from time along the seismic traces so each element d_i of the data space (recorded data) $\mathbf{d}$, and each element m_j of the model space $\mathbf{m}$ (stacked volume) will represent a whole trace. In matrix notation, equation (1) becomes

$$\mathbf{d} = \mathbf{A}\mathbf{m}. \quad (2)$$

where the abstract vector $\mathbf{m}$ is composed of offset/azimuth cubes. In operator notation, stacking can be represented as the application of the adjoint operator $\mathbf{A}'$ to the data traces (Claerbout, 1998), and by accounting for unevenness in the fold through normalization, which can be expressed by a diagonal operator $\mathbf{W}_m$:

$$\mathbf{m} = \mathbf{W}_m \mathbf{A}' \mathbf{d}. \quad (3)$$

The weights w_j^m are given by the inverse of the fold, which can be simply computed by a summation of the elements in each column of $\mathbf{A}$. Data gaps in the fold can make the weights diverge to infinity. This can be avoided by setting the weights to zero when the fold is smaller than ϵ_w:

$$w_j^m = \begin{cases} \left(\Sigma_i \, a_{ij}\right)^{-1} & \text{if } \Sigma_i \, a_{ij} \geq \epsilon_w \\ 0 & \text{elsewhere.} \end{cases} \quad (4)$$

Fold normalization is effective when the geometry is irregular, but for sizable data gaps the normalization weights tend to become large. Instability can be avoided, but gaps are going to be left in the uniformly sampled data. These gaps are likely to be spread as "smiles" by migration. They can be filled using inverse theory before migration, using information from nearby traces.

MODEL REGULARIZATION AND PRECONDITIONING

Data gaps are a challenge for simple fold normalization. They should be filled using information from traces recorded with geometry similar to the missing ones. Reflectivity can change in the earth abruptly across midpoints, but the reflection mechanism ensures that it changes gradually with the variation of the reflection

angle and azimuth. Therefore less resolution is lost if interpolation is done not with data from neighboring midpoints, but with data from the same midpoint and with different offsets and azimuths than the missing ones. In the context of least-squares inversion, smoothing along offset/azimuth in the model space (e.g. uniformly sampled offset/azimuth cubes) can be accomplished by introducing a model regularization term that penalizes variations of the seismic traces between the cubes. Denoting the *roughening* operator by $\mathbf{D_h}$, the least-squares problem can be formulated as

$$\begin{cases} 0 \approx \mathbf{d} - \mathbf{Am} \\ 0 \approx \epsilon_D \mathbf{D'_h D_h m} \end{cases} , where$$

$$\mathbf{D_h} = \frac{1}{1-\rho_D} \begin{bmatrix} (1-\rho_D)\mathbf{I} & 0 & 0 & \vdots & 0 \\ -\rho_D \mathbf{I} & \mathbf{I} & 0 & \vdots & 0 \\ 0 & -\rho_D \mathbf{I} & \mathbf{I} & \vdots & 0 \\ \cdots & \cdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & -\rho_D \mathbf{I} & \mathbf{I} \end{bmatrix} \quad (5)$$

The coefficient ρ_D must be between 0 and 1. It determines the range over which we smooth the offset/azimuth cubes. The smaller the value we set for ρ_D, the narrower the smoothing range is. The problem is easy to precondition because $\mathbf{D_h^{'} D_h}$ is easy to invert, since it is already factored in a lower block-diagonal operator $\mathbf{D_h}$ and in an upper block-diagonal operator $\mathbf{D_h^{'}}$, that can be inverted by recursion. Therefore, denoting the preconditioned model vector with $\mathbf{p} = \mathbf{D_h^{'} D_h m}$, the preconditioned least-squares problem is

$$\begin{cases} 0 & \approx & \mathbf{d} - \mathbf{A}\left(\mathbf{D_h^{'} D_h}\right)^{-1} \mathbf{p} \\ 0 & \approx & \epsilon_D \mathbf{I p} \end{cases} \quad (6)$$

Fold variations can be accounted by introducing a diagonal scaling factor and by applying the same theory discussed in the previous section. The weights for the regularized and preconditioned problem are

$$\mathbf{W_I^{-1}} = \frac{\mathbf{diag}\left\{\left[\left(\mathbf{D_h D_h^{'}}\right)^{-1} \mathbf{A^{'} A} + \epsilon_D \mathbf{I}\right] \mathbf{1}\right\}}{\mathbf{diag(1)}}. \quad (7)$$

The solution of the problem obtained by normalizing the preconditioned adjoint is

$$\widetilde{\mathbf{m}} = \left(\mathbf{D_h^{'} D_h}\right)^{-1} \mathbf{W_I} \left(\mathbf{D_h D_h^{'}}\right)^{-1} \mathbf{A^{'} d}. \quad (8)$$

The main drawback of the method described above is that smoothing over offset/azimuth cubes by the inverse of the simple roughening operator expressed in (5) may result in loss of resolution when geological dips, which are not flattened by NMO with the same velocity as flat events, are present. However, the method can be generalized by substitution of the identity matrix in the lower diagonal of $\mathbf{D_h}$ with AMO (Biondi et al., 1998), which correctly transforms a common offset-azimuth cube into an equivalent cube with a different offset and azimuth. Since the cubes to be transformed are uniformly sampled we can use a computationally-efficient Fourier-domain formulation of AMO. The roughening operator that includes AMO is $\widetilde{\mathbf{D_h}}$, which differs from $\mathbf{D_h}$ only in that the identity operators in the subdiagonal elements $\mathbf{D_h}(i+1,i)$ are replaced by the AMO operators $\mathbf{T_{h_{i,i+1}}}$ that transform. the offset-azimuth cube i into the offset-azimuth cube $i+1$. The $\widetilde{\mathbf{D_h^{'}}}\widetilde{\mathbf{D_h}}$ operator can also be easily inverted by recursion and thus the least-squares problem obtained by substituting $\widetilde{\mathbf{D_h}}$ for $\mathbf{D_h}$ in (5) can also be preconditioned and normalized.

IMAGING OF A 3-D LAND DATA SET

We tested the geometry regularization methods presented in the previous section on a land data set shot with a cross-swath geometry fairly narrow-azimuth for land data. The processing sequence comprised the following steps: a) NMO, b) geometry regularization, c) inverse NMO, d) 3-D prestack common-azimuth wave-equation migration, with the imaging step designed to preserve relative amplitudes, as discussed by Sava and Biondi (2001).

We evaluated the relative performances of three different regularization methods: a) normalization by partial stack fold [equation (3)]; we will simply call this method *normalization*. b) normalization of the regularized and preconditioned solution without AMO [equation (8)]; we will simply call this method *regularization*, c) normalization of the regularized and preconditioned solution with AMO [equation (8) with $\mathbf{D_h} = \widetilde{\mathbf{D_h}}$]; we will call this method *AMO regularization*. Our first tests produced common offset-azimuth cubes at zero azimuth (i.e. the data azimuth was ignored) because of the fairly limited azimuthal range at far offsets. However, we have indications that taking into account the data azimuth for the far-offset traces may be beneficial. To avoid offset aliasing in the downward continuation at the reservoir level (3.2 km), the offset axis was resampled at 65m by simple interpolation before migration. Finer sampling would be necessary to migrate shallower events without aliasing the higher frequencies.

Geometry regularization results

Figure 1 compares the results of geometry regularization of the three methods discussed above for one line. Figure 1a shows the normalization results, Figure 1b shows the regularization results, and Figure 1c shows the AMO regularization results. Comparing the in-line sections at one offset (3.38 km) shows the advantages of both regularization and AMO regularization over simple normalization. The amplitudes after normalization are still fairly uneven, and thus likely to produce artifacts during migration. The amplitudes are better balanced in the data obtained using regularization. The steeply dipping reflection from the fold at the reservoir level is better preserved in the AMO regularization results than in the simple regularization results. The reason is quite apparent when examining the data as a function of offset for one particular midpoint location. The dipping event is smiling upward after NMO, and thus it is attenuated by simple smoothing over offset.

Figure 2 shows a detail of the same line. As for Figure 1, Figure 2a shows the results for normalization, Figure 2b shows the results for regularization, and Figure 2c shows the results for AMO regularization. An acquisition gap is clearly visible in the middle of the constant-offset (2.275 km) section in panel (a). Simple normalization cannot fill the gap. On the contrary, the gap is filled in the regularized results, that exploit the information from the neighboring offsets. The gap in the dipping event is better filled by the AMO regularization because the information from neighboring offsets is moved to the missing data consistently with their kinematics. The differences in behavior between the two regularization methods are apparent in the time slices in the figure. The AMO regularization shows better what is actually a curved event, while the curvature is lost by the simple regularization scheme.

Amplitude-preserving migration results

We migrated the data after geometry regularization using common-azimuth migration and produced different prestack migrated images with the common image gathers depending on the in-line offset ray parameter. Figure 3 shows depth slices from the migration of the normalization and AMO regularization results. Meandering channels are visible. The slices on the left are taken for a narrow reflection angle, the slices on the right are taken for a wide reflection angle. The images obtained using normalization are noisier and show more clearly the oblique acquisition footprints. In the narrow angle image, the noise is so overwhelming that no chan-

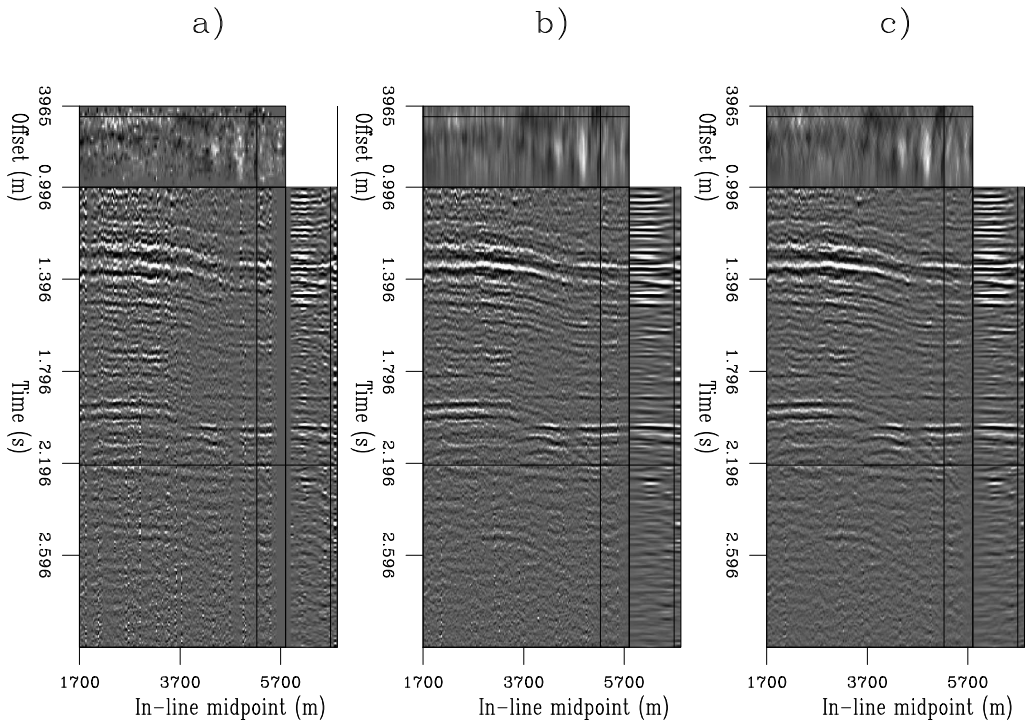

Figure 1: A prestack line after geometry regularization with: a) normalization, b) regularization, c) AMO regularization.

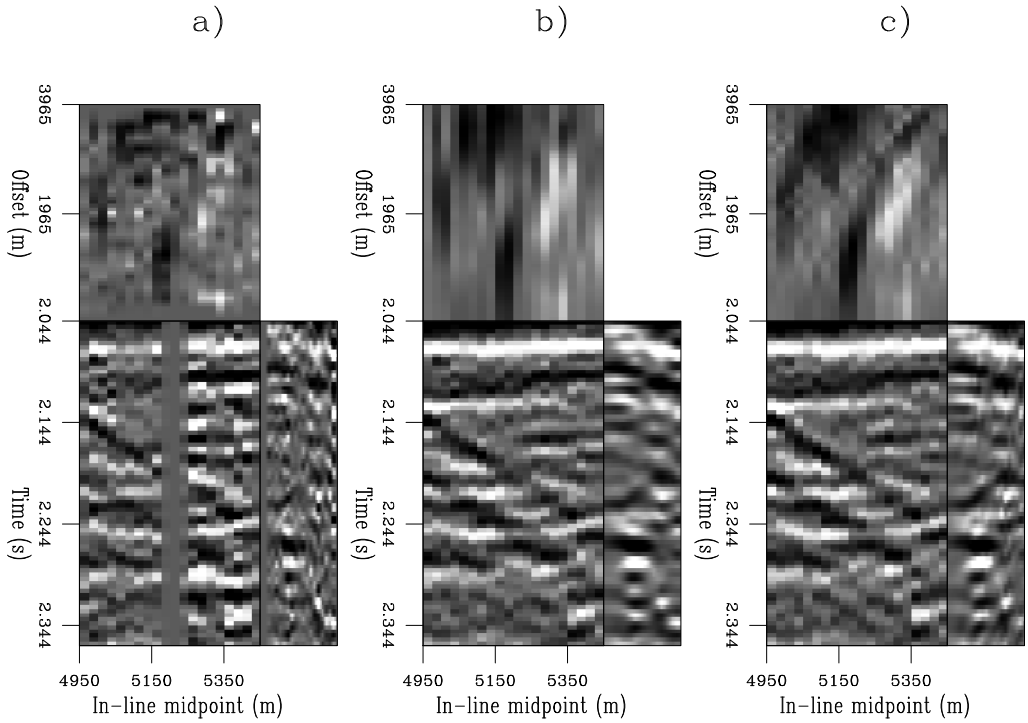

Figure 2: A detail of the prestack line shown in Figure 1. Notice the data gap in the middle of the common-offset (2.275 km) section, and how it has been interpolated differently by regularization (panel b) and AMO regularization (panel c).

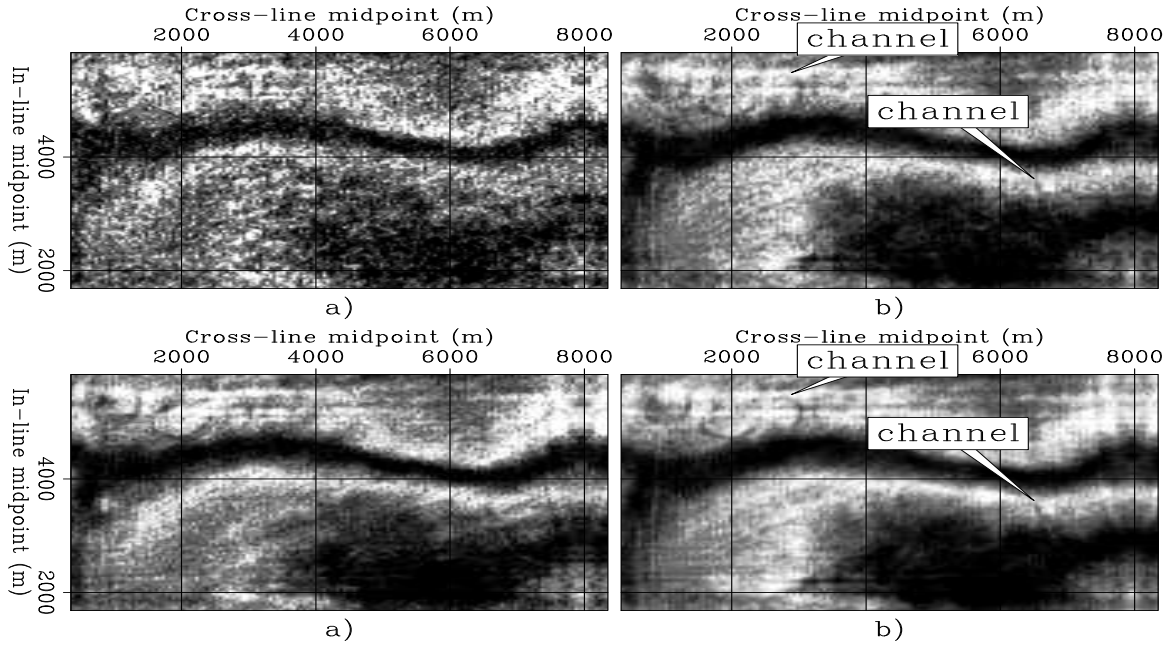

Figure 3: Depth slices, at a depth of 1.91 km, obtained by migration after geometry regularization with: normalization (top) and AMO regularization (bottom). Narrow reflection angle (left), and wide reflection angle (right).

nels are visible. Differences are noticeable in the imaging of both channels, where indicated by the arrows.

CONCLUSIONS

The proposed method for regularizing the geometry of 3-D prestack data set performed well in a real-data test. The regularization methods fill the acquisition gaps by using the information from neighboring offsets/azimuths and provide a better input to migration than the simple normalization by the partial stack fold. The inclusion of the AMO operator in the regularization assures better preservation of the steeply dipping event, thus yielding higher-resolution images than when the AMO operator is not applied.

ACKNOWLEDGMENTS

We would like to thank Trino Salinas from Ecopetrol for bringing the data set to SEP during his visit this summer, Ecopetrol for making the data available, and all the SEP sponsors for their constant support.

REFERENCES

Albertin, U., Jaramillo, H., Yingst, D., Bloor, R., Chang, W., Beasley, C., and Mobley, E., 1999, Aspects of true amplitude migration: 69th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1358–1361.

Audebert, F., 2000, Restored amplitudes AVA-gathers in the multi-angle domain: 62nd Mtg. Eur. Assoc. Expl Geophys., Abstracts, P–142.

Biondi, B., Fomel, S., and Chemingui, N., 1998, Azimuth moveout for 3-D prestack imaging: Geophysics, **63**, no. 2, 574–588.

Bloor, R., Albertin, U., Jaramillo, H., and Yingst, D., 1999, Equalised prestack depth migration: 69th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1358–1361.

Chemingui, N., and Biondi, B., 1997, Equalization of irregular data by iterative inversion: Expanded Abstracts, 1115–1118.

Chemingui, N., 1999, Imaging irregularly sampled 3D prestacked data: Ph.D. thesis, Stanford University.

Claerbout, J., 1998, Geophysical estimation by example: Environmental soundings image enhancement: http://sepwww.stanford.edu/sep/prof/.

Duijndam, A. J. W., Volker, A. W. F., and Zwartjes, P. M., 2000, Reconstruction as efficient alternative for least squares migration: 70th Ann. Internat. Meeting, Soc. Expl. Geophys., Expanded Abstracts, 1012–1015.

Duquet, B., Marfurt, K., and Dellinger, J., 1998, Efficient estimates of subsurface illumination for Kirchhoff prestack depth migration: 68th Ann. Internat. Meeting, Soc. Expl. Geophys., Expanded Abstracts, 1541–1544.

Fomel, S., 2001, Three-dimensional seismic data regularization: Ph.D. thesis, Stanford University.

Nemeth, T., Wu, C., and Schuster, G. T., 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, no. 1, 208–221.

Ronen, S., 1987, Wave-equation trace interpolation: Geophysics, **52**, no. 7, 973–984.

Rousseau, V., Nicoletis, L., Svay-Lucas, J., and Rakotoarisoa, H., 2000, 3D true amplitude migration by regularisation in angle domain: 62nd Mtg. Eur. Assoc. Expl Geophys., Abstracts, B–13.

Sava, P., Biondi, B., and Fomel, S., 2001, Amplitude-preserved common image gathers by wave-equation migration: 71st Ann. Internat. Mtg, Soc. Expl. Geophys., Expanded Abstracts, 296–299.