Tau domain migration velocity analysis using angle CRP gathers and geologic constraints

Robert G. Clapp and Biondo Biondi, Stanford University*

SUMMARY

We present an alternative approach for ray based migration velocity analysis. Instead of using Kirchoff migration and offset domain Common Reflection Point (CRP) gathers we use wave equation ray parameter CRP gathers. By performing tomography in vertical travel-time (tau) space, we avoid estimating mapping velocity, instead concentrating on focusing velocity. By introducing anisotropic preconditioning oriented along bedding planes, we quickly guide the inversion towards a geologically reasonable model. We illustrate the benefits of our tomography method by comparing it to more traditional methods on a synthetic anticline model.

INTRODUCTION

Tomography is inherently non-linear, therefore a standard technique is to linearize the problem by assuming a stationary ray field (Stork and Clayton, 1991). Unfortunately, we must still deal with the coupled relationship between reflector position and velocity (Al-Chalabi, 1997; Tieman, 1995). As a result, the back projection operator must attempt to handle both repositioning of the reflector *and* updating the velocity model (van Trier, 1990) . The resulting back projection operator is sensitive to our current guess at velocity and reflector position. In vertical travel time space reflector movement is significantly less. (Biondi et al., 1998) showed that by reformulating the problem in this space complex velocity functions could be more quickly and better resolved.

In ray-based migration velocity analysis Kirchoff migration is normally used to construct CRP gathers and residual moveout. Unfortunately conventional implementation of Kirchoff methods have trouble handling complex wave behavior. Clayton and Stolt (1981) and later Prucha et al. (1999) showed that wave equation methods could also form CRP gathers in terms of offset ray parameter.

Tomography problems are also often under-determined. By adding an additional model regularization term to our objective function (Toldi, 1985) we can stabilize the inversion. In theory, this regularization term should be the inverse model covariance matrix (Tarantola, 1987). The question is how to obtain an estimate of the model covariance matrix. Clapp et al. (1998) constructed a series of small plane wave annihilators, called steering filters, using *a priori* information to produce changes to our velocity model that was more geologically reasonable.

In this paper we show how to apply steering filters to smooth the slowness, rather than the change in slowness. We use wave equation CRP gathers as the basis for ray based tomography and reconfirm that doing tomography in the tau domain leads to significantly improved inversion results.

ANGLE DOMAIN CRP GATHERS

Ray based tomography assumes a linear relationship ($\mathbf{T}$) between the change in slowness ($\Delta\mathbf{s}$) and travel time errors ($\Delta\mathbf{t}$). When doing migration velocity analysis we migrate prestack data with our current guess at velocity, or slowness ($\mathbf{s_0}$). From the migrated volume we pick reflector positions and moveout errors. The next step is to construct ray paths that are consistent with our migrated data. With Kirchoff migration this can be difficult. Our CRP gathers are in offset, a data space coordinate system. To calculate the raypath associated with a given travel time error we are forced to interpolate between rays and/or do computationally intensive matching at reflector locations.

An interesting alternative is to use wave equation, rather than Kirchoff, migration. The obvious disadvantage is speed and flexibility. Kirchoff migration is well suited for creating sparsely space CRP gathers that we need for velocity analysis. On the other hand, wave equation methods are better able to handle complex velocity structures that Kirchoff, with its reliance on ray based travel time tables, can have problems with.

In addition, the CRP gathers we obtain from wave equation methods are in terms of the more model-friendly ray parameter px_h, rather than offset. As a result our ray tracing step simply becomes shooting from our imaging point at the angle given by:

$$\frac{\partial t}{\partial h} = px_h = \frac{2\sin\theta\cos\phi}{V\left(z,\vec{\mathbf{m}}\right)}, \tag{1}$$

where θ is the aperture angle, ϕ is the geologic dip, and $V\left(z,\vec{\mathbf{m}}\right)$ is the velocity at the CRP location, Figure 1.

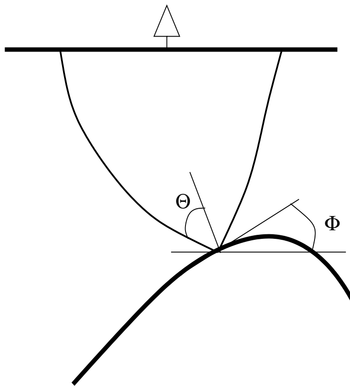

Figure 1: How the takeoff angle for a ray-pair are defined.

Another advantage of a wave equation approach is that assuming that you have correctly defined the macro velocity structure (suh as a salt body) the ray based methods would correctly handle complex wavefield behavior through the macro structure. A Kirchoff approach, which relies on finding rays that match some predetermined surface positions, would not be as fortunate.

TAU TOMOGRAPHY

One of the biggest advantages of time imaging for velocity estimation is the relative stablity of reflector positions. In depth, reflector position will move significantly with change in the velocity above it. Our tomography operator is forced to choose how much a given moveout error is due to reflector positioning and how much is do to velocity errors. As a result, the null space of the depth tomography operator becomes large. Biondi et al. (1998) proposed minimizing this problem by reformulating our tomography problem in terms vertical travel time, rather than depth. In (τ, x) space reflector movement is minimized, but complex velocity structures are still possible. As a result our ray-based tomography operator is less sensitive to our current velocity estimate.

STEERING FILTER REGULARIZATION

Velocity estimation, like most geophysical problems, is under-determined. As a result we often parameterize our model space in terms of splines, or introduce a regularization operator to the inversion problem. The second option amounts adding a second term to our objective function,

$$Q(\Delta\mathbf{s}) = \|\Delta\mathbf{z}_{\mathbf{p_{hx}}} - \mathbf{T_{tau}}\Delta\mathbf{s}\|^2 + \epsilon\|\mathbf{A}\Delta\mathbf{s}\|^2 \tag{2}$$

or in terms of fitting goals,

$$\begin{aligned} \Delta\mathbf{z}_{\mathbf{p_{hx}}} &\approx \mathbf{T_{tau}}\Delta\mathbf{s} \\ \mathbf{0} &\approx \epsilon\mathbf{A}\Delta\mathbf{s}, \end{aligned} \tag{3}$$

where $\Delta\mathbf{z}_{\mathbf{p_{hx}}}$ our are positioning errors in terms of ray parameter, $\Delta\mathbf{s}$ is our change in slowness, $\mathbf{T_{tau}}$ is our tomography operator, $\mathbf{A}$ is a regularization operator, and ϵ is tuning parameter controlling how important model smoothness is compared to fitting moveouts.

Often an isotropic operator is chosen for $\mathbf{A}$. The disadvantage of this choice is it tends to create geologically unreasonable, blobby features in our model. A more interesting choice for $\mathbf{A}$ was introduced in Clapp et al. (1998). This showed how we construct a space varying operator composed of small plane wave annihilation filters oriented along the dip that we would like to see in our model space, Figure 2.

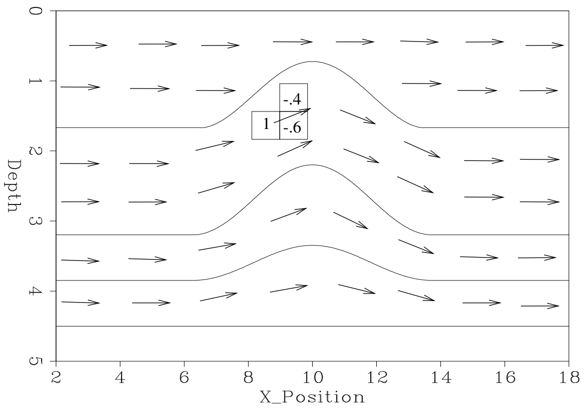

Figure 2: Composite steering filter operator. Each model point has its own steering filter oriented along the desired velocity gradient at that location.

An important consideration is that what we really want to shape is not ($\Delta\mathbf{s}$) but $\mathbf{s}$, as a result we need to modify our fitting regularization fitting goal,

$$\begin{aligned} \mathbf{0} &\approx \epsilon\mathbf{A}(\mathbf{s_0} + \Delta\mathbf{s}) \\ -\epsilon\mathbf{A}\mathbf{s} &\approx \epsilon\mathbf{A}\Delta\mathbf{s}. \end{aligned} \tag{4}$$

Unfortunately this regularized inversion problem converge slowly. By taking advantage of our ability to create stable, inverse operators (Claerbout, 1998), we can rewrite our regularized problem to an equivalent preconditioned problem,

$$\begin{aligned} \Delta\mathbf{z}_{\mathbf{p_{hx}}} &\approx \mathbf{T_{tau}}\mathbf{A}^{-1}\mathbf{p} \\ -\epsilon\mathbf{A}\mathbf{s} &\approx \epsilon\mathbf{p}, \end{aligned}$$

where $\mathbf{p} = \mathbf{A}\Delta\mathbf{s}$. This new preconditioned will converge significantly faster than it regularized counterpart.

RESULTS

To test the method we constructed a simple anticline model, and then did isotropic modeling to create a synthetic dataset, Figures 3 and 4. As our initial guess at velocity we used a $v(z)$ velocity function taken from the edge of the anticline. The resulting wave equation migration result, Figure 5 show CRP gathers with significant residual moveout.

We performed hree non-linear iterations of three different velocity updating schemes. In the first, we used we termed the 'conventional' approach, by doing tomography in depth, and regularizing $\Delta\mathbf{s}$. As Figure 6 shows we have improved our reflector positioning within the antilcine structure compared to our initial estimate but we have some focusing problems, if hurt the position outside the anticline, have significantly mispositioned our reflectors. In the second approach, 'tau-isotropic', we did tau domain tomography using a isotropic smoother on $\mathbf{s}$. As you can see in Figure 6 our focusing and moveouts are significantly improved over the 'conventional' approach. For our final approach we did tau tomography and used steering filters to regularize our slowness field. As you can see the result migration image shows better position and focusing of the reflectors. We have eliminated almost all of residual moveout and positioned are reflectors significantly better than the two other updating schemes.

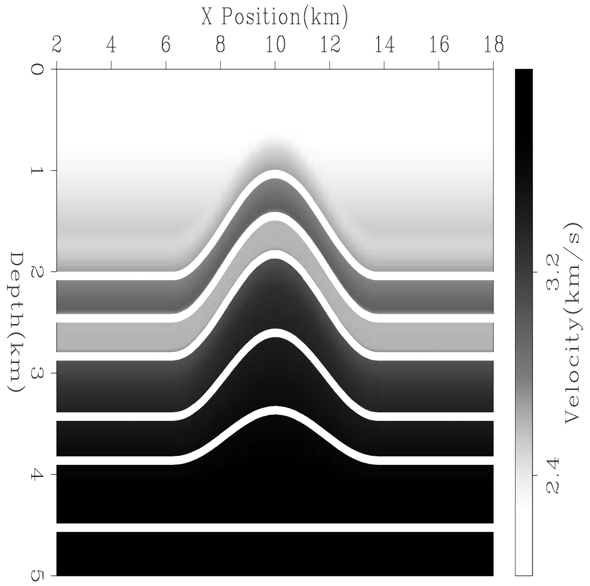

Figure 3: The velocity and reflectors used to create the synthetic dataset.

CONCLUSIONS

We showed that wave equation derived CRP gathers are well suited to ray based tomography. We also showed that performing tomography in the tau rather than depth domain and regularizing our slowness model using anisotropic preconditioner significantly improve our velocity estimate and in turn the migration result.

REFERENCES

Al-Chalabi, M., 1997, Parameter nonuniqueness in velocity versus depth functions: Geophysics, **62**, no. 03, 970–979.

Biondi, B. L., Clapp, R. G., Fomel, S. B., and Alkhalifah, T. A., 1998, Robust reflection tomography in the time domain: Robust reflection tomography in the time domain:, 68th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1847–1850.

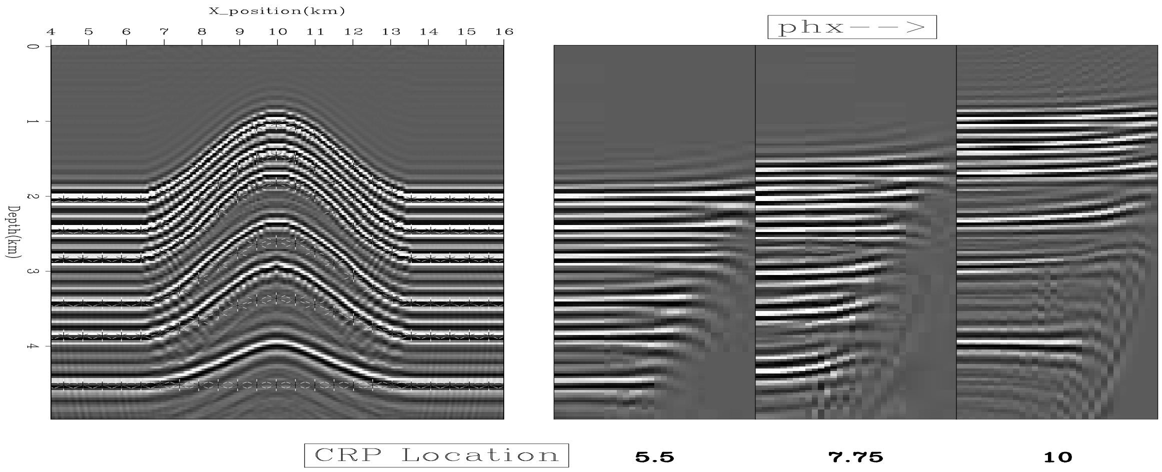

Figure 5: The migration result using a v(z) velocity function. The left panel is the zero ray parameter image, the right is CRPs located at 5.5, 7.75, and 10. The '*' show the correct reflector position.

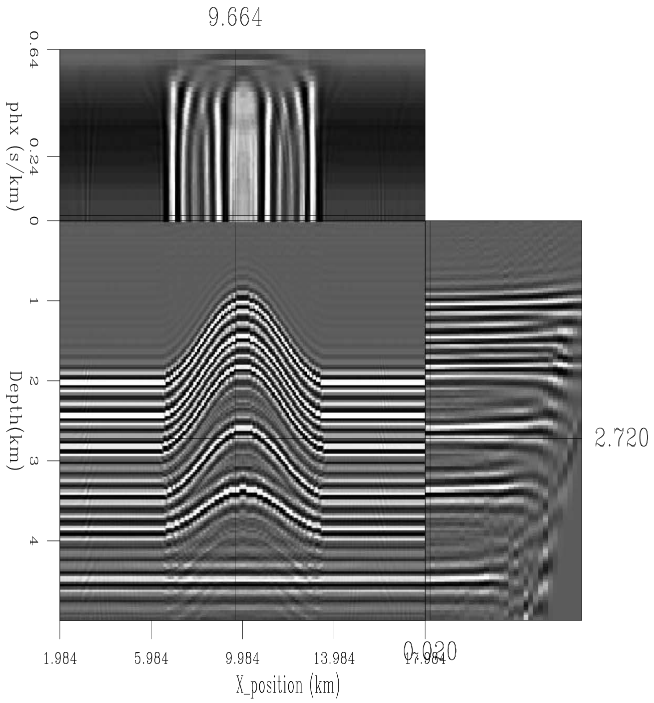

Figure 4: The result of migrating the synthetic data with the correct velocity. Note that CRPs are flat.

Claerbout, J. F., 1998, Multi-dimensional recursive filtering via the helix: Geophysics, **63**, no. 5, 1532–1541.

Clapp, R. G., Biondi, B. L., Fomel, S. B., and Claerbout, J. F., 1998, Regularizing velocity estimation using geologic dip information: Regularizing velocity estimation using geologic dip information:, 68th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1851–1854.

Clayton, R. W., and Stolt, R. H., 1981, A born-wkbj inversion method for acoustic reflection data: Geophysics, **46**, no. 11, 1559–1567.

Prucha, M. L., Biondi, B. L., and Symes, W. W., 1999, Angle-domain common image gathers by wave-equation migration: Angle-domain common image gathers by wave-equation migration:, 69th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 824–827.

Stork, C., and Clayton, R. W., 1991, An implementation of tomographic velocity analysis: Geophysics, **56**, no. 4, 483–495.

Tarantola, A., 1987, Inverse problem theory: Elsevier.

Tieman, H. J., 1995, Migration velocity analysis: Accounting for the effects of lateral velocity variations: Geophysics, **60**, no. 01, 164–175.

Toldi, J., 1985, Velocity analysis without picking: SEP–**43**.

van Trier, J., 1990, Tomographic determination of structural velocities from depth migrated seismic data: Ph.D. thesis, Stanford University.

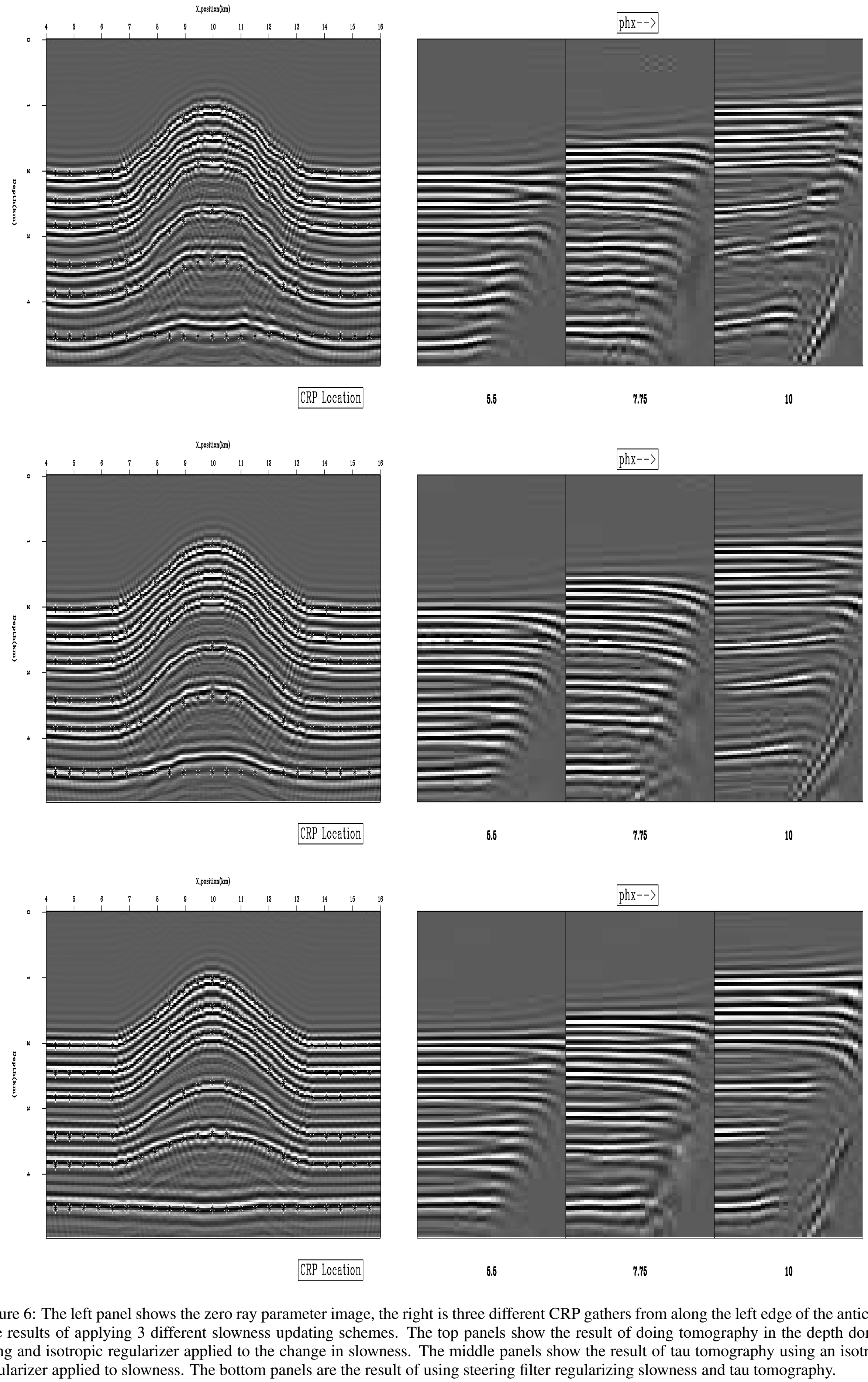

Figure 6: The left panel shows the zero ray parameter image, the right is three different CRP gathers from along the left edge of the anticline. The results of applying 3 different slowness updating schemes. The top panels show the result of doing tomography in the depth domain using and isotropic regularizer applied to the change in slowness. The middle panels show the result of tau tomography using an isotropic regularizer applied to slowness. The bottom panels are the result of using steering filter regularizing slowness and tau tomography.