Two-dimensional textures and prediction-error filters

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For many years it has been true that our most powerful signal-analysis techniques are in *one*-dimensional space, while our most important applications are in *multi*-dimensional space. The helical coordinate system introduced by Claerbout (Geophysics, 1998, vol 63, no 5) makes a giant step towards overcoming this difficulty. Figure 1 shows how convolution in two dimensions is equivalent to that in one dimension.

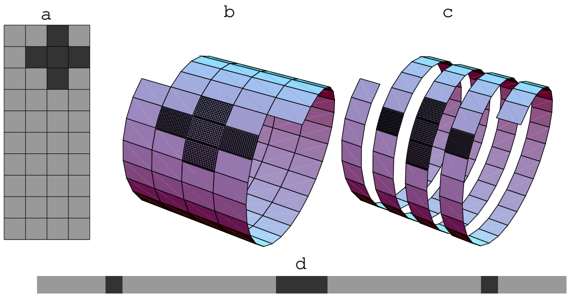

Figure 1: The helix maps 2-D convolution to 1-D. A dense filter on the helix `b` is equivalent to a sparse one on the strip `d`.

Here we examine elementary signal-processing applications of 2-D prediction-error filters (PEFs). A 2-D PEF is the same as a 1-D PEF constrained to be "1" on the first element at the end of a helix. We test both everyday 2-D textures and seismic data. We will see that some textures are easily modeled with prediction-error filters (PEFs) while others are not. Figures 2 to 9 all used the same 10×10 filter shape. No attempt was made to optimize filter size or shape or any other parameters.

Results are shown with various familiar textures on the left as training data sets. From these training data sets using the helix transformation, a prediction-error filter (PEF) is estimated (using a conjugate-direction fitter). The center frame is synthetic data made by deconvolving (polynomial division) random numbers by the PEF. The right frame is the more familiar process, convolving the PEF on the training data set. You can notice the filter size by knowing that the output is taken to be zero where the filter is only partially on the data. Theoretically, the right frame tends towards a white spectrum.

Since a PEF tends to the inverse of the spectrum of its input, results similar to ours could probably be found using Fourier transforms, smoothing spectra, etc. We used PEFs because of their flexibility. The filters can be any shape. They can dodge around missing data, or we can use them to estimate missing data. We avoid periodic boundary assumptions. The PEF's operate only internal to known data, not off edges so they are readily adaptible to nonstationarity. Thinking of these textures as time slices, the textures can easily be required

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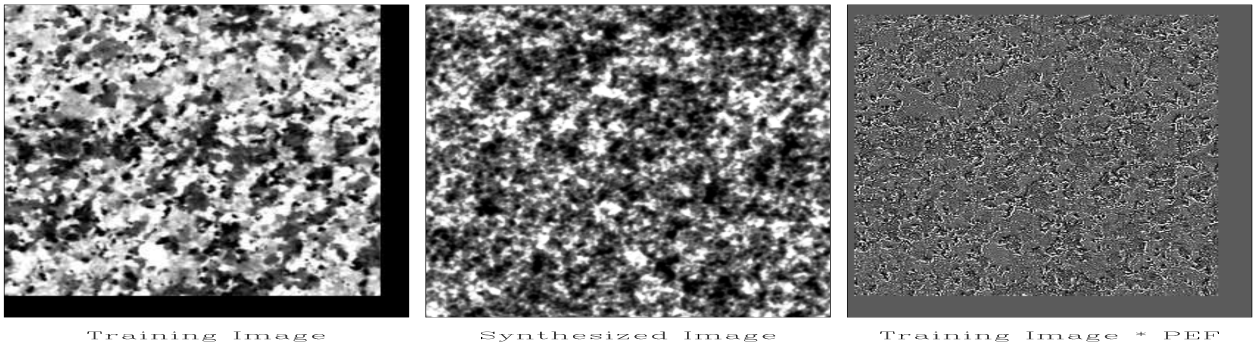


Figure 2: Synthetic granite matches the training image quite well. The prediction error is spectrally white as theoretically expected but seems to outline the grains.

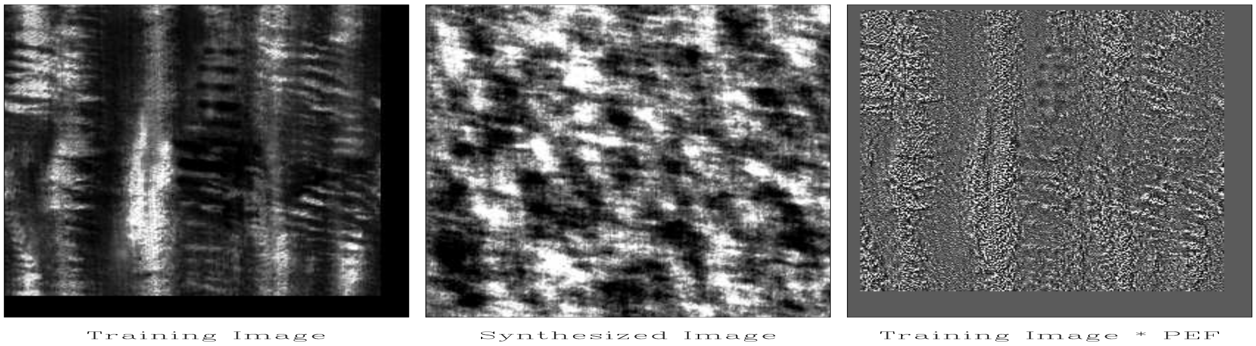


Figure 3: Tree bark. Synthetic tree bark lacks long vertical stripes. Perhaps the PEF is too short.

Figure 4: Synthetic wood grain has too little white. This is because of the nonsymmetric brightness histogram of natural wood. Again, the PEF output looks random as expected.

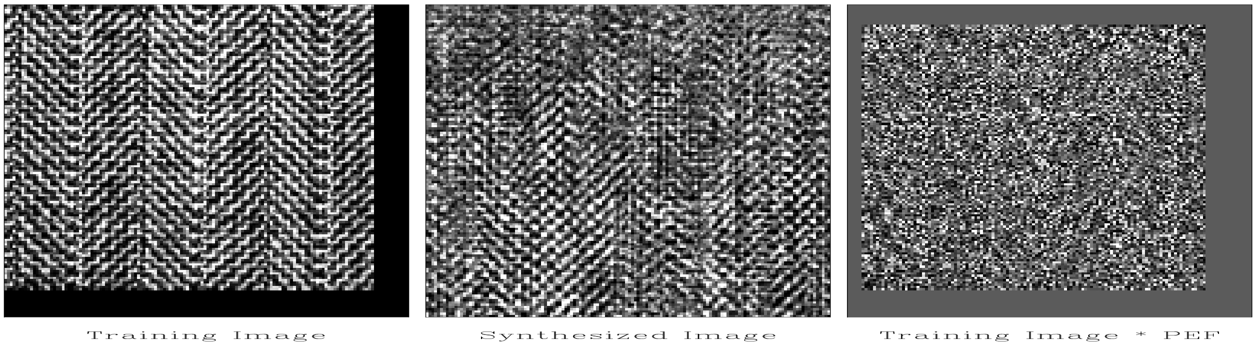


Figure 5: A banker's suit (left). A student's suit (center). My suit (right).

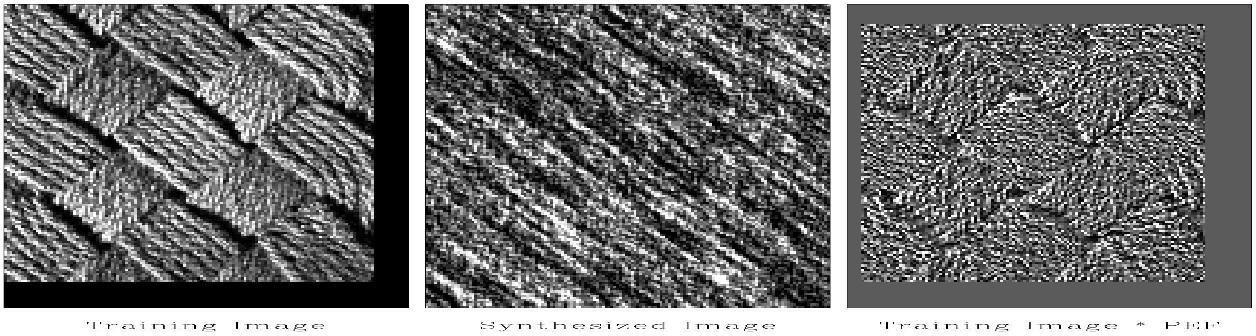


Figure 6: Basket weave. The synthetic data fails to segregate the two dips into a checkerboard pattern. The PEF output looks structured perhaps because the filter is too small.

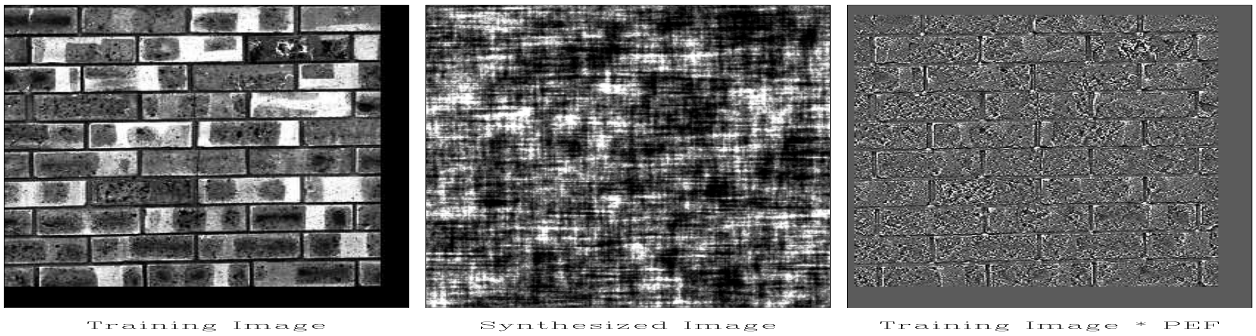


Figure 7: Brick. Synthetic brick edges are everywhere and do not enclose blocks containing a fixed color. PEF output shows the mortar.

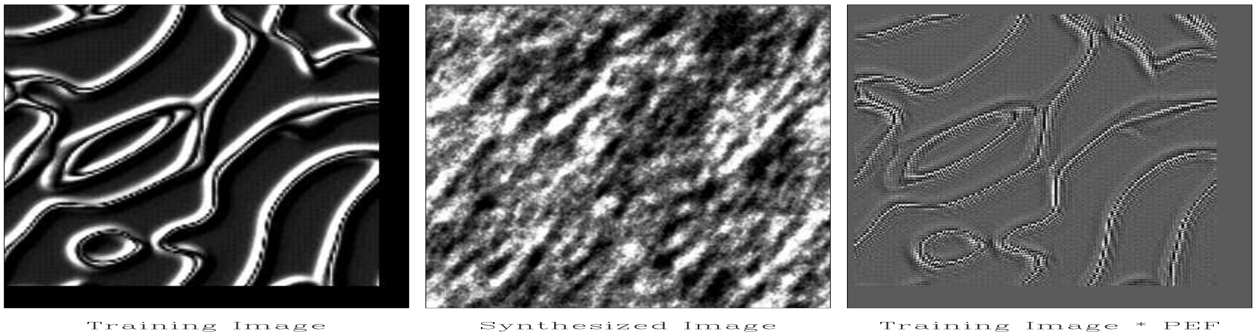


Figure 8: Ridges. A spectacular failure of the stationarity assumption. All dips are present but in different locations. The ridges have been sharpened by the deconvolution. We cannot explain why the nonsymmetric ridges have become symmetric.

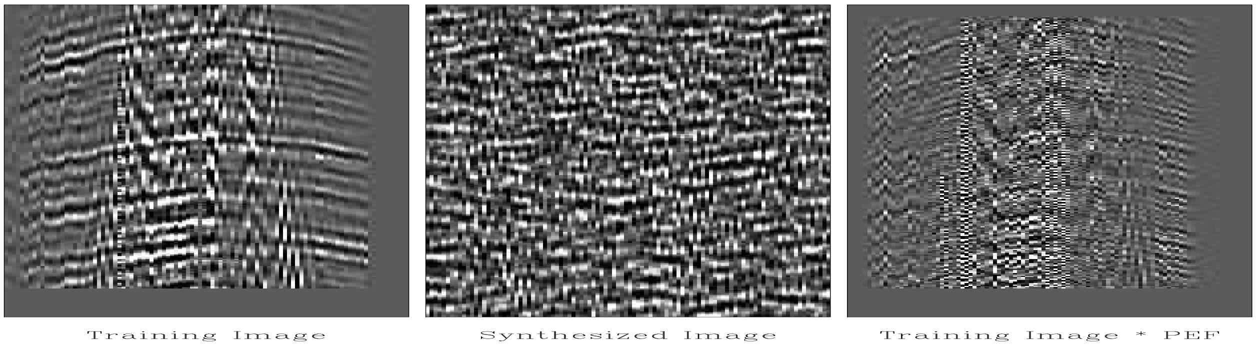
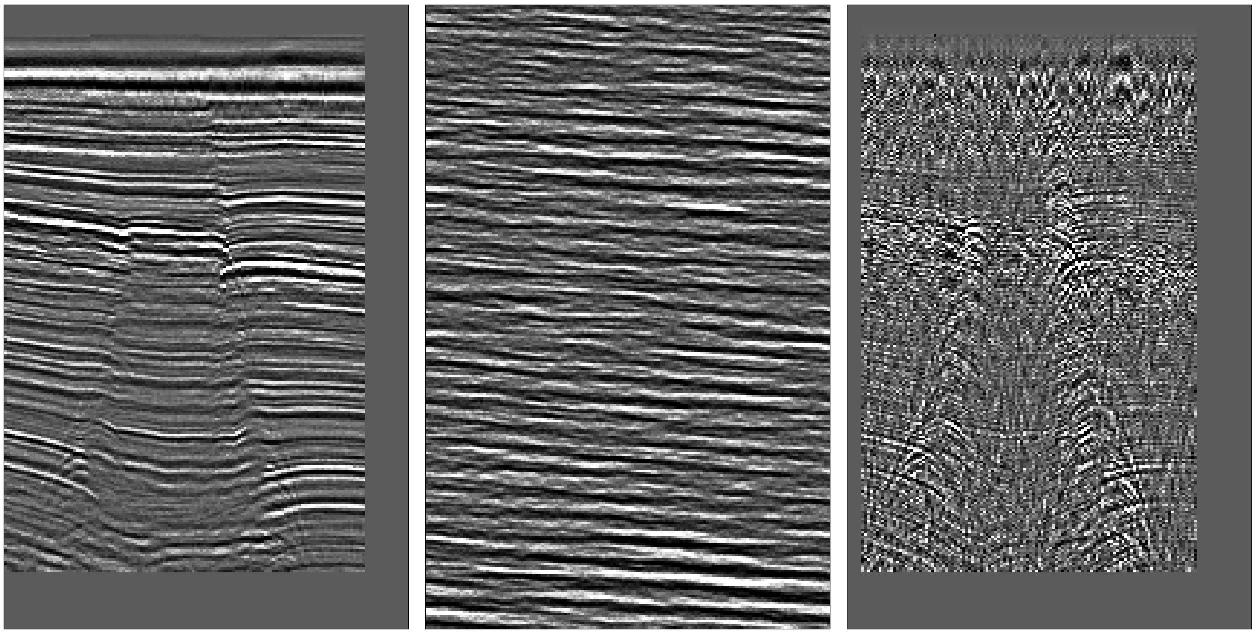
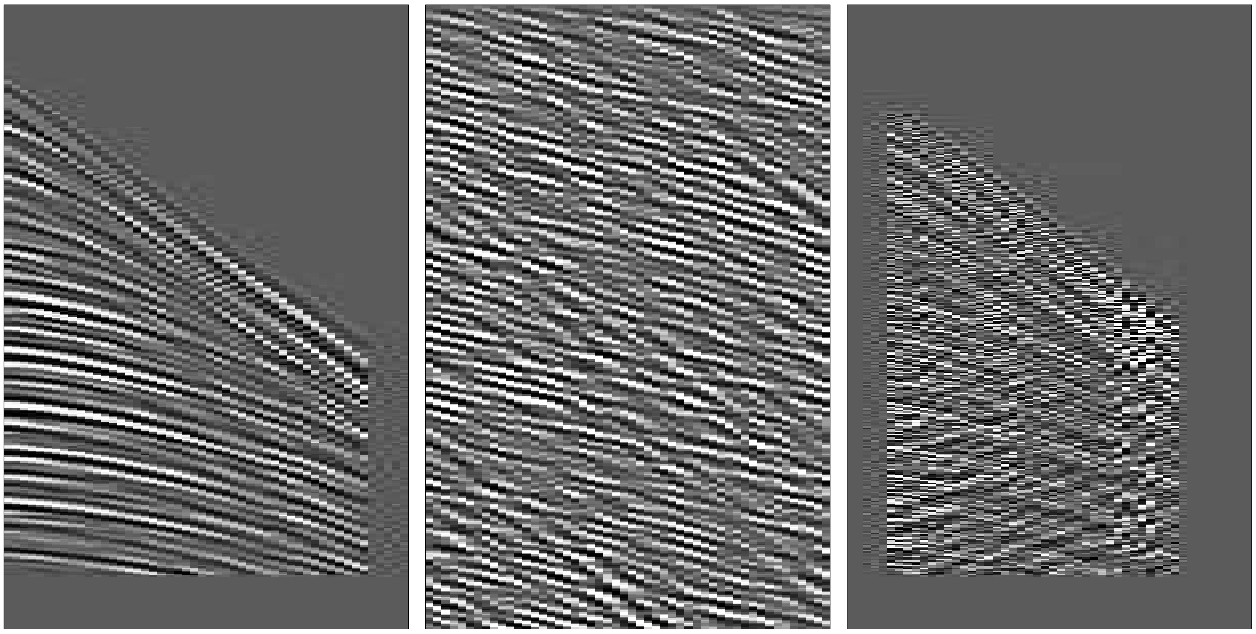


Figure 9: Alberta split spread. Stationary model. Deconvolution with enhanced statics.



Training Image Synthesized Image Training Image * PEF

Figure 10: Gulf of Mexico seismic section, modeled, and deconvolved. Strong horizontal layering has been suppressed giving a better view of the hyperbolas. Do you see any prospects in the synthetic data? The decon filter is the same 10×10 used on the everyday textures.



Training Image Synthesized Image Training Image * PEF

Figure 11: Shot profile. Homogenized. Deconvolved.

to pass thru specific values at well locations. Additional details available in the author's textbook freely available on the web.[2]

[2]http://sepwww.stanford.edu/sep/prof/.