Regularization strategies for time-lapse full-waveform inversion

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ABSTRACT

In this chapter, we study regularization strategies for time-lapse full-waveform inversion in VTI media. We show that total-variation regularization on the model differences improves the inversion results by removing fine-scale fluctuation. For integrated reservoir monitoring where geomechanical information is available, we can construct a geomechanics constrained regularization on the model differences, based on the third order elastic theory. With extra information on the reservoir compaction, we can impose a regularization on vertical velocity changes based on the RTM image alignment.

TIME-LAPSE FULL-WAVEFORM INVERSION

We formulate time-domain full waveform inversion as an optimization problem with the following objective function (Tarantola, 1984; Virieux and Operto, 2009)

$$J(\mathbf{m}) = \frac{1}{2}\|\mathbf{S}\mathbf{u}\,(\mathbf{m}) - \mathbf{d}\|_2^2, \tag{1}$$

where $\mathbf{m}$ is the subsurface model (velocity, anisotropic parameter, etc), $\mathbf{S}$ is the measurement operator, $\mathbf{u}$ is the synthetic wavefield and $\mathbf{d}$ is the observed data. For vertical transversely isotropic (VTI) media, the wavefield is computed by solving the following equation:

$$\frac{1}{v_z^2}\partial_t^2 \begin{pmatrix} p \\ r \end{pmatrix} - \begin{pmatrix} 1+2\varepsilon & \sqrt{1+2\delta} \\ \sqrt{1+2\delta} & 1 \end{pmatrix} \begin{pmatrix} \partial_x^2 + \partial_y^2 & 0 \\ 0 & \partial_{z'}^2 \end{pmatrix} \begin{pmatrix} p \\ r \end{pmatrix} = f, \tag{2}$$

where v_z is the acoustic velocity, ε and δ are two Thomsen parameters, p and r are vertical and horizontal stress components, and $\mathbf{f}$ is the source function. The synthetic wave field $\mathbf{u} = [p, r]^T$.

We estimate the subsurface model $\mathbf{m}^*$ by minimizing the following objective function

$$\mathbf{m}^* = \text{argmin}_{\mathbf{m}} J(\mathbf{m}). \tag{3}$$

Time-lapse FWI estimates the production-induced change in the subsurface using seismic data from a survey before production (baseline survey), and repeated surveys during production (monitor survey). Common time-lapse FWI strategies include parallel difference (estimating baseline and monitor model separately), sequential difference (using baseline model as the starting model for monitor model estimation), double difference (inverting the differential data) and joint inversion approach (estimating baseline and monitor model simultaneously).

In this chapter, we use a the joint inversion approach with the following objection function:

$$\begin{aligned} J(\mathbf{m}_{\mathrm{b}}, \mathbf{m}_{\mathrm{m}}) = & \frac{1}{2}\|\mathbf{S}_{\mathrm{b}}\mathbf{u}_{\mathrm{b}}\left(\mathbf{m}_{\mathrm{b}}\right) - \mathbf{d}_{\mathrm{b}}\|_2^2 \\ & + \frac{1}{2}\|\mathbf{S}_{\mathrm{m}}\mathbf{u}_{\mathrm{m}}\left(\mathbf{m}_{\mathrm{m}}\right) - \mathbf{d}_{\mathrm{m}}\|_2^2 \\ & + \frac{\alpha}{2}\|\mathbf{W}\mathbf{R}\left(\mathbf{m}_{\mathrm{m}} - \mathbf{m}_{\mathrm{b}} - \Delta\mathbf{m}_{\mathrm{prior}}\right)\|_a^b, \end{aligned} \tag{4}$$

where b is a substript indicating baseline variables, m is a subscript for monitor variables, α is the strength of the regularization term, $\mathbf{W}$ is a weighting function, $\mathbf{R}$ is regularization on the model difference $\mathbf{m}_{\mathrm{m}} - \mathbf{m}_{\mathrm{b}}$ and $\Delta\mathbf{m}_{\mathrm{prior}}$ is the a prior time-lapse model change. Regularization plays an important role in time-lapse full-waveform inversion (FWI), because of the non-repeatability issues. In practice, $\mathbf{R}$ can be an identity operator, promoting minimum norm solution. $\mathbf{R}$ can also be the gradient operator for the recovery of smooth time-lapse change.

In the following sections, we study several different approaches to construct the regularization term $\|\mathbf{W}\mathbf{R}\left(\mathbf{m}_{\mathrm{m}} - \mathbf{m}_{\mathrm{b}} - \Delta\mathbf{m}_{\mathrm{prior}}\right)\|_a^b$. We first study total-variation regularization, a common technique for noise removal. Next, we study the feasibility to regularize 4D FWI with geomechanics. Regularization based on baseline and monitor RTM image alignment is also discussed, and is potentially useful when knowledge about reservoir compaction is available.

TOTAL-VARIATION REGULARIZATION

Total-variation (TV) regularization (Rudin et al., 1992) is a technique that preserves sharp edges and removes fine-scale fluctuation in model simultaneously. The TV regularization is defined as,

$$TV(\mathbf{m}) = \int_{\boldsymbol{\Omega}} \sqrt{|\nabla\mathbf{m}|^{\mathbf{2}}}\mathbf{dx} = \|\mathbf{R}\mathbf{m}\|_{\mathbf{1}}^{\mathbf{1}}, \tag{5}$$

where Ω is the model space, and $\mathbf{Rm} \equiv \sqrt{|\nabla\mathbf{m}|^{\mathbf{2}}}$.

The TV regularization is not differentiable when the model is constant, and a positive constant ε is added to avoid the singularity in the derivative,

$$TV(\mathbf{m}) = \int_{\boldsymbol{\Omega}} \sqrt{|\nabla\mathbf{m}|^{\mathbf{2}} + \varepsilon^{\mathbf{2}}}\mathbf{dx}. \tag{6}$$

The value of the extra parameter ε needs to be set properly, which increases the difficulty in the application of TV regularization in 3D FWI.

In practice, we can approximate the gradient operator $\mathbf{R}$ by different norms. One popular choice is to replace $\sqrt{|\nabla \mathbf{m}|^2 + \varepsilon^2}$ by $|\nabla_x \mathbf{m}| + |\nabla_y \mathbf{m}| + |\nabla_z \mathbf{m}| + \varepsilon$. In this case, the TV regularization term becomes

$$TV_{\mathbf{fro}}(\mathbf{m}) = \int_{\mathbf{\Omega}} \left(|\nabla_{\mathbf{x}} \mathbf{m}| + |\nabla_{\mathbf{y}} \mathbf{m}| + |\nabla_{\mathbf{z}} \mathbf{m}| + \varepsilon\right) \mathbf{dx}, \tag{7}$$

where ε is irrelevant to the optimization problem, which reduces the difficulty in minimizing $TV_{\mathbf{fro}}(\mathbf{m})$.

Our time-lapse FWI objective function for this subsection is thus constructed as,

$$\begin{aligned} J(\mathbf{m}_{\mathrm{b}}, \mathbf{m}_{\mathrm{m}}) = & \frac{1}{2} \|\mathbf{S}_{\mathrm{b}} \mathbf{u}_{\mathrm{b}} \left(\mathbf{m}_{\mathrm{b}}\right) - \mathbf{d}_{\mathrm{b}}\|_2^2 \\ & + \frac{1}{2} \|\mathbf{S}_{\mathrm{m}} \mathbf{u}_{\mathrm{m}} \left(\mathbf{m}_{\mathrm{m}}\right) - \mathbf{d}_{\mathrm{m}}\|_2^2 \\ & + \frac{\lambda_1}{2} \|\mathbf{m}_{\mathrm{m}} - \mathbf{m}_{\mathrm{b}}\|_2^2 \\ & + \lambda_2 TV_{\mathbf{fro}}(\mathbf{m}_{\mathrm{m}} - \mathbf{m}_{\mathrm{b}}), \end{aligned} \tag{8}$$

where the first 2 lines of equation 8 are data fitting terms, the third line is minimum norm regularization, and the last line corresponds to total-variation regularization. The strength of the regularization terms λ_1 and λ_2 still need to be carefully choosen.

We use a simple synthetic model to demonstrate the effect of total-variation regularization. A layered model is constructed as baseline (Figure 1), and has time-lapse changes in Figure 2. We run FWI for 50 iterations using L-BFGS. The results from seismic data with offset up to 20 km are shown in Figure 3.

Results suggest that change in velocity is relatively easy to resolve. On the other hand, we can only correctly estimate $\Delta\eta$ at the shallow area, while results at the deeper area are completely unreliable. Minimum norm regularization leads to better results as shown in shown in Figure 4 by suppressing the time-lapse changes with negligible effect on the objective function. Total-variation (Figure 5) removes high frequency artifacts while maintaining the "blockly" model.

REGULARIZATION BASED ON GEOMECHANICS

The estimation of model prior $\mathbf{\Delta m}_{\mathrm{prior}}$ in equation 4 is challenging for integrated reservoir monitoring. In principle, we can estimate $\mathbf{\Delta m}_{\mathrm{prior}}$ from reservoir simulation results, based on the relation between velocity and differential pressure,

$$v = v_{\mathrm{inf}}(1 - Ae^{-P/P_0}), \tag{9}$$

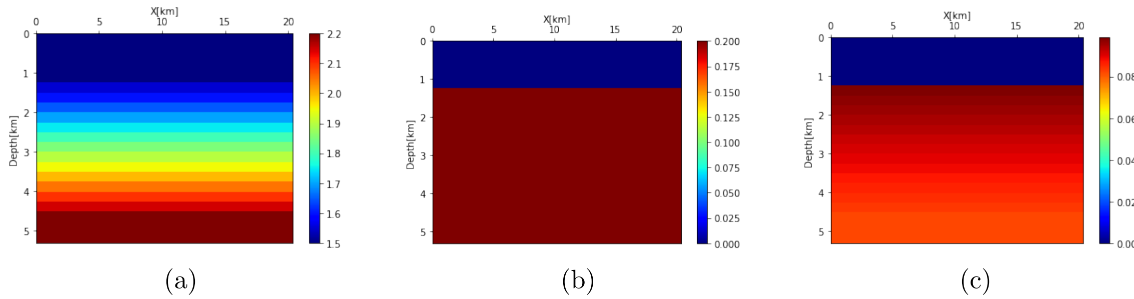


Figure 1: Baseline vertical velocity, ε, and δ. [**CR**]

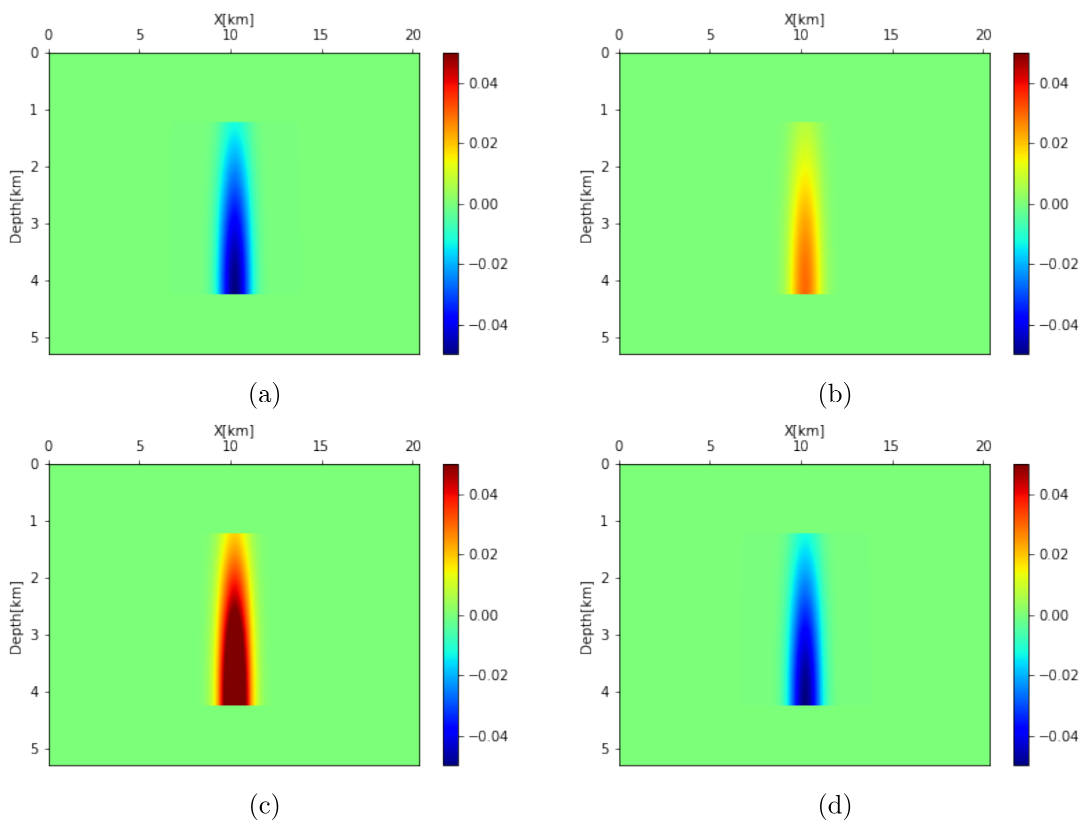


Figure 2: Time-lapse change in (a) vertical velocity, (b) ε, (c) δ and (d) η. [**CR**]

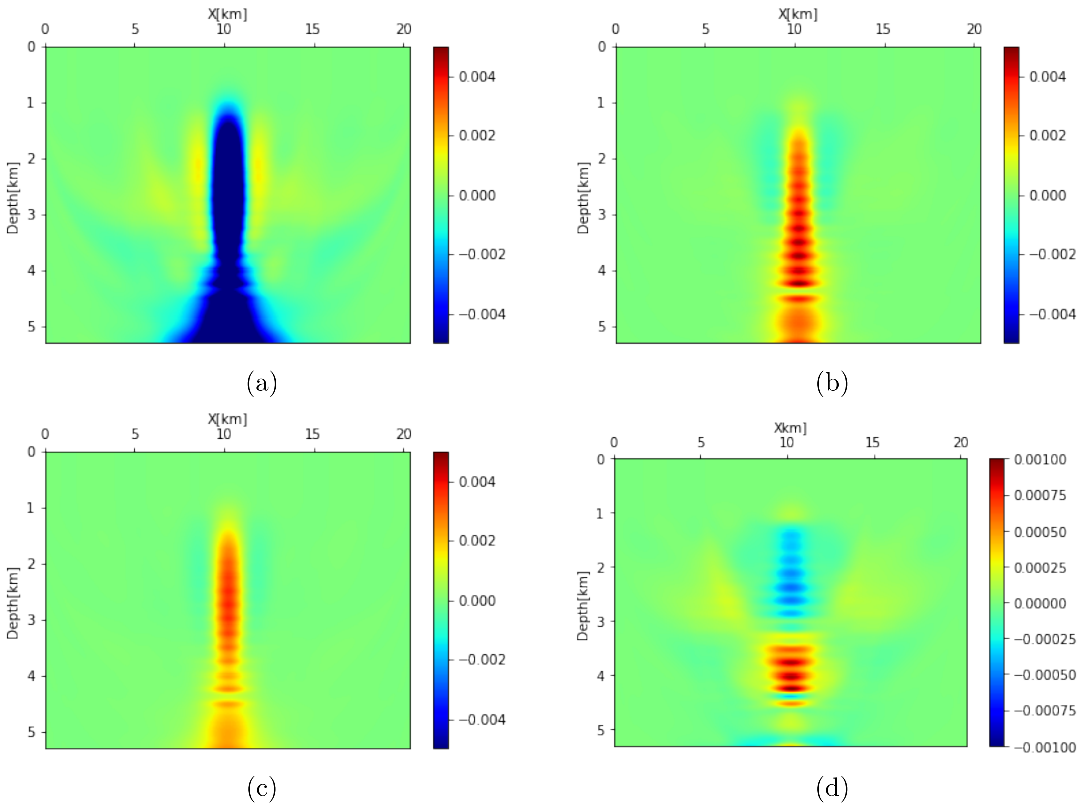
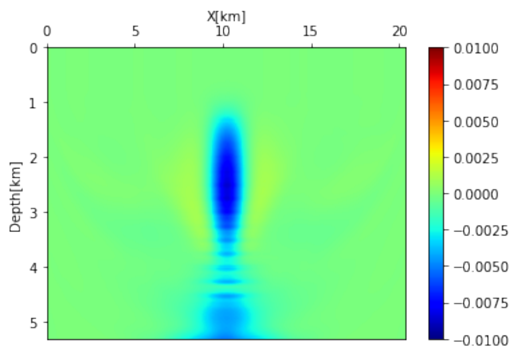
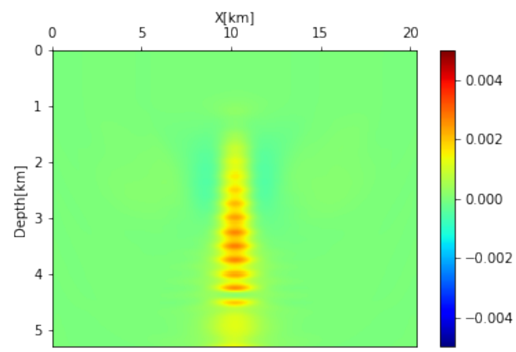


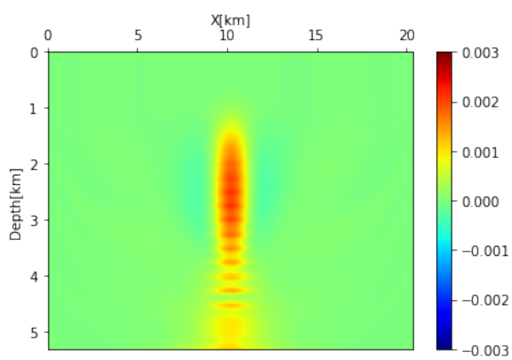
Figure 3: Estimated time-lapse change in (a) vertical velocity, (b) ε, (c) δ and (d) η. Offset up to 20 km, no regularization applied. [**CR**]



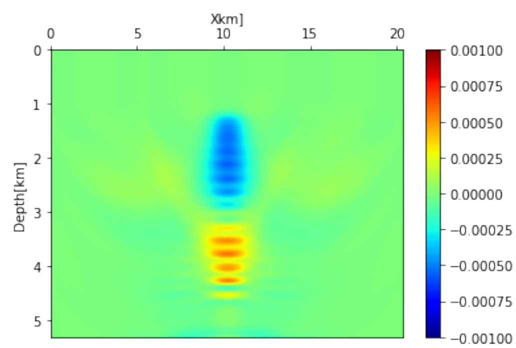
(a)



(b)



(c)



(d)

Figure 4: Estimated time-lapse change in (a) vertical velocity, (b) ε, (c) δ and (d) η. Offset up to 20 km, minimum norm regularization applied. [**CR**]

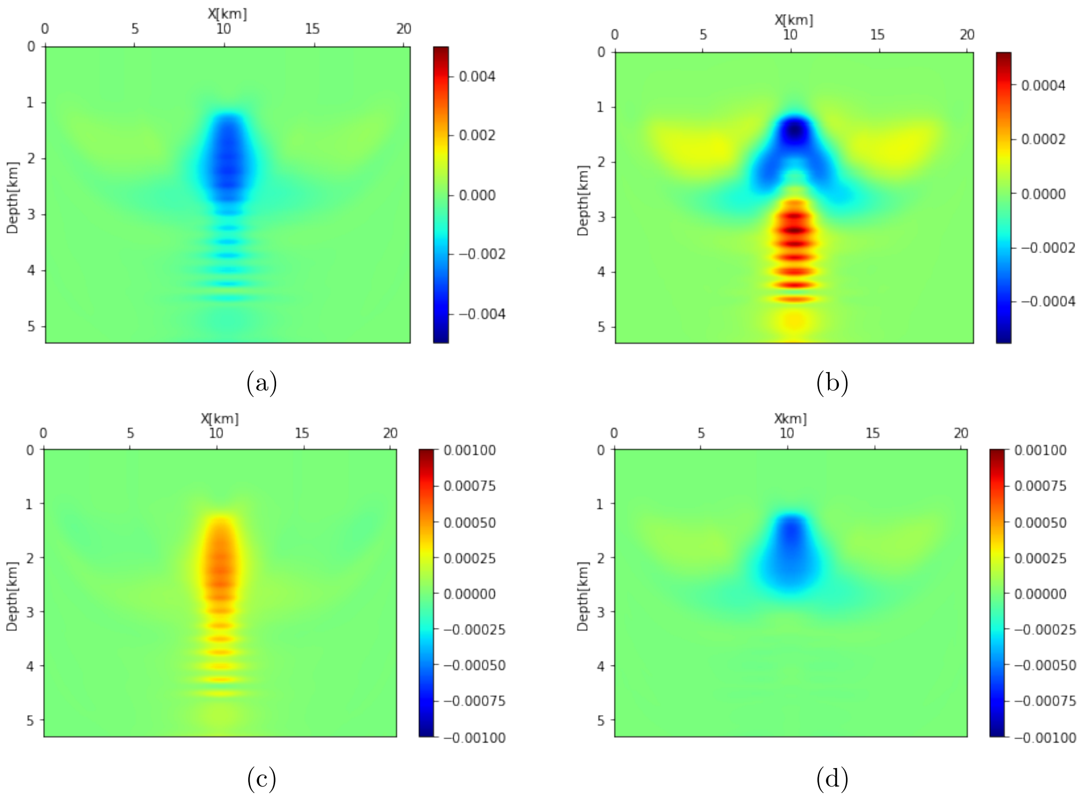


Figure 5: Estimated time-lapse change in (a) vertical velocity, (b) ε, (c) δ and (d) η. Offset up to 20 km, total-variation regularization applied. [**CR**]

where $v_{\rm inf}$, A and P_0 are fitting constants. We can also estimate $\Delta\mathbf{m}_{\rm prior}$ from geomechanical modeling (Landr and Stammeijer, 2004) using the following:

$$\frac{dv}{v} = -R\varepsilon_{\rm zz}, \tag{10}$$

where $\frac{dv}{v}$ is the fractional change in velocity and $\varepsilon_{\rm zz}$ is the vertical strain. The ratio R depends on the rock properties. Without sufficient data to fix the values of those fitting parameters, we cannot effectively integrate reservoir simulation or geomechanical modeling results into our time-lapse FWI objective function.

In this subsection we propose a non-local, non-convex regularization term on the time-lapse model change. We assume a linear relation between time-lapse velocity change and the prior information from geomechanical modeling, as follows:

$$\Delta\mathbf{m} \propto \Delta\mathbf{p}_{\rm prior}, \tag{11}$$

where $\Delta\mathbf{p}_{\rm prior}$ can be vertical strain, pressure change, etc. We formulate a shaping regularization term as

$$\left\| \frac{\Delta\mathbf{m}}{\|\Delta\mathbf{m}\|_2} - \frac{\Delta\mathbf{p}_{\rm prior}}{\|\Delta\mathbf{p}_{\rm prior}\|_2} \right\|_2^2, \tag{12}$$

where the scaling factor between velocity change and attribute change is eliminated by the normalization.

The regularization term in Equation 12 promotes time-lapse change $\Delta\mathbf{m}$ to have the same shape as the prior $\Delta\mathbf{p}_{\rm prior}$. As long as the linear relation in equation 11 holds, the regularization term in Equation 12 goes to zero.

With our proposed regularization strategies, the time-lapse FWI objective function can be written as:

$$\begin{aligned} J(\mathbf{m}_{\rm b}, \mathbf{m}_{\rm m}) = & \frac{1}{2}\|\mathbf{S}_{\rm b}\mathbf{u}_{\rm b}\left(\mathbf{m}_{\rm b}\right) - \mathbf{d}_{\rm b}\|_2^2 \\ & + \frac{1}{2}\|\mathbf{S}_{\rm m}\mathbf{u}_{\rm m}\left(\mathbf{m}_{\rm m}\right) - \mathbf{d}_{\rm m}\|_2^2 \\ & + \frac{\gamma}{2}\left\| \frac{\mathbf{m}_{\rm m} - \mathbf{m}_{\rm b}}{\|\mathbf{m}_{\rm m} - \mathbf{m}_{\rm b}\|_2} - \frac{\Delta\mathbf{p}_{\rm prior}}{\|\Delta\mathbf{p}_{\rm prior}\|_2} \right\|_2^2, \end{aligned} \tag{13}$$

where each term in the objective function is nonlinear, non-convex.

The non-convex nature of the FWI objective could potentially lead to the convergence to a local minimum (the cycle-skipping issue) and adding another nonlinear regularization term could be problematic.

REGULARIZATION BASED ON IMAGE ALIGNMENT

The vertical velocity is poorly constrained by surface seismic data for multi-parameter FWI in VTI medium. As a result, we may have different vertical velocity models that explain the observed travel time of the reflection data within the limits of the seismic resolution (Figure 6).

For time-lapse seismic data sets, data from the monitor survey has a time shift relative to the data from the baseline survey. To the first order, the time shifts are caused by reservoir compaction/dilation and velocity changes,

$$\frac{\Delta t}{t} = \frac{\Delta z}{z} - \frac{\Delta v}{v}, \tag{14}$$

where $\Delta t/t$ is the time strain, $\Delta z/z$ is the physical strain related to dialation/compaction and $\Delta v/v$ is the relative change in velocity. The ambiguity between physical strain and velocity changes further increase the difficulty to estimate changes in vertical velocity.

Prior information about reservoir compaction/overburden dilation may help us reduce the ambiguity and get better estimation of the vertical velocity changes. Inspired by recent studies from the Genesis field (Rickett et al., 2007), we consider a special case where reservoir compaction is negligible in depth and the time shift is caused mostly by vertical velocity change: $\Delta z/z \ll \Delta v/v$. The RTM images for baseline and monitor should align in depth, provided that we have an accurate estimate of the vertical velocity models for both vintages. The misalignment between baseline image and monitor image (Ma et al., 2017) should be explained by the vertical velocity change.

We use a two-step approach to construct the regularization term: estimate the relative shift in RTM images and then project it to the changes in vertical velocity. The first step is to estimate the image misalignment between baseline and monitor. The image misalignment can be estimated with a dynamic programming approach (Liner and Clapp, 2004; Hale, 2013). It can also be estimated based on the cross-correlation of two images (Hale, 2007). Once we have estimated the shift between baseline and monitor images, we can convert it to changes in vertical velocity by minimizing the following term,

$$\left\| \int_0^{z_0} \frac{\Delta v_z}{v_z^2} dz - \frac{\Delta z}{v_z(z_0)} \right\| \to 0, \tag{15}$$

where z_0 is the reference depth where we estimate the image misalignment and Δz is the relative shift between baseline RTM image and monitor RTM image, and Δv_z is the time-lapse change in vertical velocity that we want to estimate. Without further information, the misalignment Δz will be projected to changed in velocity Δv_z from the reflector to the surface.

Assuming that the change in vertical velocity is linearly correlated with vertical strain, $\Delta v_z \propto \Delta \varepsilon_{zz}$, and $\Delta \varepsilon_{zz}$ is available from geomechanical modeling, we can construct a better regularization of the vertical velocity,

$$\left\| \alpha \int_0^{z_0} \frac{\Delta \varepsilon_{zz}}{v_z^2} dz - \frac{\Delta z}{v_z(z_0)} \right\| \to 0 \tag{16}$$

$$\Delta v_z = \alpha \Delta \varepsilon_{zz} \tag{17}$$

where we minize the regularization term with respect to a scalar α, and $\Delta\varepsilon_{zz}$ is fixed.

Numerical examples are under development and are not available in this current report.

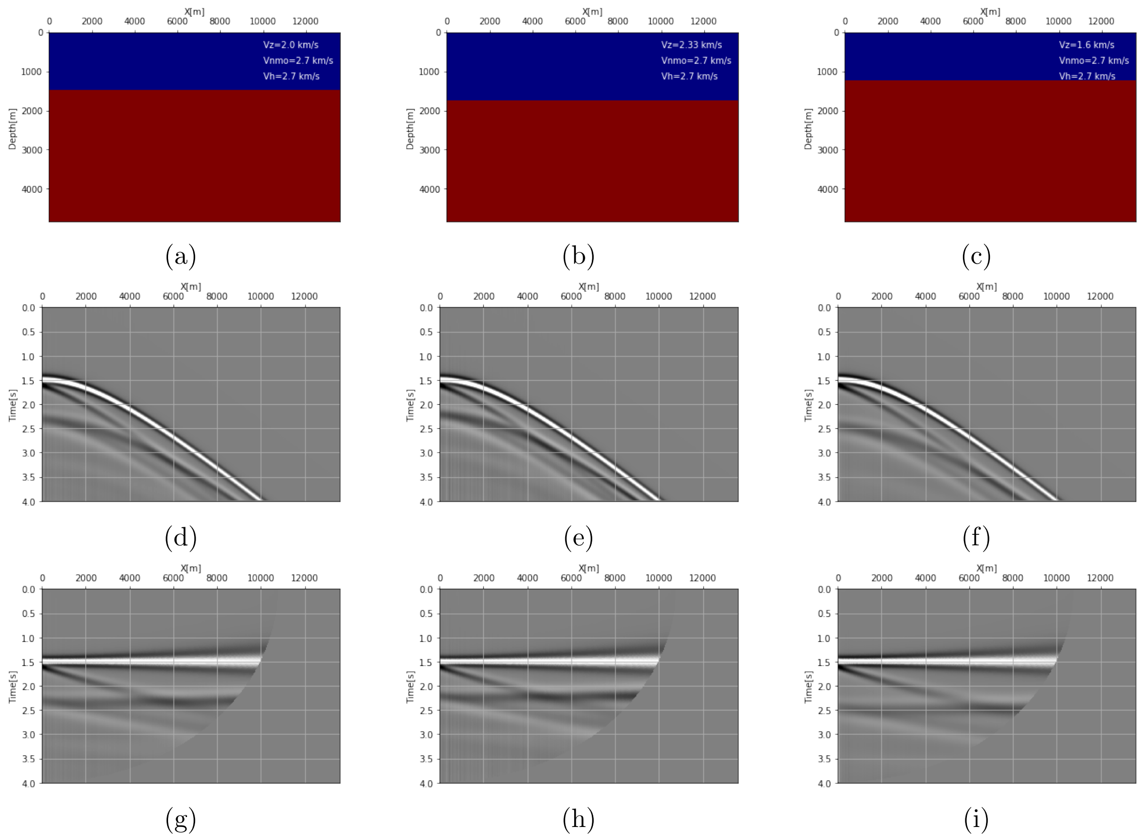


Figure 6: Top row: two layer velocity model, we have different vertical velocities and different depth of reflectors. Middle row: shot gathers from the two layer models. Bottom row: NMO correction of the shot gathers. [**CR**]

CONCLUSIONS

In this chapter, we study the feasible regularization strategies for time-lapse full-waveform inversion. Total-variation regularization is tested to remove fine-scale fluctuation. We propose a geomechanics constrained regularization on model difference. Regularization based on RTM image alignment is proprosed and numerical example is under developement.

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