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Wavefield extrapolation in laterally-varying tilted TI media

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ABSTRACT

A new wavefield extrapolation method has been developed that allows the propagation of waves in an anisotropic medium. The anisotropic medium considered here is transversely isotropic (TI) with an axis of symmetry. Our method applies an asymmetric explicit correction filter after the normal isotropic extrapolation operator. It is stable and suitable for laterally varying TI media. This new scheme is useful to extrapolate wavefields in a vertical transversely isotropic (VTI) medium in tilted coordinates. The explicit correction operator, designed by a weighted least-square method, is stable and accurate for the desired wavenumbers. Impulse responses from this scheme and the anisotropic phase-shift method are compared to illustrate the algorithm.

INTRODUCTION

Anisotropy has been shown to exist in many sedimentary rocks (Thomsen, 1986). If it is neglected in wavefield-extrapolation operators, reflectors in the subsurface, especially steeply dipping reflectors, will be mispositioned. Most sedimentary rocks can be approximated by a transversely isotropic medium with a symmetry axis. The symmetry axis can be vertical or tilted, and the corresponding media are called VTI or tilted TI media, respectively.

Although Kirchhoff migration can incorporate anisotropy into migration, it fails to handle the multi-pathing problem. Wave-equation-based methods are able to handle the multi-pathing problem and image the complicated subsurface structure. However, it is still challenging to image steeply dipping reflectors in the subsurface, such as a salt flank. Wavefield extrapolation in tilted coordinates (Etgen, 2002; Shan and Biondi, 2004) is useful for these steeply dipping reflectors. The energy related to these steeply dipping reflectors propagates almost horizontally and is greatly affected by the anisotropy of the sediment. In tilted coordinates, VTI media become tilted TI media in the extrapolation direction. It is useful therefore to develop a wavefield-extrapolation scheme for tilted TI media.

During the last decade, methods have been developed to incorporate anisotropy into wavefield extrapolation in TI media. As with isotropic extrapolation operators, anisotropic extrapolation operators include the implicit method (Ristow and Ruhl, 1997), phase-shift-plus-interpolation (PSPI) (Rousseau, 1997), non-stationary phase-shift (Ferguson and Margrave, 1998), explicit operator (Uzcategui, 1995; Zhang et al., 2001a,b), and reference anisotropic

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phase-shift with an explicit correction filter (Baumstein and Anderson, 2003).

In this paper, we incorporate anisotropy into wavefield extrapolation by adding an explicit anisotropic correction operator to the normal isotropic extrapolation operator. This new extrapolation scheme is capable of propagating waves in an anisotropic, heterogeneous medium with strong lateral variation. The explicit correction operator is designed by weighted, least-squares fitting to the true anisotropic phase-shift operator in the wavenumber domain (Thorbecke, 1997). In our method, we handle the lateral velocity variation by using a mixed-domain isotropic operator and the lateral anisotropic parameter variation by using explicit correction operator. At each depth level, we don't need to run the explicit correction operator for isotropic points. Therefore, it is efficient for a medium with both isotropic and anisotropic points. Furthermore, it is useful for VTI media with tilted coordinates where for each depth level most points are isotropic.

ANISOTROPIC PHASE-SHIFT IN TILTED TI MEDIA

In a VTI media, the phase velocity of qP- and qSV-waves in Thomsen's notation can be expressed as (Tsvankin, 1996):

$$\frac{V^2(\theta)}{V_{P0}^2} = 1+\varepsilon\sin^2(\theta)-\frac{f}{2}\pm\frac{f}{2}\sqrt{\left(1+\frac{2\varepsilon\sin^2(\theta)}{f}\right)^2-\frac{2(\varepsilon-\delta)\sin^2(2\theta)}{f}}, \tag{1}$$

where θ is the phase angle of the propagating wave, and $f = 1-(V_{S0}/V_{P0})^2$. V_{P0} and V_{S0} are the qP- and qSV- wave velocities in the vertical direction, respectively. ε and δ are anisotropy parameters defined by Thomsen (1986):

$$\varepsilon = \frac{C_{11}-C_{33}}{2C_{33}}, \delta = \frac{(C_{11}+C_{44})^2-(C_{33}-C_{44})^2}{2C_{33}(C_{33}-C_{44})},$$

where C_{ij} are elastic moduli. In equation (1), $V(\theta)$ the is qP-wave phase-velocity when the sign in front of the square root is positive, and the qSV-wave phase velocity for a negative sign.

If we rotate the symmetry axis from vertical to a tilted angle φ, we obtain the phase velocity of a tilted TI medium whose symmetry axis forms an angle φ with the vertical direction:

$$\frac{V^2(\theta,\varphi)}{V_{P0}^2} = 1+\varepsilon\sin^2(\theta-\varphi)-\frac{f}{2}\pm\frac{f}{2}\sqrt{\left(1+\frac{2\varepsilon\sin^2(\theta-\varphi)}{f}\right)^2-\frac{2(\varepsilon-\delta)\sin^2 2(\theta-\varphi)}{f}}. \tag{2}$$

Here, in contrast to equation (1), ε and δ are now defined in a direction tilted by the angle φ from the vertical direction. V_{P0} is the qP-wave velocity in the direction parallel to the symmetry axis.

For plane-wave propagation, the phase angle θ is related to the wavenumbers k_x and k_z by:

$$\sin\theta = \frac{V(\theta,\varphi)k_x}{\omega}, \qquad \cos\theta = \frac{V(\theta,\varphi)k_z}{\omega}, \tag{3}$$

where ω is the temporal frequency. Squaring equation (2) and substituting (3) into (2), we can obtain a dispersion relation equation:

$$d_4k_z^4 + d_3k_z^3 + d_2k_z^2 + d_1k_z + d_0 = 0, \tag{4}$$

where the coefficients d_0, d_1, d_2, d_3, and d_4 are as follows:

$$\begin{aligned}
d_0 &= (2+2\varepsilon\cos^2\varphi - f)\left(\tfrac{\omega}{V_{P0}}\right)^2 k_x^2 - \left(\tfrac{\omega}{V_{P0}}\right)^4 - \left[(1-f)(1+2\varepsilon\cos^2\varphi) + \tfrac{f}{2}(\varepsilon-\delta)\sin^2 2\varphi\right]k_x^4,\\
d_1 &= [2\varepsilon(1-f)\sin 2\varphi - f(\varepsilon-\delta)\sin 4\varphi]k_x^3 - \left(\tfrac{\omega}{V_{P0}}\right)^2 2\varepsilon\sin 2\varphi k_x,\\
d_2 &= [f(\varepsilon-\delta)\sin^2 2\varphi - 2(1-f)(1+\varepsilon) - 2f(\varepsilon-\delta)\cos^2 2\varphi]k_x^2 + \left(\tfrac{\omega}{V_{P0}}\right)^2 (2+2\varepsilon\sin^2\varphi - f),\\
d_3 &= [f(\varepsilon-\delta)\sin 4\varphi + 2\varepsilon(1-f)\sin 2\varphi]k_x,\\
d_4 &= f - 1 + 2\varepsilon(f-1)\sin^2\varphi - \tfrac{f}{2}(\varepsilon-\delta)\sin^2 2\varphi.
\end{aligned}$$

The dispersion relation equation (4) is a quartic equation. It can be solved analytically (Abramowitz and Stegun, 1972) or numerically by Newton's Method (Stoer and Bulirsch, 1992). Equation (4) has four roots, which are related to up-going and down-going qP- and qSV- waves, respectively. For a medium without lateral change in the velocity V_{P0} and anisotropy parameters ε and δ, the wavefield can be extrapolated by the phase-shift method (Gazdag, 1978):

$$P(z+\Delta z) = P(z)e^{-ik_z^a\Delta z}, \tag{5}$$

where k_z^a is one of the roots of equation (4).

EXTRAPOLATION OPERATOR IN LATERALLY VARYING MEDIA

The phase-shift method is effective, but it is not suitable for a strongly heterogeneous medium, where strong lateral changes are present in velocity as well as in the anisotropy parameters, ε and δ. This can be remedied by PSPI (Rousseau, 1997), explicit operator (Zhang et al., 2001a), or reference anisotropic phase-shift with an explicit correction filter (Baumstein and Anderson, 2003).

In this paper, we use an explicit anisotropic correction filter in addition to the normal isotropic operator. For each z step, we first regard the medium as an isotropic medium and extrapolate the wavefield using an isotropic operator with the velocity in the direction parallel to the symmetry axis. The isotropic operator can be the split-step method (Stoffa et al., 1990), the general screen propagator (Huang and Wu, 1996), or Fourier finite difference (FFD) (Ristow and Ruhl, 1994). Then we correct the wavefield with an explicit correction operator.

After we extrapolate the wavefield with an isotropic operator, the resulting error relative to anisotropic extrapolation is:

$$F(k_x) = e^{i\Delta z\Delta\phi(k_x)}, \tag{6}$$

where $\Delta\phi(k_x)$ is the difference between the isotropic wavenumber, k_z^i, and the anisotropic wavenumber, k_z^a, that satisfies

$$\Delta\phi(k_x) = k_z^a(V_{P0},\delta,\varepsilon,\varphi) - k_z^i(V_{P0}), \tag{7}$$

where,

$$k_z^i(V_{P0}) = \pm\sqrt{\left(\frac{\omega}{V_{P0}}\right)^2 - k_x^2},$$

and k_z^a is one of the four roots of the dispersion-relation equation (4). We design the explicit correction operator by weighted least squares. The obtained explicit operator is approximately the same as $F(k_x)$ in the wavenumber domain for desired wavenumbers.

EXPLICIT CORRECTION OPERATOR

Explicit extrapolation operators have been proved useful in isotropic wavefield extrapolation (Holberg, 1988; Blacquiere et al., 1989; Hale, 1991b,a; Thorbecke, 1997). They are also applied in wavefield extrapolation for TI media (Zhang et al., 2001a). For an isotropic or VTI medium, the extrapolation operator is symmetric and can be approximated by a cosine function series. For a tilted TI medium, k_z is not an even function of k_x, and the extrapolation operator is asymmetric. Thus, we need both the sine and cosine function series to approximate the correction operator in the wavenumber domain. In equation (6), $F(k_x)$ is not an even function, but can be divided $F(k_x)$ into two parts: even function $F^e(k_x)$ and odd function $F^o(k_x)$,

$$F^e(k_x) = \frac{1}{2}(F(k_x)+F(-k_x)), \tag{8}$$

$$F^o(k_x) = \frac{1}{2}(F(k_x)-F(-k_x)). \tag{9}$$

To design the explicit correction operator, we specify $F^e(k_x)$ in the form

$$F^e(k_x) = \sum_{n=0}^{N} a_n \cos(n\Delta x k_x), \tag{10}$$

and $F^o(k_x)$ in the form

$$F^o(k_x) = \sum_{n=1}^{N} b_n \sin(n\Delta x k_x), \tag{11}$$

where a_n, b_n are complex coefficients. These coefficients can be determined by the following weighted least-squares fitting goals:

$$\mathbf{W}(\mathbf{A}\mathbf{a}-\mathbf{f^e}) \approx \mathbf{0}, \tag{12}$$

$$\mathbf{W}(\mathbf{Bb}-\mathbf{f^o}) \approx \mathbf{0}, \tag{13}$$

where

$$\mathbf{a} = (a_0, a_1, \cdots, a_N)^T,$$

$$\mathbf{b} = (b_1, b_2, \cdots, b_N)^T.$$

$\mathbf{A}$ is an $(M+1)\times(N+1)$ matrix with elements $A_{mn} = \cos(mn\Delta k_x \Delta x)$, $m = 0,1,2,\cdots,M$, and $n = 0,1,2,\cdots,N$. $\mathbf{B}$ is an $M \times N$ matrix with elements $B_{mn} = \sin(mn\Delta k_x \Delta x)$, $m = 1,2,\cdots,M$, and $n = 1,2,\cdots,N$. $\mathbf{f^e}$ is a vector with elements $F^e(m\Delta k_x)$, $m = 0,1,2,\cdots,M$. $\mathbf{f^o}$ is a vector with elements $F^o(m\Delta k_x)$, $m = 1,2,\cdots,M$. $\mathbf{W}$ is a diagonal matrix with proper weights for the wavenumber k_x. One way to solve the fitting goal (12) is to do QR decomposition (Golub and Van Loan, 1996) of the matrix $\mathbf{WA}$: $\mathbf{WA} = \mathbf{QR}$, where $\mathbf{Q}$ is an orthogonal matrix and $\mathbf{R}$ is an upper triangular matrix. Then the coefficient vector $\mathbf{a}$ is given by

$$\mathbf{a} = \mathbf{R}^{-1}\mathbf{Q}^T\mathbf{W}\mathbf{f^e}. \tag{14}$$

We can solve the fitting goal in equation (13) and obtain the coefficient vector $\mathbf{b}$ in the same way. After we have the coefficient vectors $\mathbf{a}$ and $\mathbf{b}$, we can combine them into the coefficients for the explicit correction operator. From Fourier transform theory, it is well known that the inverse Fourier transform of the function $\cos(n\Delta x k_x)$ and $\sin(n\Delta x k_x)$ are:

$$\mathcal{F}^{-1}\{\cos(n\Delta x k_x)\} = \frac{1}{2}(\delta(x-n\Delta x)+\delta(x+n\Delta x)), \tag{15}$$

$$\mathcal{F}^{-1}\{\sin(n\Delta x k_x)\} = \frac{1}{2i}(\delta(x-n\Delta x)-\delta(x+n\Delta x)). \tag{16}$$

Thus, the inverse Fourier transform of the function $a_n\cos(n\Delta x k_x)+b_n\sin(n\Delta x k_x)$ is

$$\mathcal{F}^{-1}\{a_n\cos(n\Delta x k_x)+b_n\sin(n\Delta x k_x)\} = \frac{1}{2}(a_n+ib_n)\delta(x-n\Delta x)+\frac{1}{2}(a_n-ib_n)\delta(x+n\Delta x).$$

Therefore, the explicit correction operator is:

$$(c_{-N}, c_{-(N-1)}, \cdots, c_{-1}, c0, c_1, \cdots, c_{(N-1)}, c_N), \tag{17}$$

where $c_0 = a_0$, and

$$c_{-n} = \frac{1}{2}(a_n - ib_n), \;\; n = 1,2,\cdots,N,$$

$$c_n = \frac{1}{2}(a_n + ib_n), \;\; n = 1,2,\cdots,N.$$

In 3-D, based on the following trigonometric identity,

$$\cos(n\theta) = 2\cos(\theta)\cos[(n-1)\theta]-\cos[(n-2)\theta], \tag{18}$$

we can run McClellan transformations (McClellan and Parks, 1972; McClellan and Chan, 1977; Hale, 1991a) for the cosine terms. Similarly, based on the trigonometric identity:

$$\sin(n\theta) = 2\cos(\theta)\sin[(n-1)\theta]-\sin[(n-2)\theta], \tag{19}$$

we can design a recursive operator similar to McClellan transformations for the sine terms.

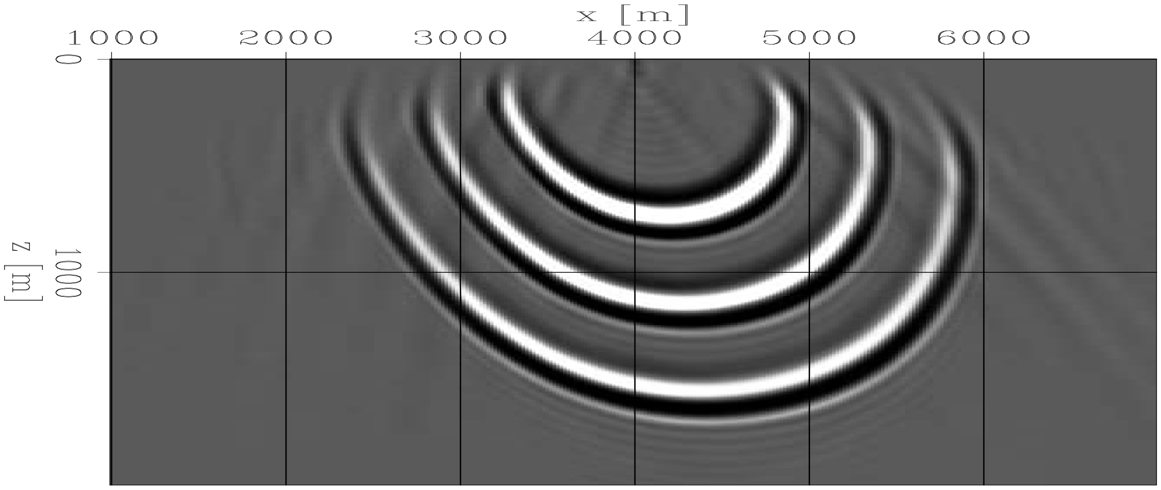

Figure 1: Impulse response of isotropic phase-shift with an anisotropic correction operator. guojian1-ico [CR]

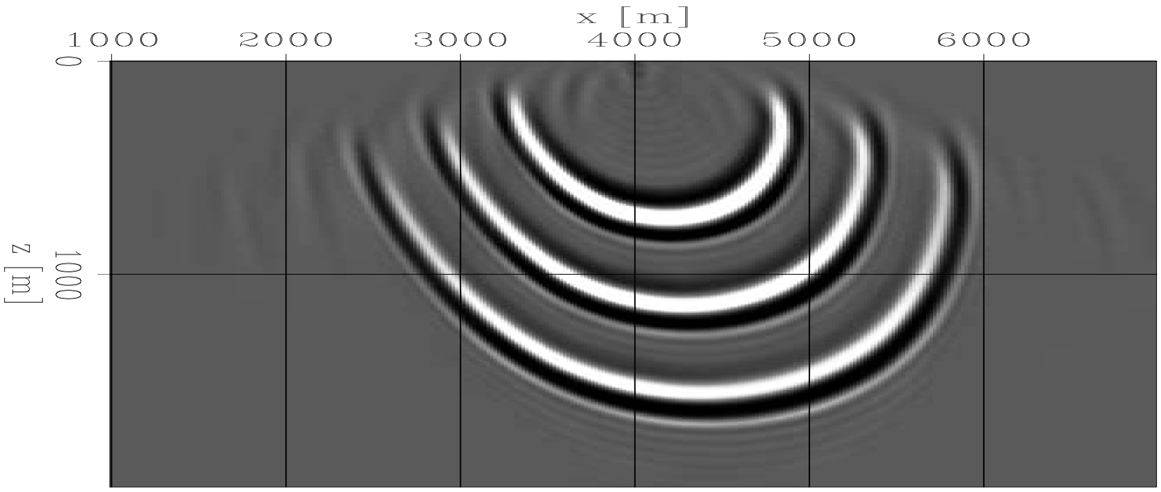

Figure 2: Impulse response of isotropic FFD with an anisotropic correction operator. guojian1-icoffd [CR]

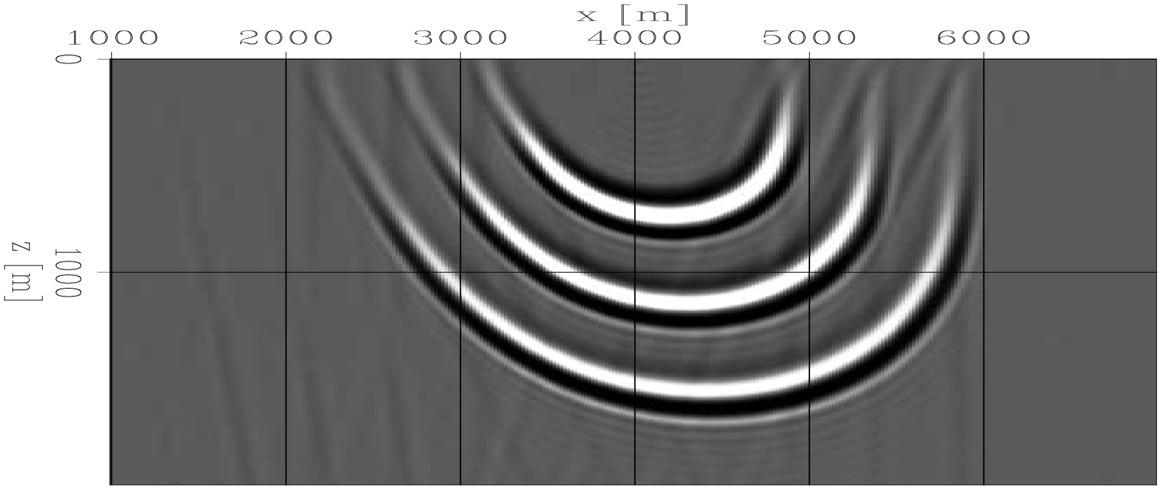

Figure 3: Impulse response of anisotropic phase-shift. guojian1-phsift [CR]

IMPULSE RESPONSE TESTS

The performance of an extrapolation operator can be verified by its impulse response. Figures 1-3 are the impulse responses of the qP-wave in the same medium using three different methods. The symmetry axis of the medium is tilted 30° from the vertical direction. The qP- and qSV-wave velocities in the direction parallel to the symmetry axis are 2000 m/s and 1100 m/s, respectively. The anisotropic parameters ε and δ are 0.4 and 0.2, respectively. The impulse location is at 4000 m.

Figure 1 is the impulse response of isotropic phase-shift with an anisotropic correction operator. Figure 2 is the impulse response of isotropic FFD with an anisotropic correction operator. The reference velocity for the FFD is 1500 m/s. We use 39 points for the explicit correction operator in Figures 1 and 2. Since the medium is homogeneous, we can also extrapolate the wavefield with anisotropic phase-shift (equation (5)). Figure 3 is the impulse response of anisotropic phase-shift. Comparing Figures 1, 2 and 3, the impulse response of the isotropic operator with an anisotropic correction operator is the same as that of the anisotropic phase-shift method for propagating angles up to almost 80°. It is different from anisotropic phase-shift for higher angles because the explicit correction operator is not same as the one for anisotropic phase-shift for wavenumbers close to the evanescent area.

CONCLUSION

We describe a new anisotropic wavefield-extrapolation scheme. This new scheme uses an explicit anisotropic correction filter in addition to the normal isotropic extrapolation operator. It can extrapolate wavefields in a laterally varying TI medium with a symmetry axis. It is effective since it uses only the normal isotropic extrapolation operator in isotropic regions. The comparison of impulse responses shows that the new scheme is accurate for angles to almost 80° in a homogeneous medium. More work is needed to test the scheme on complicated models.

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