Near-Surface Scholte-Wave Velocities at Ekofisk from Short Noise Recordings by Chaotic Wavefield Gradiometry

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We propose a new approach for imaging the near surface using chaotic inter-
terface waves present in the ambient seismic field. Unlike seismic interfer-
ometry, our technique does not rely on cross-correlations to obtain the Green’s
function between two seismic receivers. Rather, it relies on the local mea-
surements of phase-velocity obtained directly from the ratio between second-
order temporal and spatial derivatives of the wavefield. We process 10 min-
utes of ambient seismic noise recording made at a large and dense array in-
stalled over Ekofisk. We image a subsidence-induced geomechanical imprint
on the Scholte-wave phase velocities in the near surface. This result compares
very well to the results from seismic-noise cross-correlation tomography, but
is obtained using an order of a magnitude shorter recording.
1. Introduction

Geophysicists have long attempted to image the subsurface using chaotic wavefields. Here, we define a chaotic wavefield as a wave field where many wavefronts interfere as they travel in different directions. Such a wavefield can, for example, be formed either by many sources acting randomly in space and time or by multiple scattering from a single source in a complex medium. Almost all approaches have centered on cross-correlating long ambient seismic recordings made at two stations to obtain the interstation seismic response. After the pioneering work of Aki [1957] and Claerbout [1968], seismic interferometry found wide application in the retrieval and imaging of surface waves on a global, regional and local scale [Campillo and Paul, 2003; Shapiro and Campillo, 2004; Shapiro et al., 2005; Gerstoft et al., 2006; Yao et al., 2006; Stewart, 2006; Lin et al., 2008; Dellinger and Yu, 2009; Bussat and Kugler, 2011; de Ridder and Dellinger, 2011]. Cross-correlating recordings made at two stations is an operation that ignores the amplitude dependency of nearby recordings.

The advent of ever larger and denser arrays covering the Earth’s surface start to provide complete recordings of wavefields that at low frequencies are not aliased in either time or space. Such recordings essentially entail a direct measurement of local medium properties, because the temporal and spatial derivatives of the wavefield are related by the medium properties through the wave equation.

Wavefield gradiometry is not a new concept in seismology. Langston [2007a, b, c] proposed to invert the first-order spatial and temporal derivatives of the observed wavefield amplitudes for a set of wavefield coefficients. These derivatives can be related to the local ray parameter, local wave directionality, local geometrical spreading, and local radiation...
pattern. This technique was successfully applied on recordings of the Embayment Seismic Excitation Experiment in a one-dimensional linear array [Langston, 2007a, c], in two dimensions using local earthquakes recorded by a local array near Moscow Tennessee [Langston, 2007b] and on several earthquakes recorded by USAArray [Liang and Langston, 2009].

The fundamental assumption of this approach is that the wavefield at each point consists of noninterfering plane waves [Langston, 2007a, b]. This assumption limits the use of wavefield gradiometry to nonchaotic wavefields where specific arrivals can be identified and isolated. Coherent interfering seismic noise leads to deteriorating results [Poppeliers et al., 2013]. In this paper, we first propose a strategy for wavefield gradiometry for a chaotic wavefield. This strategy relies on posing a scalar wave equation to model a wavefield composed of single-mode interface waves. We then apply the method to image Scholte waves present at low frequencies in the ambient seismic noise recordings made by an ocean bottom cable (OBC) array at Ekofisk.

2. Chaotic Wavefield Gradiometry

Propagation of a single dispersive interface wave mode in two dimensions is governed by a two-dimensional scalar wave equation for each frequency component of the wavefield:

\[ c^2(x, \omega) \nabla^2 \hat{u}(x, \omega) + \omega^2 \hat{u}(x, \omega) = -\hat{s}(x, \omega), \tag{1} \]

where \( u \) is a scalar field variable (for example, pressure or the vertical component of particle velocity) that is observable in time and space, and \( \hat{u} \) is its Fourier transformed dual; \( s \) is a generalized source term, and \( \hat{s} \) is its Fourier transformed dual; \( \nabla^2 \) is the Laplace operator acting on the two spatial dimensions, and \( c \) is the phase velocity. If we applied a narrow
bandpass filter (with central frequency $\omega'$), we can neglect the frequency dependence of the phase velocity. The source distribution is generally unknown for chaotic wavefields. But in the absence of strong local sources, we can neglect the source distribution within the area of recording, $\hat{s}(x, \omega) = 0$. In the time domain, a filtered recording would obey the following:

$$\left[ \nabla^2 u(x, t) \right] c^2_{\omega'}(x) = \left[ \partial^2_t u(x, t) \right].$$  \hspace{1cm} (2)

where $\partial^2_t$ is the second-order derivative acting on the time dimension, and $c_{\omega'}$ is the phase velocity for this central frequency.

If recordings of a wavefield are made on a sufficiently dense regular or irregular grid, and we can evaluate the spatial and temporal second-order derivatives by finite differences, the phase velocity would be their ratio. Let $u(x, t)$ denote the discrete recordings of a continuous wavefield, and $D_{tt}$ and $D_{xx}$ are (usually sparse) matrices containing finite-difference approximations of the second-order derivative operators applied to the wavefield in time and space, respectively. Then, for each time sample $u_i = u(x, t_i)$, equation 2 can be written in matrix form as follows:

$$W \text{diag} \{ D_{xx} u_i \} \ c^2_{\omega'}(x) = WD_{tt} u_{i-1,i,i+1},$$  \hspace{1cm} (3)

where $D_{tt}$ operates on the current and two adjacent time samples (for a second-order finite difference approximation to the second-order derivative in time). The notation $\text{diag} \{ \}$ denotes a diagonal operator specifying the elements on the diagonal between $\{ \}$. We can discard locations with poor measurements or where we have a poor approximation of the finite difference operators using a masking operator, $W$, which has the structure of a diagonal matrix with ones and zeros on the diagonal elements. Equation 3 has the
structure \( \mathbf{F}_i \mathbf{m} = \mathbf{b}_i \), where \( \mathbf{m} = c_w^2(x) \), \( \mathbf{F}_i = \text{diag} \{ \mathbf{D}_{xx} \mathbf{u}_i \} \), and \( \mathbf{b}_i = \mathbf{D}_{tt} \mathbf{u}_{i-1,i,i+1} \). We formed these data-fitting equations for a set of \( N_t \) time samples in the data. An estimate for \( \mathbf{m} \) is found by least-squares through inverting the following:

\[
\sum_{i=1}^{N_t} \mathbf{F}_i^\dagger \mathbf{W} \mathbf{F}_i \mathbf{m} = \sum_{i=1}^{N_t} \mathbf{F}_i^\dagger \mathbf{W} \mathbf{b}_i, \tag{4}
\]

where \( \dagger \) denotes matrix adjoint. The solution for each element of \( \mathbf{m}(x) \) is essentially an independent division of the second-order temporal derivative of the wavefield by the second-order spatial derivative of the wavefield. This ratio is evaluated at each point in space and averaged over all recording time by linear regression. We implicitly assumed that the velocity does not change significantly across the length of the spatial finite-difference stencil.

We split \( \mathbf{m} \) into a homogeneous background and a perturbation, \( \mathbf{m} = m_0 + \Delta \mathbf{m} \). This separation is a tool in our strategy to overcome the error of the finite-difference approximation of the second-order spatial derivatives, described in the next section. The homogeneous background is estimated by computing the ratio between the second-order temporal derivative and second-order spatial derivative averaged over all time and all points in space as follows:

\[
m_0 = \frac{1}{X} \sum_{j=1}^{N_s} \text{diag} \left\{ \sum_{i=1}^{N_t} \mathbf{F}_i^\dagger \mathbf{W} \mathbf{b}_i \right\}^{-1}_j \text{diag} \left\{ \sum_{i=1}^{N_t} \mathbf{F}_i^\dagger \mathbf{W} \mathbf{F}_i \right\}_j, \tag{5}\]

where \( j \) is the index over the elements of the diagonals, \( N_s \) is the total number of stations, and \( X \) is the cardinality of the set in the masking matrix, \( \mathbf{W} \). We believe the medium parameters should not vary rapidly as a function of space. Therefore, we wish to constrain the model by a second-order Tikhonov regularization (penalizing spatial second-order derivatives) by adding the model-styling goal \( \epsilon \nabla^2 \mathbf{m} = 0 \), where \( \epsilon \) determines the
importance of the model-styling goal versus the data-fitting equations. The Laplacian for
regularization is evaluated using the same finite-difference approximation $D_{xx}$ as before.
The least-squares estimator for $\Delta m$ is given by inverting the following:

$$
\Delta m = \left[ \sum_{i=1}^{N_t} F_i^\dagger W^\dagger W F_i + \epsilon^2 D_{xx} \right]^{-1} \left( \sum_{i=1}^{N_t} F_i^\dagger W^\dagger W b_i - \sum_{i=1}^{N_t} F_i^\dagger W^\dagger W F_i m_0 \right).
$$

(6)

Notice that the solution becomes a simple linear regression for each point in space when
we set $\epsilon = 0$. Finally we can retrieve the phase velocity, $c_\omega'(x)$, from the inverted model
perturbation using $c_\omega'(x) = \sqrt{m_0 + \Delta m(x)}$. This method for imaging a chaotic wavefield
does not employ cross-correlations.

3. Imaging the near-surface at Ekofisk oil field using short ambient-seismic
recordings

Ekofisk oil field has had a Life of Field Seismic (LoFS) four-component optical-sensor
array installed since 2010 [Eriksrud, 2010]. The array has dense in-line and sparse cross-
line station spacing (50 m and 300 m, respectively). The main objective of this installation
is to record during seismic surveying for production-related time-lapse surveying [Folstad
et al., 2010]. However, even during periods of no active source seismic acquisition, the
pressure sensors of the installation record strong seismic energy in the frequency band
0.35 to 1.35 Hz known as microseism noise [de Ridder, 2014]. This microseism noise at
Ekofisk is found to be dominated by fundamental-mode Scholte waves propagating along
the seafloor. Below 0.8 Hz, these waves are recorded not aliased in both the in-line and
cross-line directions de Ridder [2014]. The Scholte waves propagating through the near
surface at Ekofisk exhibit azimuthal anisotropy [de Ridder et al., 2014; Kazinnik et al.,
2014], but for the purposes of this study, we neglect this effect.
A 10-minute recording was bandpass filtered between 0.6 and 0.8 Hz and subsequently down-sampled to a 0.1-second sampling rate. The spectra of the recordings are shaped to be as a Hann taper, so the original spectrum of the ambient seismic field does not bias the subsequent measurements. Figure 1a contains an amplitude map showing a time sample of the wavefield. We observe a chaotic wavefield: a wavefield that is composed of many interfering waves that propagate in different directions, such that the wavefield amplitudes may be modeled as forming a stochastic process. For each time sample, we evaluated the second-order temporal derivative using a second-order regular finite-difference stencil applied to the current plus the two adjacent time samples, $(+1, -2, +1)$. The second-order spatial derivative at the location of each station was evaluated using an irregular finite-difference stencil using the recordings at nearby stations. The coefficients for this operator are different for each station and are computed by inverting a second-order Taylor series expansion on the geometric distribution of the nearby stations [Huiskamp, 1991].

Figure 1b shows the values of the second-order derivative in time versus the second-order derivative in space for a station near UTM (514,6261) km and a station near UTM (514,6263) km. These two stations are located over areas known to exhibit phase velocities above and below the average phase velocity, respectively. A simple linear regression shows that the ratio between the second-order temporal and spatial derivatives of the wavefield is distinctively different between both stations. Over the fast area we found a 95% confidence interval for the phase velocity of 506.1-508.8 m/s and over the slow area, we found 490.5-492.9 m/s). The coefficient of determination for each regression is respectively 0.966 and 0.957. This coefficient indicates that linear regression is a good
statistical model for the relationship between the second-order derivative in time and the second-order derivative in space of the chaotic wavefield.

For each station in the Ekofisk array, we selected the 36 nearby stations that lie farthest away from but within a radius of 400 m. If fewer then 36 stations were available for a central node, we discarded the regression equation for this station using the masking operator. These criteria ensured that the finite-difference stencils on top of the majority of stations were very similar and exhibited a consistent error. Consequently, we were not able to run this method near the edges of the array and near the platform where the array is interrupted. The error in finite differences can be considerable when the spacing is close to the Nyquist sampling. This error causes an underestimation of the second-order spatial finite differences and an overestimation of the phase velocity. The finite-difference stencil is therefore scaled such that the average phase velocity matches the dispersion of the fundamental-mode Scholte-waves as found by de Ridder [2014].

First, we choose $\epsilon = 0$ and consequently did a simple linear regression for each grid point in space. The result is a map of phase velocity at each station location (Figure 2a). Each measurement comes with a 95% confidence interval (Figure 2b) that indicates our velocities have an uncertainty that is generally less then 3 m/s. Finally, we found that linear regression is a good statistical model for the relationship between the second-order derivative in time and the second-order derivative in space of the chaotic wavefield as measured over the entire area. However, without regularization, the phase-velocity map (Figure 2a) exhibits unreasonably rapid spatial variations of the velocities.
Next, we applied a regularization strength $\epsilon = 5$, selected by comparing the reduction of the standard deviation of the model space versus increasing regularization strength (Figure S1). The result is a map of velocity at each station location. We interpolated this map using cubic splines in tension \cite{Wessel1998} with tension coefficient $10^{-7}$. Figure 3a contains the inversion by chaotic wavefield gradiometry. Figure 3b contains the inversion by chaotic wavefield gradiometry and interpolated using cubic splines in tension. The tension coefficient was chosen as weak as possible. But, the nature of the interpolation method produces a smoother map.

We compared the phase-velocity map obtained using chaotic wavefield gradiometry with a phase-velocity map obtained using seismic interferometry (Figure 3c) \cite{deRidder2014}. In that study, a little over 40 hours of ambient seismic recordings were cross-correlated to produce virtual seismic sources. These sources were imaged using an eikonal tomography technique to produce the phase velocity map in Figure 3c.

The phase velocity image obtained using chaotic wavefield gradiometry (Figure 3b) matches the long wavelength features in the image obtained by seismic interferometry and eikonal tomography (Figure 3c) quite well. The velocity map in Figure 3b is much smoother than the velocity map in Figure 3c. There may be several causes for this difference. We used splines in tension to interpolate the velocity map from Figure 3a, which suppresses spatially small velocity variations. The regularization of the inversion enforces a smooth velocity map. Lastly, we assumed the velocity does not change significantly across the length of the spatial finite-difference stencil, making it impossible to image velocity variations below the length of the spatial finite-difference stencil.
The positive and negative anomalies of the phase velocity image obtained using chaotic wavefield gradiometry (Figure 3a) were smaller than the corresponding anomalies in the image obtained by seismic interferometry and eikonal tomography (Figure 3c). This disparity could be a result of the error in the spatial finite-difference stencil, the regularization of the chaotic wavefield gradiometry, or a spurious feature of the eikonal tomography after seismic interferometry.

Rapid pressure depletion in the early phase of production and weakening due to subsequent water injection caused more than 9 meters of sea floor subsidence over the Ekofisk field [Hermansen et al., 1997; Lyngnes et al., 2013]. The phase velocity maps of the fundamental-mode Scholte-waves at Ekofisk generally show a high velocity anomaly in the center of the array surrounded by a ring of low velocities. At the southern end of the array, we found higher phase velocities again. The ring of low velocities coincides with the edge of the subsidence bowl where the sea floor is in a state of extensional stress in the direction away from the center of the subsidence bowl [de Ridder et al., 2014; Kazinnik et al., 2014]. These extensional stresses lower the shear strength in the near surface and resulted in lower Scholte-wave velocities.

4. Conclusions

We presented a new method to extract surface-wave phase velocities from ambient seismic recordings: chaotic wavefield gradiometry. The method is based on evaluating the second-order spatial and temporal derivatives in the two-dimensional scalar wave equation to directly infer phase velocities. This method does not employ cross-correlations.
in contrast with conventional methods to image chaotic ambient seismic noise based on
seismic interferometry.

We applied the method on field data recorded by a permanent ocean bottom cable
array installed over Ekofisk. We produced qualitatively accurate phase-velocity maps
using very short recordings (as little as 10 minutes). We imaged a subsidence-induced
geomechanical imprint on the Scholte wave velocities over Ekofisk. The results obtained
by chaotic wavefield gradiometry compared very well to the results from seismic-noise
cross-correlation tomography. However, they were obtained from as little as 10 minutes of
recording instead of 40 hours of recording. Because this method also requires very little
computational resources it may offer a good opportunity for real-time monitoring using
ambient seismic noise.

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References


Eriksrud, M. (2010), Towards the optical seismic era in reservoir monitoring, First Break, 28, 105–111.


Lyngnes, B., H. Landa, K. Ringen, and N. Haller (2013), Life of Field Seismic at Ekofisk - Utilizing 4D seismic for evaluating well target, in 75th *Conference and Exhibition, EAGE, Extended Abstracts*, p. We 12 09.


Figure 1. a) Snapshot of seismic noise filtered between 0.6 and 0.8 Hz. b) Scatter plot of the values of the second-order spatial versus the second-order time derivative for two locations; a station near (514, 6261) km (red) and a station near (514, 6263) km (blue); see arrows in (a). The red and blue lines in (b) indicate the 95% confidence intervals for the slope of a linear regression.

Figure 2. a) Phase velocity by linear regression. b) 95% Confidence Bounds for the linear regression in (a). c) Coefficient of determination for the linear regression in (a).
Figure 3. a) Phase velocity image at 0.7 Hz obtained using chaotic wavefield gradiometry using 10 minutes of ambient seismic recordings. b) The phase velocity image at 0.7 Hz from (a) interpolated by cubic splines in tension. c) Phase velocity image at 0.7 Hz obtained using eikonal tomography on virtual seismic sources obtained from cross-correlations of almost 40 hours of ambient seismic recordings. (Reproduced from de Ridder (2014).)