Rock Physics

Sensitivity of seismic parameters to critical reservoir properties

\[ \text{e.g. porosity, lithofacies, pore fluid type, saturation, pore pressure} \]

Introduction

- With recent improvement in seismic data quality need/ability to quantify their interpretation in terms of:
  - Hydrocarbon detection
  - Reservoir characterization
  - Reservoir monitoring (4D seismics)
Reference

- Avseth, Mukerji & Mavko 2005 *Quantitative Seismic Interpretation*, Cambridge Univ. Press

(MUST read WANG, 2001 !! Download from www)

**REVIEW** of physical properties influencing Vp and Vs

![Diagram showing the effects of different rock properties on P and S-wave velocities and their ratios. (After Talham and McCormack, 1991.)](image)
Relationship between velocity (impedance) and porosity and lithofacies

• Rock physics analysis of well logs and cores, together with a geological model, leads to sensible relationships.
• Use of classical empirical trends (e.g. Wyllie time average relationship) is now frowned upon

Bounds on elastic moduli

• *Effective-medium models* predict whole rock elastic moduli of a mixture of grains and pores.
• Need to know:
  – *Volume fractions* of constituents
  – *Elastic moduli* of various constituents
  – *Geometric details* of relationships between constituents
Voigt and Reuss bounds

- When we only specify volume fractions and elastic moduli of constituents can only predict upper and lower bounds on the moduli of the composite rock.
- These bounds are useful, since well logs give accurate information on volumes and moduli but are poor on grain and pore microstructure/geometry.

Voigt upper bound

- There is no way that nature can put together a mixture of constituents that is elastically stiffer than the simple arithmetic average of the constituent moduli.

The Voigt upper bound on the effective elastic modulus, $M_V$, of a mixture of $N$ material phases is

$$M_V = \sum_{i=1}^{N} f_i M_i$$

(1.1)

with

- $f_i$, the volume fraction of the $i$th constituent
- $M_i$, the elastic modulus of the $i$th constituent
Voigt upper bound

- The Voigt upper bound is sometimes called the isostrain average.
- It gives the ratio of the average stress to average strain when all constituents are assumed to have the same strain.

Reuss lower bound

- There is no way nature can put together a mixture of constituents that is elastically softer than the harmonic average of the constituent moduli.

The Reuss lower bound of the effective elastic modulus, $M_R$, is

$$\frac{1}{M_R} = \sum_{i=1}^{N} \frac{f_i}{M_i}$$
Reuss lower bound

- The Reuss lower bound is sometimes called the isostress average.
- It gives the ratio of the average stress to average strain when all constituents are assumed to have the same stress.
- Describes the effective moduli of a suspension of solid grains in a fluid.

Voigt (upper) and Reuss (lower) bounds
Hashin-Shtrikman bounds

- Give the narrowest range of elastic moduli bounds without specifying anything about geometries of grains

Physical interpretation: Assembly of spheres each with the same volume fraction of fluid and grain.

Hashin-Shtrikman bounds

The best bounds for an isotropic elastic mixture, defined as giving the narrowest possible range of elastic moduli without specifying anything about the geometries of the constituents, are the Hashin–Shtrikman bounds (Hashin and Shtrikman, 1963). For a mixture of two constituents, the Hashin–Shtrikman bounds are given by

\[
K_{\text{HS}}^\pm = K_1 + \frac{f_2}{(K_2 - K_1)^{-1} + f_1(K_1 + 4\mu_1/3)^{-1}}
\]

\[
\mu_{\text{HS}}^\pm = \mu_1 + \frac{f_2}{(\mu_2 - \mu_1)^{-1} + 2f_1(K_1 + 2\mu_1)/[5\mu_1(K_1 + 4\mu_1/3)]}
\]

with

- \(K_1, K_2\): bulk moduli of individual phases
- \(\mu_1, \mu_2\): shear moduli of individual phases
- \(f_1, f_2\): volume fractions of individual phases
Mineral - Fluid Mixtures

Figure 1.3: Conceptual illustration of upper and lower bounds to bulk and shear moduli for a mixture of two materials, one of which is a fluid.

When all constituents are gases or liquids with zero shear modulus, then the Reuss average gives the effective moduli of the mixture, exactly.

In contrast to the Reuss average which describes a number of real physical systems, real isotropic mixtures can never be as stiff as the Voigt bound (except for the single-phase end members).

Observed Vp vs $\Phi$ relationships

Figure 1.5: P-wave velocity versus porosity for a variety of water-saturated sediments, compared with the Voigt-Reuss bounds. Data are from Yin (1992), Han (1988) and Hamilton (1986).
Diagenesis and Depositional trends

**Velocity trends**

- Usually an increase in porosity results in decrease in $V_p$ and $V_s$.
- In sandstones, clay content causes scatter around the $V_v$ $\Phi$ trend. In consolidated sst clay decreases $V$ and increases $V_p/V_s$. In unconsolidated rocks clay can sometimes stiffen the rock.
Velocity trends

• Variations in pore shape (round vs. crack-like aspect ratios) cause changes in trends. Crack-like pores produce softer rocks.

Fluid Substitution

• Understanding and predicting how seismic velocity / impedance depend on pore fluids.
• Gassmann’s (1951) relations predict how rock modulus changes with a change in pore fluids.
Fluid substitution

- Two effects of the fluid
  - Change in rock bulk density
  - Change in rock compressibility (NB shear modulus is zero in a fluid)

Compressibility of dry rock

\[
\frac{1}{K_{\text{dry}}} = \frac{1}{K_{\text{mineral}}} + \frac{\phi}{K_0} \tag{1.7}
\]

where \( \phi \) is the porosity, \( K_{\text{dry}} \) is the dry rock bulk modulus, \( K_{\text{mineral}} \) is the mineral bulk modulus, and \( K_0 \) is the pore space stiffness defined by:

\[
\frac{1}{K_0} = \frac{1}{v_{\text{pore}}} \frac{\partial v_{\text{pore}}}{\partial \sigma} \tag{1.8}
\]

Here, \( v_{\text{pore}} \) is the pore volume, and \( \sigma \) is the increment of hydrostatic confining stress from the passing wave. Poorly consolidated rocks, rocks with microcracks, and rocks at low effective pressure are generally soft and compressible and have a small \( K_0 \). Stiff rocks that are well cemented, lacking microcracks, or at high effective pressure have a large \( K_0 \). In terms of the popular but idealized ellipsoidal crack models, low-aspect-ratio cracks have small \( K_0 \) and rounder large-aspect-ratio pores have large \( K_0 \). In simple terms we can write approximately \( K_0 \approx \alpha K_{\text{mineral}} \) where \( \alpha \) is aspect ratio. (This approximation is best at low porosity.)
Compressibility of a saturated rock

Similarly, the compressibility of a saturated rock can be expressed as

\[ \frac{1}{K_{\text{sat}}} = \frac{1}{K_{\text{mineral}}} + \frac{\phi}{K_{\phi} + K_{\text{fluid}}K_{\text{mineral}}/K_{\text{mineral}} - K_{\text{fluid}}} \]

or approximately as

\[ \frac{1}{K_{\text{sat}}} \approx \frac{1}{K_{\text{mineral}}} + \frac{\phi}{K_{\phi} + K_{\text{fluid}}} \]

where \( K_{\text{fluid}} \) is the pore-fluid bulk modulus.

Effects of fluid substitution

- In a stiff rock (with large pore-space stiffness) will have a small sensitivity to fluids.
- In a soft rock (with small pore-space stiffness) will have a large sensitivity to fluids.
Gassmann’s Equation

\[ \frac{K_{\text{sat}}}{K_{\text{mineral}} - K_{\text{sat}}} = \frac{K_{\text{dry}}}{K_{\text{mineral}} - K_{\text{dry}}} + \frac{K_{\text{fluid}}}{\phi(K_{\text{mineral}} - K_{\text{fluid}})} \]  

(1.11)

and the companion result

\[ \mu_{\text{sat}} = \mu_{\text{dry}} \]  

(1.12)

Gassmann’s equations (1.11) and (1.12) predict that for an isotropic rock, the rock bulk modulus will change if the fluid changes, but the rock shear modulus will not.

These dry and saturated moduli, in turn, are related to P-wave velocity \( V_p = \sqrt{K + (4/3)\mu}/\rho \) and S-wave velocity \( V_s = \sqrt{\mu}/\rho \), where \( \rho \) is the bulk density given by

\[ \rho = \phi \rho_{\text{fluid}} + (1 - \phi) \rho_{\text{mineral}} \]  

(1.13)

Gassmann’s Equation

• \( K_{\text{dry}} \) = Frame bulk modulus, measured at irreducible saturation conditions of wetting fluids
• \( K_{\text{fluid}} \) can be calculated using Wood’s equation

\[ 1/ K_{\text{fluid}} = S_{\text{water}}/ K_{\text{water}} + S_{\text{oil}}/ K_{\text{oil}} + S_{\text{gas}}/ K_{\text{gas}} \]
Basic Assumptions

- The rock is macroscopically homogeneous
- All pores are interconnected
- Pores filled with frictionless fluid
- Rock-fluid system closed
- Pore fluid does not interact with the solid to soften or harden the frame

Gassmann’s Equations

- **Low frequency** approximation
- Derived under the assumption that wave-induced pressures throughout the pore space have time to equilibrate during the passage of the wave.
- Gassmann’s equations work well for seismic data (<100Hz) **BUT** less well for fluid effects in lab ultrasonic experiments (~10⁴ Hz).
Pore fluid properties

- Batzle-Wang formulae describe the empirical dependence of gas, oil and brine properties on:
  - Temperature
  - Pressure
  - Composition

Gassmann fluid substitution recipe

The most common scenario is to begin with an initial set of velocities and densities \( V_1^0, V_2^0, \) and \( \rho_1^0 \) corresponding to the rock with an initial set of fluids, which we call “Fluid 1.” These velocities often come from well logs, but might also be the result of an inversion or theoretical model. Then fluid substitution is performed as follows:

**Step 1:** Extract the dynamic bulk and shear modulus from \( V_1^0, V_2^0, \) and \( \rho_1^0 \):

\[
K_1^0 = \rho_1^0 \left( V_1^0 \right)^2 - \frac{4}{3} \rho_1^0 \left( V_1^0 \right)^3
\]

\[
\mu_1^0 = \rho_1^0 \left( V_1^0 \right)^2
\]

**Step 2:** Apply Gassmann’s relation, equation (1.11), to transform the bulk modulus

\[
\frac{K_{\text{mixed}} - K_1^0}{\phi(K_{\text{mixed}} - K_2^0)} = \frac{K_2^0}{\phi(K_{\text{mixed}} - K_2^0)}
\]

where \( K_{\text{mixed}}^0 \) and \( K_2^0 \) are the rock bulk modulus saturated with fluid 1 and fluid 2, and \( K_1^0 \) and \( K_2^0 \) are the bulk moduli of the fluids themselves.

**Step 3:** Leave the shear modulus unchanged:

\[
\mu_{\text{mixed}}^0 = \mu_2^0
\]

**Step 4:** Remember to correct the bulk density for the fluid change:

\[
\rho_{\text{mixed}}^0 = \rho_1^0 + \phi \rho_2^0 - \rho_1^0
\]

**Step 5:** Recalculate the velocities:

\[
V_{\text{mixed}}^0 = \sqrt{\frac{K_{\text{mixed}}^0 + 4 \mu_{\text{mixed}}^0}{\rho_{\text{mixed}}^0}}
\]

\[
V_{\text{mixed}}^0 = \sqrt{\frac{K_{\text{mixed}}^0}{\rho_{\text{mixed}}^0}} + \frac{2 \mu_{\text{mixed}}^0}{\rho_{\text{mixed}}^0}
\]

NB since replacing fluid in the pores can eliminate dry rock modulus
Biot (1956) Equation

- Biot (1956) extended Gassmann’s equation to FULL range of frequencies
- Same as Gassmann at zero frequency; predicts ~ 3% velocity change for most reservoir rocks

Importance of Vp/Vs

- Vs as well as Vp often allows for better lithofacies, pore fluid type/saturation and pore pressure determination.
- Changes in fluid saturation produce significant changes in Vp/Vs
- Changes in pore pressure produce little changes in Vp/Vs
Observed Vp vs Vs relationships

Figure 1.20 Vp vs Vs for water-saturated sandstones, with porosities \( \phi \) ranging from 4% to 40%. Effective pressures 5-50 MPa, clay fraction 0.5-0.05. Arrows show direction of increasing porosity, clay, pore pressure. Data are from Han (1986), Blight (1992) and Yin (1992).

Figure 1.21 Plot of Vp vs. Vs for water-saturated and gas-saturated sandstones, with porosities \( \phi \) ranging from 4% to 40%. Effective pressures 5-50 MPa, clay fraction 0.5-0.05. Arrows show direction of increasing porosity, clay, pore pressure. The trend of saturation is perpendicular to that for porosity, clay, pore pressure.

Figure 1.29 Saturation and pressure discrimination are very similar in different attribute domains. A. Plot of Vp vs. Vs sandstone data, showing the value of combining P- and S-wave data for separating lithologies, pore pressure, and saturation (same as Figure 1.21). B. Same rock samples as in A, but plotted as P- and S-wave impedances. C. Same rock samples plotted as \( Vp/Vs \) vs. Vs.
(Vs/Vp) = \sqrt{(0.5 - \sigma)/(1-\sigma)]}

**Fig. 5.2** Poisson’s ratio vs acoustic impedance