Chapter 5

Future directions

There are many ways that the method of CDR tomographic inversion could be improved. An obvious improvement would be to replace the gridded velocity model with a velocity model made up of smooth basis functions. Other improvements might include using a measure of dip-bar focusing in the objective function, and damping less in regions where there are more dip bars, in order to allow sharp velocity variations at strong reflectors.

There exist several untested applications of CDR tomography. These include extension to three-dimensional data sets, and analysis of converted-wave data.

5.1 Defining the velocity with basis functions

Chapters 3 and 4 show how a gridded velocity model can be used in the determination of interval velocity. Whether this model is the best one to use, though, is not clear. When a gridded model is used, the inversion problem is extremely underdetermined; it is necessary to use damping terms in order to transform the underdetermined problem into an overdetermined one. Even when damping terms are used, however, the inversion remains poorly conditioned; much computer time is expended in solving linear least-squares problems. Another problem with the gridded velocity model is that the large number of grid points increases the amount of computer time and memory needed for the inversion.

Both problems, that of underdetermination and that of expense, can be solved if smooth basis functions are used in place of the gridded model. Only a few basis functions (as opposed to the many boxes in the gridded model) would be needed to describe the interval velocity model, thus solving the problem of underdetermination. No damping parameters
would be necessary in a smooth-basis-function model, since the model would already be as smooth as the basis functions themselves. The expense of ray tracing through such a smooth model would presumably also be less.

A conceptual difficulty with the basis-function model is that smooth basis functions force the velocity model to be smooth, whereas experience suggests that the earth is blocky, with sharp discontinuities between areas of smoothly varying velocity. It might therefore be argued that a gridded velocity model should be used, so that these sharp discontinuities can be properly modeled. However, the CDR tomographic method (as described so far) is able to determine only the low-frequency components of the velocity field. Unless high-frequency components such as discontinuities can be accurately determined, a smooth-basis-function model seems adequate.

The main practical problem with the basis-function model is determining $\mathbf{A}$, the Fréchet matrix. When a gridded velocity model is used, the cost of determining the Fréchet matrix is about 4 times the cost of simple ray tracing (see Appendix D). If it proves necessary to use finite differencing to determine the corresponding matrix for the basis-function model, the expense will be many times greater. It is possible that the paraxial ray approximation (Červený and Pšenčík, 1984a) could be used to circumvent this problem.

### 5.2 Using horizon coherency in the objective function

We know from observation that seismic data tends to be reflected from distinct horizons within the earth rather than from random diffractors. The CDR tomographic method contains no assumptions about the existence of distinct horizons, and thus fails to make use of this important empirical observation.

For instance, an inspection of Figure 3.13 (page 55) shows that the horizons formed by the migrated dip bars are not coherent at all locations of the depth section. The presence of incoherencies suggests that the velocity model is not entirely accurate, but the CDR tomographic method does not try to adjust the velocity model in order to make the horizons more coherent. By measuring and trying to minimize the degree of incoherency along horizons, it should be possible to find a better velocity model.

I will not propose a specific coherency measure. But, in order to incorporate any given coherency measure into the inversion, it is necessary to find a Fréchet matrix giving the
change in coherency with respect to changes in the velocity model. This task is not as difficult as it first seems. A similar quantity, the change in dip-bar depth with respect to changes in the velocity model, is already determined (see Appendix D). All that remains is to define a coherency measure, and to find an expression for the change in coherency with respect to changes in the depths of individual dip bars.

5.3 Damping and dip-bar density

We know from theory and from observation that strong reflections are a sign of sharp velocity variations. Such sharp velocity variations may be usefully incorporated in the velocity model at locations where the presence of many dip bars suggests that there is a strong reflecting horizon. In a gridded model this could be done by lowering the damping parameters in areas of the grid that contain many dip bars. Sharp jumps in the velocity model at these locations would result, and the velocity model would be smoother everywhere else. Such a model, with smoothly varying regions separated by discontinuities, seems more realistic than a model consisting of a single smoothly varying velocity function.

It is uncertain that this approach would be successful. There may be ambiguities between the locations of the dip bars (reflecting horizons) and variations in the velocity model (Stork and Clayton, 1986). This approach would be difficult to implement if the model were based on smooth basis functions rather than on grids.

5.4 Converted waves

CDR tomographic inversion can be applied to converted-wave data. Assume that a velocity model \( v_p \) has already been determined by inverting conventional P-wave data. Converted-wave data can then be used to determine the S-wave velocity model \( v_s \). Suppose (without loss of generality) that the source is a P-wave source, and that S waves are received at the geophones. The only change, then, in the inversion procedure is that for the computation of \( x_{err} \), rays from the shot are traced through the known P-wave model \( v_p \), while rays from the geophone are traced through the trial S-wave model \( v_s \). When the Fréchet matrix \( A \) is calculated, only the geophone rays are used.

One difficulty with using converted-wave data is that the data must be free from any residual non-converted (PP or SS) waves. The tomographic inversion method is unable to
Figure 5.1: Typical 3-D marine survey. In such a survey, the ship collects data along parallel straight lines. As a result, CDR analysis will give only the in-line components of the ray parameters.

distinguish converted from non-converted waves, just as it is unable to distinguish multiple reflections from primaries. Non-converted waves will therefore act as noise, and may cause the converted-wave inversion to fail.

5.5 Application to 3-D surveys

The CDR method has a major advantage over most other processing methods: because the CDR process picks only the important reflection events, the volume of data that must be manipulated is greatly reduced. This advantage is especially significant when the data are from a 3-D seismic survey. In principle, no problems should arise in applying CDR tomography to the inversion of three-dimensional data. The definition of $z_{err}$ does not change; the difference between the two-dimensional and three-dimensional inversion problems is seemingly only a matter of scale. Difficulty occurs, however, in that 3-D data is not usually collected in a way conducive to CDR analysis.

Consider the standard marine survey shown in Figure 5.1. From such a survey, the east-west components of $p_s$ and $p_g$ are easily determined, but there is no easy way to determine the north-south components. Most other 3-D survey geometries have analogous
problems. Looking at Figure 5.1 in another way, both components of the midpoint ray parameter \( p_y \) can be determined (if the survey lines are close enough together), but only the east-west component of the offset ray parameter \( p_h \) can be found.

It may be possible to find the unknown components of \( p_x \) and \( p_e \) by using supplemental data from cross-line surveys. In that case, CDR tomographic inversion is conceptually simple, since the definition of \( x_{err} \) is easily extended from two dimensions to three. Other CDR processing techniques are extendible to three dimensions as well. For example, the averaged CDR velocity \( v_{CDR} \) can be determined from 3-D CDR data, although a complication exists: in three dimensions, the CDR velocity is over-determined, so it is necessary to discard some information. If some of the ray-parameter information is thrown out, an explicit if complicated expression can be found for \( v_{CDR} \) (Vasil'ev and Urupov, 1978). It may be better, however, to determine CDR velocity by some other technique. For instance, \( v_{CDR} \) might be defined to be the constant velocity that minimizes \( x_{err} \). Once the CDR velocity is determined, it can be used in velocity filtering (see section 2.4.2, page 33). I likewise expect that CDR time migration can be extended to three dimensions, once \( v_{CDR} \) is defined, but I have not worked out the formulas.

Even if no cross-line data are available, CDR tomographic inversion may still be feasible. If only three of the four ray-parameter components are known (both components of \( p_y \) and one component of \( p_h \), for instance), then these components define a suite of shot and geophone rays, rather than a single shot ray and a single geophone ray. This suite of rays can be used to find a suite of values for \( x_{err} \); the true value of \( x_{err} \) may be defined to be the minimum value in that suite. Many rays must therefore be traced to determine \( x_{err} \) for each set of picked parameters. As a result, CDR tomography is more expensive when only three of the four ray-parameter components are available.

The main difficulty in 3-D CDR tomography will be the measurement of the ray parameters, given the difficulties imposed by the geometry of the survey. Although data from purely parallel-line surveys can be used with some difficulty, CDR tomographic inversion is cheaper and more straightforward when data from cross-line surveys are available as well.