Appendix A

Designaturing

FROM CIRCUITS TO RESPONSE CURVES

Hydrophones

The basic circuitry of a hydrophone has a capacitor (the peizo crystal that deforms under pressure) and the inherent resistance of the rest of the circuit that can be considered as a single resistor.
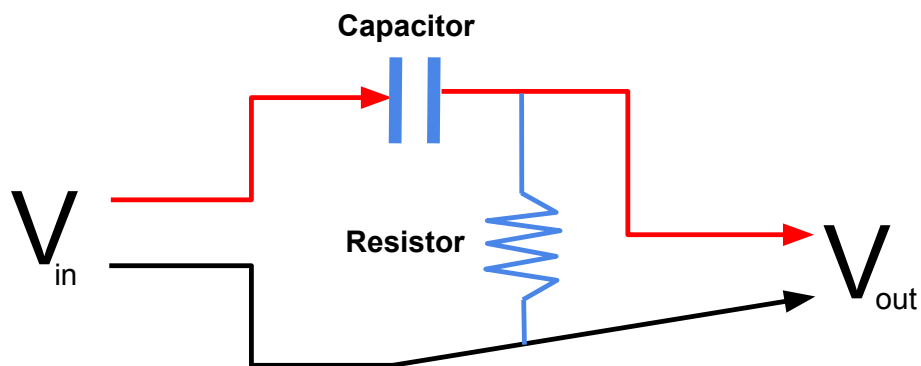

Figure A.1: Basic circuit diagram of a hydrophone. [**NR**] appendix1/. hydro-circuit

We can represent figure (A.1) with several equations that balance the current:

$$V_{in} - V_{out} = Q/C \tag{A.1}$$

$$V_{out} = IR \tag{A.2}$$

$$I = \frac{dQ}{dt} \tag{A.3}$$

Equation A.1 is the voltage on the capacitor, equation (A.2) is the current in the resistor, and equation (A.3) is the same current in the capacitor itself. What we are interested in is the output voltage V_{out} and how it relates to the input voltage for different frequencies:

$$V_{out} = f(\omega, RC)V_{in} \tag{A.4}$$

$$\tag{A.5}$$

To derive this formulation, we begin by taking the time derivative of equation (A.1) in order to eliminate Q:

$$(V_{in} - V_{out}) = Q/C$$

$$\frac{d}{dt}(V_{in} - V_{out}) = \frac{dQ}{dt}/C,$$

and then combine with equation (A.3):

$$\frac{d}{dt}(V_{in} - V_{out}) = I/C,$$

To eliminate I, we substitute in $I = V_{out}/R$ from equation (A.2):

$$\frac{d}{dt}(V_{in} - V_{out}) = V_{out}/RC$$

To calculate V_{in} from V_{out}, move all V_{out} to the right side:

$$\frac{dV_{in}}{dt} - \frac{dV_{out}}{dt} = V_{out}/RC \tag{A.6}$$

$$\frac{dV_{in}}{dt} = \frac{dV_{out}}{dt} + V_{out}/RC \tag{A.7}$$

We can solve equation A.7 in the fourier space by making the substitution $\frac{d}{dt} = i\omega$:

$$i\omega V_{in} = i\omega V_{out} + V_{out}/RC$$

$$V_{in} = V_{out} + \frac{V_{out}}{i\omega RC}$$

$$V_{in} = V_{out}\left(1 + \frac{1}{i\omega RC}\right)$$

$$i\omega RCV_{in} = V_{out}\left(i\omega RC + 1\right)$$

This means that in Fourier space we can define our transfer function as:

$$f(\omega, RC) = \frac{i\omega RC}{(i\omega RC + 1)} \tag{A.8}$$

from which we can find the original input data by solving equation (A.8) for V_{in}. From this function, we can easily find the amplitude scaling and the phase shifting that the instrument creates in the recorded data:

$$\phi(\omega) = \tan^{-1}\left[\frac{\text{real}f(\omega, RC)}{\text{imag}f(\omega, RC)}\right] \tag{A.9}$$

$$\tag{A.10}$$

Alternatively, if we aren't interested in actually analyzing the transfer function, but just want to deconvolve our instrument response from our recorded data, we can solve for V_{in} from equation A.7 using a finite difference approach:

$$\frac{V_{in}[i] - V_{in}[i-1]}{dt} = \frac{V_{out}[i] - V_{out}[i-1]}{dt} + \frac{V_{out}[i] + V_{out}[i-1]}{2RC}$$

$$V_{in}[i] = V_{in}[i-1] + V_{out}[i] - V_{out}[i-1] + \frac{(V_{out}[i] + V_{out}[i-1])dt}{2RC}$$

Geophones
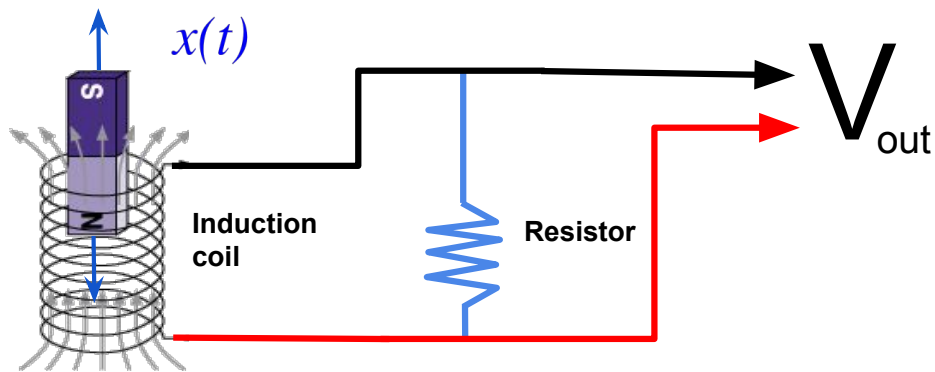

Figure   A.2:   Basic   circuit   diagram   of   a   geophone.   [**NR**]
appendix1/. geophone-circuit-0

For geophones, instead of having a piezo crystal modulate an input voltage, we have a mechanical system that creates voltage by converting motion of a magnet through a coil.

The motion $x(t)$ that occurs in the coil produces a voltage, and of course the circuit has some general resistance to account for. However, the most significant part of the geophone response curve comes from the equations of motion for the magnet relative to the coil.

Figure A.3 shows the internal and external forces at play in a vertical component geophone, and Figure A.4 shows the movement of the magnet relative to the instrument frame. We can begin our derivation of the instrument response by writing a force balance equation to balance the external (left hand side) and internal forces (right hand side).
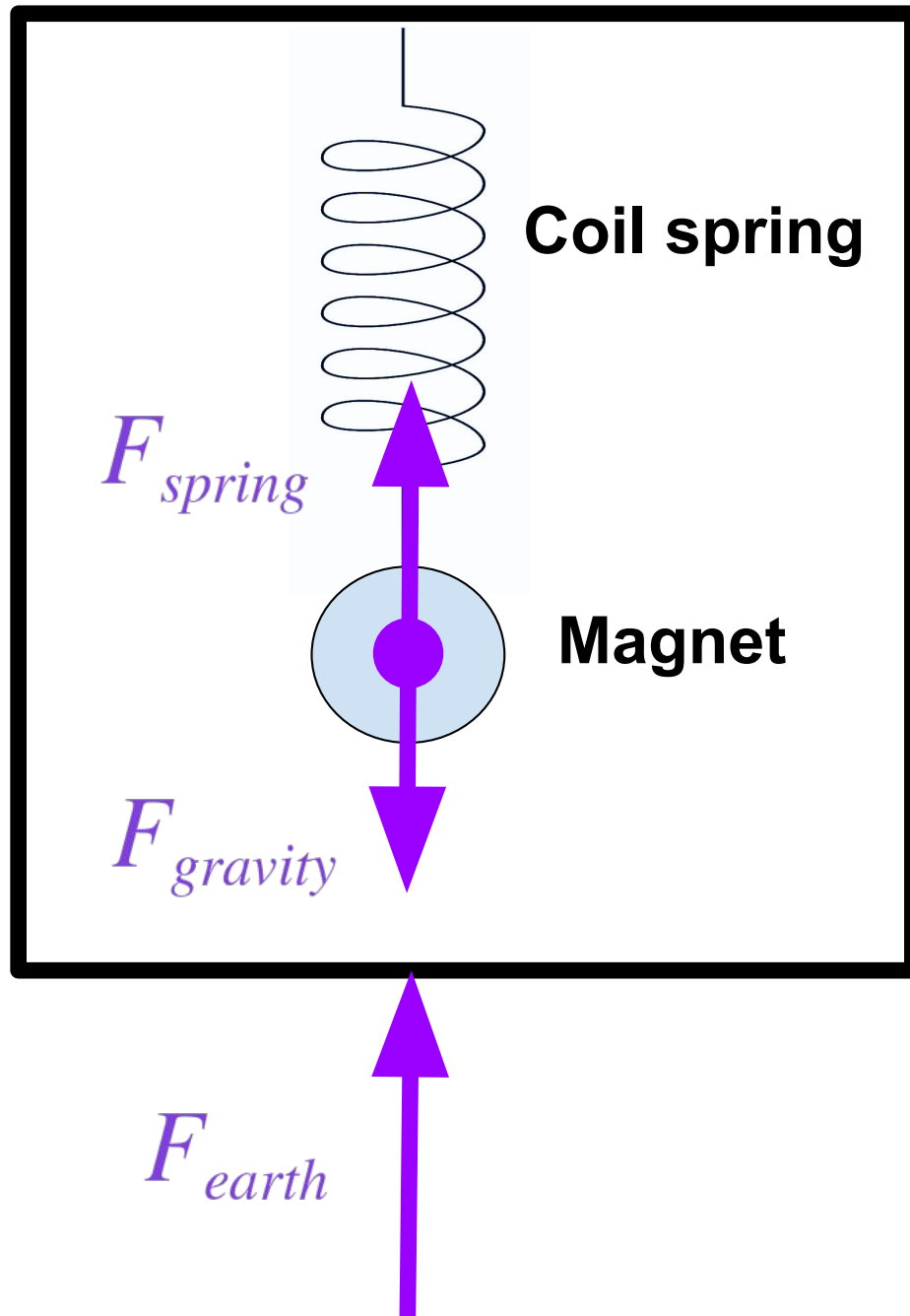
Geophone casing

Coil spring

F_{spring}

Magnet

$F_{gravity}$

F_{earth}

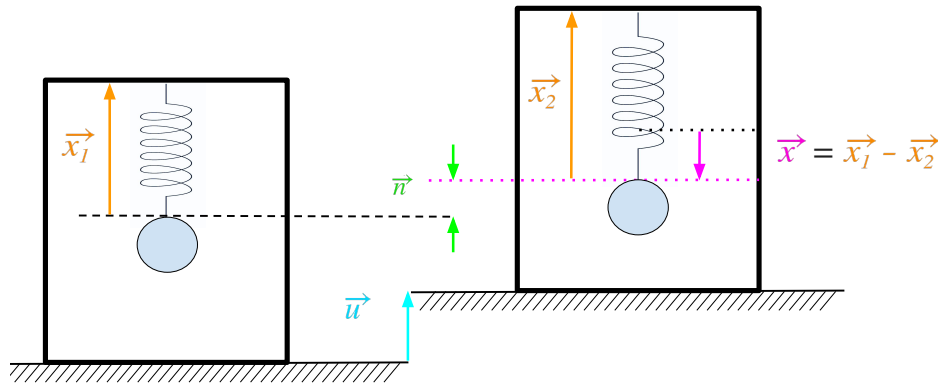Figure A.3: Diagram of internal and external static forces. [**NR**] appendix1/. geophone-physics

Figure A.4: Diagram of vertical component geophone responding to ground motion. **u** is the motion of the earth, and **x** is the motion of the magnet relative to the coil. [**NR**] appendix1/. geophone-physics2

$$m\frac{\partial^2}{\partial t^2}(u + x) = -kx \tag{A.11}$$

Where m is the mass of the magnet, and k is the spring constant. We can re-define k according the natural frequency of the spring and mass system:

$$k = \omega_0^2 m,$$

which gives us the equation of harmonic motion:

$$\frac{\partial^2 x}{\partial t^2} + \omega_0^2 x = -\frac{\partial^2 u}{\partial t^2} \tag{A.12}$$

Equation A.12 defines an undamped system, which means it would be overcome by frequencies close to the natural frequency of the system (ω_0). In order to allow for the recording of other frequencies, we add a dampening term that is proportional to the velocity of the magnet:

$$\frac{\partial^2 x}{\partial t^2} + \omega_0^2 x + 2\omega_0\lambda\frac{\partial x}{\partial t} = -\frac{\partial^2 u}{\partial t^2}, \tag{A.13}$$

where λ is the dampening ratio. A simple way to solve equation A.13 is to use the Fourier transform, which allows us to subsitute $i\omega$ with $\frac{\partial}{\partial t}$. Since the actual

measurement that we record is voltage, and voltage is a function of the velocity of the magnet, we ultimately would like to solve for $\frac{\partial x}{\partial t}$:

$$\frac{\partial}{\partial t}\frac{\partial x}{\partial t} + \omega_0^2 x + 2\omega_0\lambda\frac{\partial x}{\partial t} = -\frac{\partial^2 u}{\partial t^2}$$

$$i\omega\frac{\partial x}{\partial t} - \frac{\omega_0^2}{i\omega}\frac{\partial x}{\partial t} + 2\omega_0\lambda\frac{\partial x}{\partial t} = \omega^2 u$$

This means we can relate the output voltage $\left(\frac{\partial x}{\partial t}\right)$ with the input acceleration of the earth $\left(\frac{\partial^2 u}{\partial t^2}\right)$:

$$\frac{\frac{\partial x}{\partial t}}{\frac{\partial^2 u}{\partial t^2}} = \frac{-i\omega}{-\omega^2 + \omega_0^2 + i2\omega\omega_0\lambda} \tag{A.14}$$

DE-SIGNATURING DATA

Application to Cardamom data

In order to implement equations A.9 and A.14, we perform the division in the Fourier domain. Our algorithm works according to the following workflow:

Algorithm 3 Remove instrument response from data

1: **procedure** RESPONSEREMOVAL($data$,$response$)
2:     **for** each trace i in $data$ **do**
3:         $temp(i) = \text{FFT}(data(i))$
4:         $temp(i).amp = \sqrt{(temp(i).imag)^2 + (temp(i).real)^2}$
5:         $temp(i).phase = \tan^{-1}\left(\frac{temp(i).imag}{temp(i).real}\right)$
6:         $temp2(i).phase = temp(i).phase - response.phase$
7:         $temp2(i).amp = \frac{temp(i).amp}{response.amp}$
8:         $output(i) = \text{FFT}^{-1}(temp2(i))$
9:     **end for**
10:     Return $output$
11: **end procedure**

We perform the actual instrument response removal in the Fourier domain, and then convert back to the time domain. The response curves are functions of frequency

as we describe in equations A.13 and A.14. We can see the impact of the response removal in Figure A.5 and Figure A.6. Further, the difference in the spectra can be seen clearly in Figures A.7 and A.8, where the lower frequencies are noticably boosted.
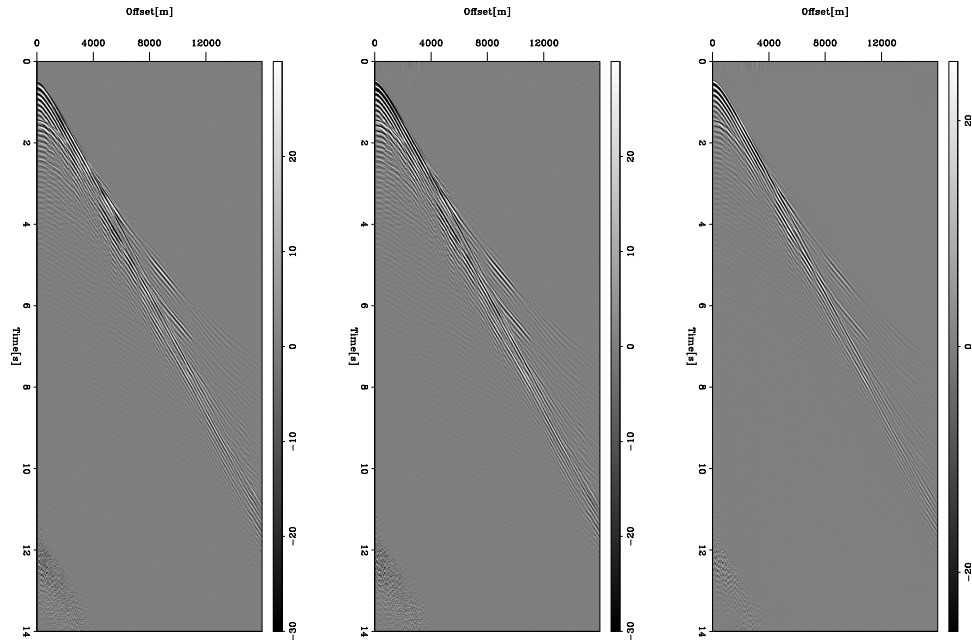
Figure A.5:   Hydrophone   data   original   (left),   designatured   (middle) and difference (right).   Clipped to emphasize differences.   [**ER**] appendix1/. hydro-response-removal-comp
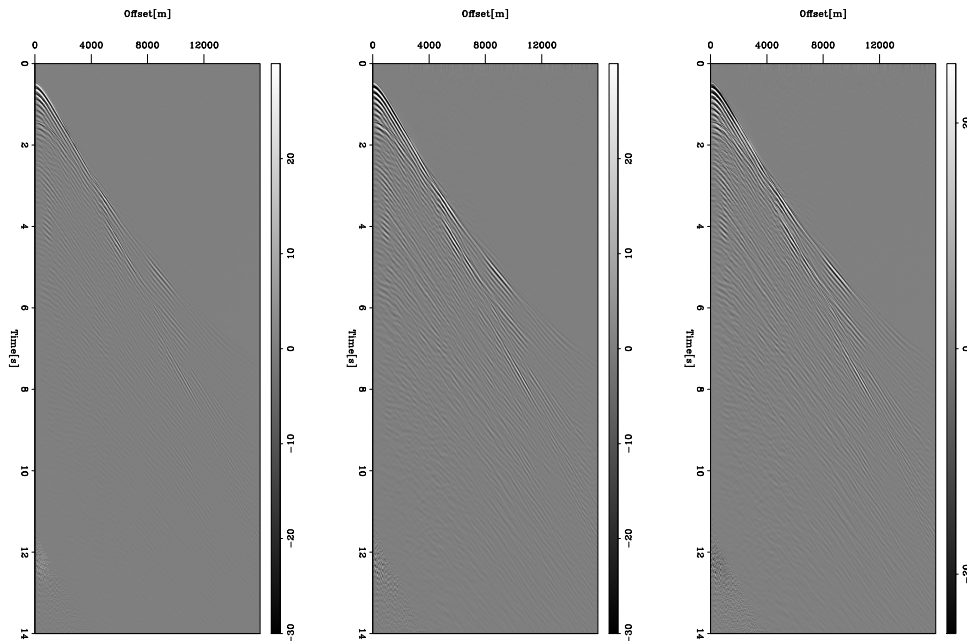
Figure A.6: Geophone data original (left), designatured (middle) and difference (right). Clipped to emphasize differences. [**ER**] appendix1/. geophone-response-removal-comp
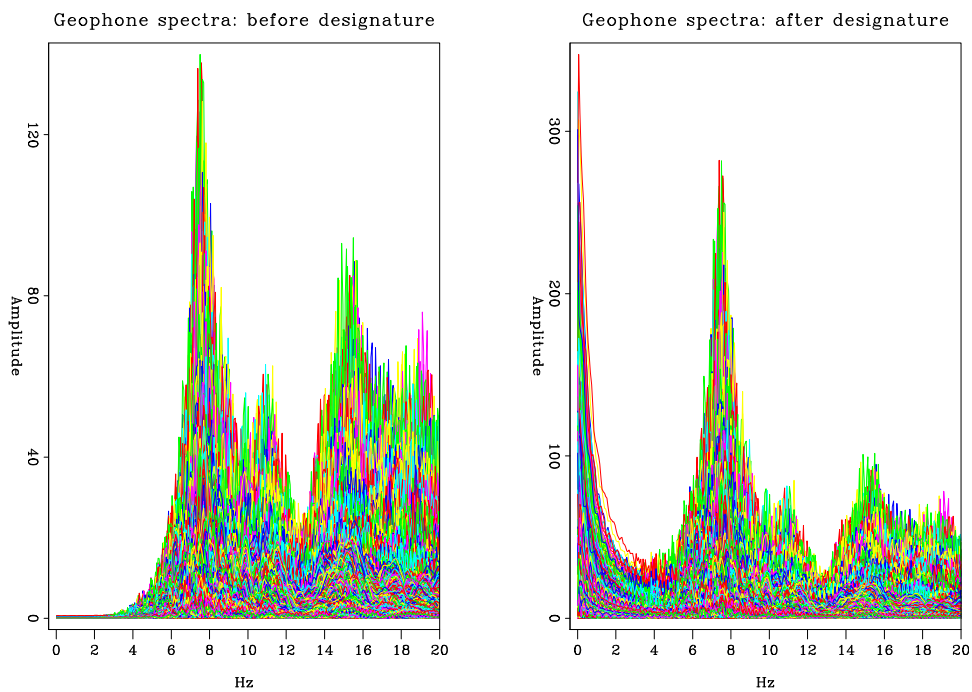

Figure A.7: Geophone data spectra before response removal (left), and after (right) for each trace. [**ER**] appendix1/. spectra-vert-compare
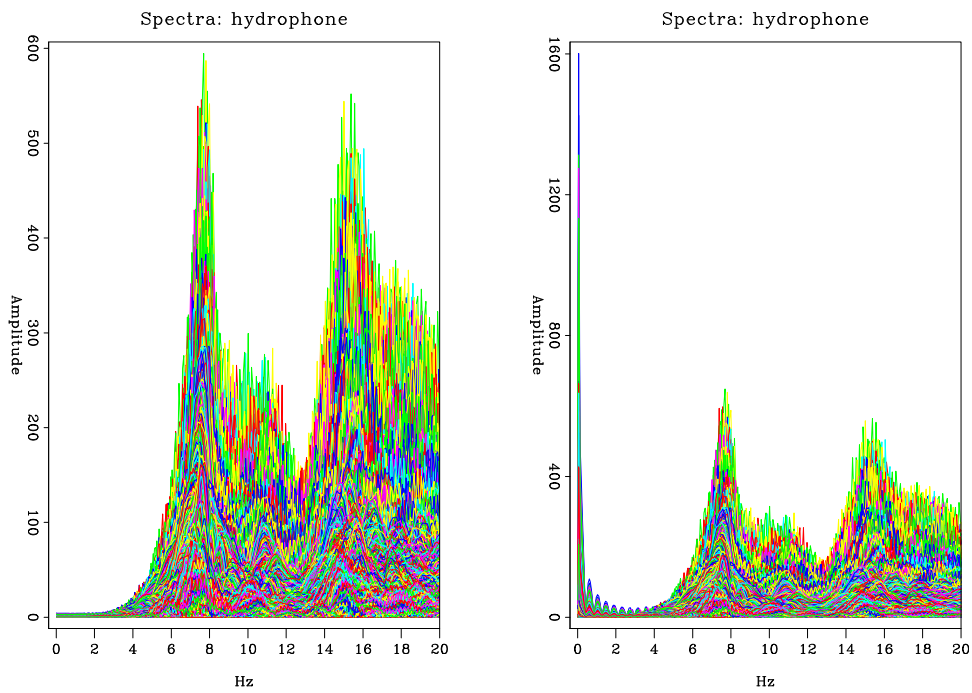
Figure A.8: Hydrophone data spectra before response removal (left), and after (right) for each trace. [**ER**] appendix1/. spectra-hydr-compare