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On Stolt stretch time migration

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ABSTRACT

We implement Stolt-stretch time migration with an analytical formulation for the optimal stretch parameter and show how it improves the quality of imaging. By a cascaded *f-k* migration approach with this algorithm, we manage to obtain time migration results on real data comparable to Gazdag's phase-shift method, with a high accuracy for steeply deeping events at a computational cost dramatically lowered.

INTRODUCTION

Time migration remains a very fast imaging process compared to prestack depth migration and therefore is still commonly used by seismic imaging contractors. Such an economical technique reveals itself useful as a first approach to a problem or for producing accurate images when the interval velocity varies only with depth. Among the many algorithms avaible for post-stack time migration, Stolt's is known as the fastest of all. It is derived from a wavefield downward-continuation in constant velocity. This constant velocity assumption yields the well-known shortcoming of Stolt's algorithm. In his classic paper, Stolt (1978) proposed as an approximation for $v(z)$ media a stretching of the time axis that is commonly called "Stolt-stretch" migration. In that context, the vertical heterogeneities of the velocity model are represented by a single nondimensional parameter W, substituted for a complicated function of several parameters. In the constant velocity case, W is equal to 1.0. In a medium where the velocity is increasing with depth, its value is constrained to lie between 0.0 and 1.0.

In practice, a frustrating drawback of the technique is that there was no constructive way to choose the parameter W. To overcome this heuristic guess, Fomel (1995) derived an explicit formulation for W based on Malovichko's formula for approximating traveltimes in vertically inhomogeneous media (Malovichko, 1978; Sword, 1987; Castle, 1988; de Bazelaire, 1988). In this paper, we implement Stolt-stretch time migration with this optimal choice for W and discuss its accuracy.

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STOLT STRETCH THEORY REVIEW

Stolt time migration can be summarized as the following sequence of transformations:

$$p_0(x,t_0) \to P_0(k_x,\omega_0) \to P(k_x,\omega) \to p(x,t)\,, \tag{1}$$

where

$$P_0(k_x,\omega_0) = P(k_x,\omega(k_x,\omega_0)) \left| \frac{d\omega(k_x,\omega_0)}{d\omega_0} \right| \tag{2}$$

The function $\omega(k,\omega_0)$ is the dispersion relation and has the following expression in the constant velocity case:

$$\boxed{\omega(k,\omega_0) = \mathrm{sign}(\omega_0)\sqrt{\omega_0^2 - v^2 k_x^2}} \tag{3}$$

The approximation suggested by Stolt (1978) for extending the method to $v(z)$ media involves a change of the time variable (Stolt-stretch):

$$s(t) = \sqrt{\frac{2}{v_0^2}\int_0^t \tau\, v_{rms}^2(\tau)\, d\tau}\,, \tag{4}$$

where $s(t)$ is the stretched time variable, v_0 is an arbitrarily chosen constant velocity, and $v_{rms}(t)$ is the root mean square velocity along the vertical ray, defined by

$$v_{rms}(t) = \frac{1}{t}\int_0^t v^2(\tau)\, d\tau\,. \tag{5}$$

This change of variable yields a transformed wave-equation for the wavefield extrapolation, in which Stolt replaces a slowly varying complicated function of several parameters (denoted by W) by its average value. Making this approximation yields a new dispersion relation in the transformed coordinate system:

$$\boxed{\hat{\omega}(k_x,\hat{\omega}_0) = \left(1-\tfrac{1}{W}\right)\hat{\omega}_0 + \frac{\mathrm{sign}(\hat{\omega}_0)}{W}\sqrt{\hat{\omega}_0^2 - W v_0^2 k_x^2}} \tag{6}$$

This factor W contains all the information about the heterogeneities of the medium. However, it has to be determined a priori, that is, before migration. This empirical choice for W was one of the drawbacks of the Stolt-stretch method. Fomel (1995) derived an analytical formulation of the Stolt-stretch parameter, based on Malovichko's formula for approximating traveltimes in vertically inhomogeneous media (Malovichko, 1978):

$$t_0 = \left(1 - \frac{1}{S(t)}\right) t + \frac{1}{S(t)}\sqrt{t^2 + S(t)\frac{(x-x_0)^2}{v_{rms}^2(t)}}\,, \tag{7}$$

where the function $S(t)$ defines the so-called parameter of heterogeneity:

$$S(t) = \frac{1}{v_{rms}^4 t} \int_0^t v^4(t) dt \tag{8}$$

Fomel proved that, for a given depth (or vertical traveltime), the optimal value of W is

$$\boxed{W(t) = 1 - \frac{v_0^2 s^2(t)}{v_{rms}^2(t) t^2} \left(\frac{v^2(t)}{v_{rms}^2(t)} - S(t) \right)}\,, \tag{9}$$

where $v_{rms}(t)$ is the root mean square velocity along the vertical ray, and $t = \int_0^z \frac{dz}{v(z)}$ the vertical traveltime. The value of W used during Stolt migration is the average along the vertical profile of these $W(t)$. In the case of an homogeneous constant-velocity model, W is equal to 1.0, whereas it has to be less than 1.0 if the velocity increases monotonically with depth.

We can sum up the application of Stolt-stretch algorithm with the optimal parameter W by the following sequence of steps:

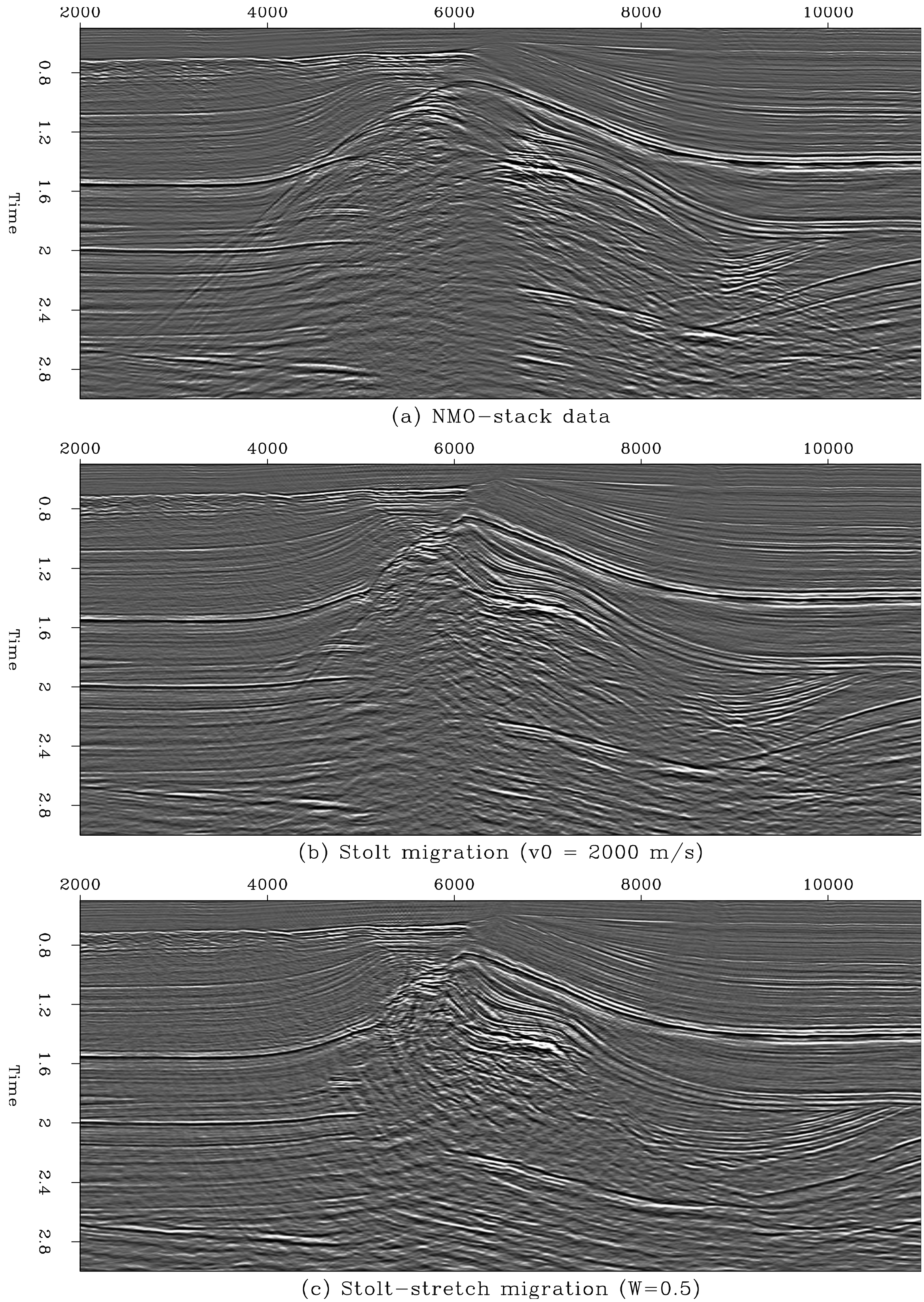
1. Stretch the time axis and determine
the value of W along the vertical profile
2. Interpolate stretched time to a regular grid
3. 2-D FFT
4. Apply Stolt migration with the dispersion relation (6)
5. 2-D inverse FFT
6. Unstretch the time axis

APPLICATION

Following the study by Larner et al. (1989), we selected a dataset that includes steep dips in order to test the accuracy of our algorithms. The data is courtesy of Elf Aquitaine, was recorded in the North Sea, and shows a salt dome (Figure 3). Figure 1 shows (a) the data after NMO-stack and (b) after poststack Stolt migration, using a constant velocity of 2000 m/s. We notice that Stolt's method obviously yields undermigrated events on both sides of the salt body. Using a higher velocity to focus them better would have created overmigration artifacts at shallow reflectors. Stolt-stretch migrated section (c) using $W = 0.5$ should be compared with figure 2a.

Using the Stolt-stretch method with the optimal choice for W derived from equation (9) yields a better focusing of events at all depths (Figure 2a), compared to other values of W (Figures 1b and 1c, respectively for W equals 1.0 and 0.5). The $v(z)$ model used for migration is shown in Figure 4a and was obtained by averaging laterally the reference velocity model.

The reference method of migration for our study is the phase-shift approach proposed by Gazgag (1978). It is known to be perfectly accurate for all dips up to 90° in a $v(z)$ velocity field. A comparison between the phase-shift migration result (Figure 2b) and the section



(a) NMO−stack data

(b) Stolt migration (v0 = 2000 m/s)

(c) Stolt−stretch migration (W=0.5)

Figure 1: (a) Section of the North Sea data, after NMO-stack. (b) Section migrated using Stolt's method with v_0=2000 m/s. (c) Section migrated using Stolt-stretch with an arbitrary value $W = 0.5$ for the paramater of heterogeneity. stoltex-data-stolt-ststr [ER]

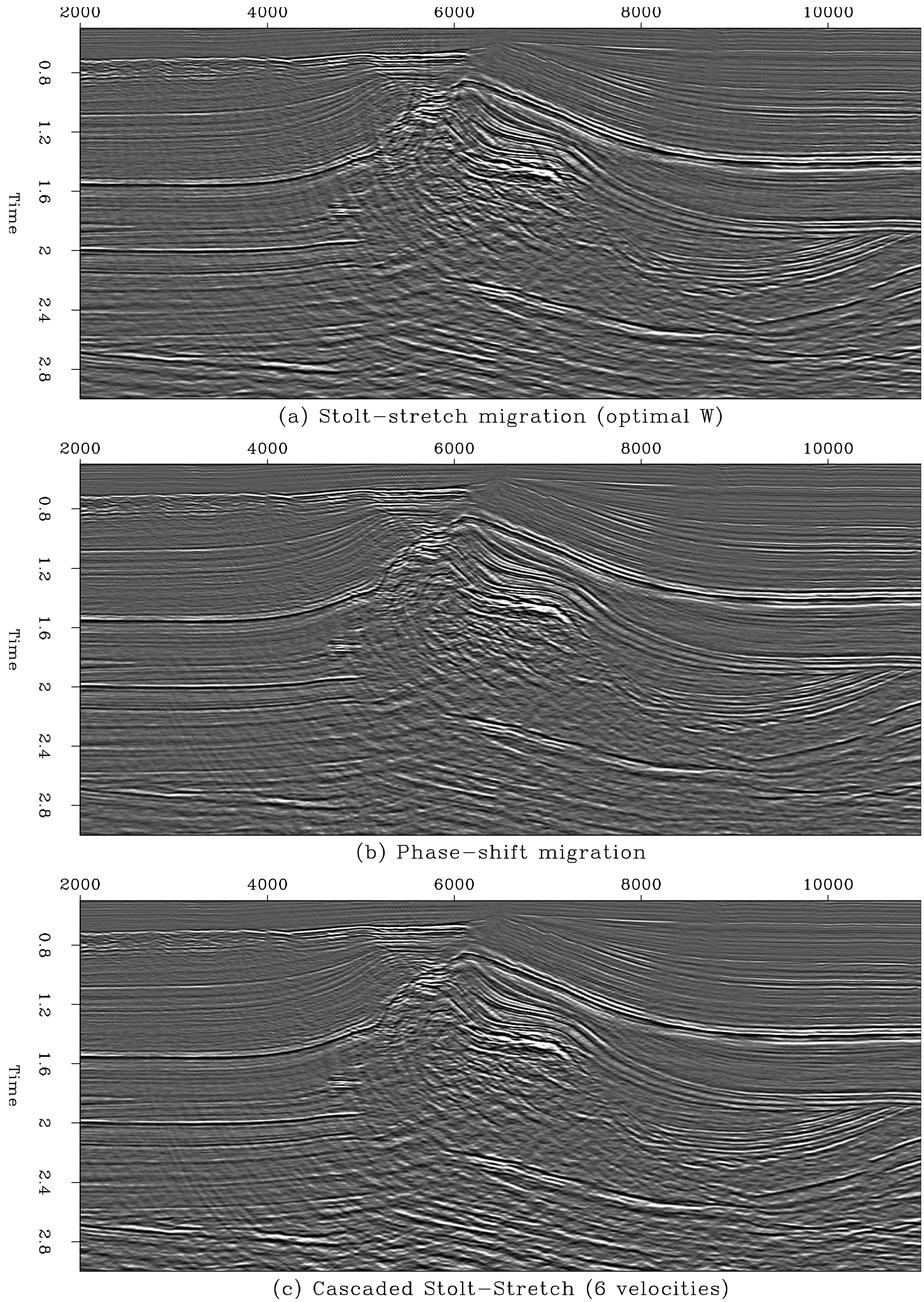


Figure 2: (a) Section migrated with the Stolt-stretch method using the optimal value (≈ 0.67) for the parameter W. (b) Section migrated with the phase-shift method. (c) Section migrated using the cascaded Stolt-stretch approach (6 velocities). stoltex-data-ststr-pshift-casc [ER]

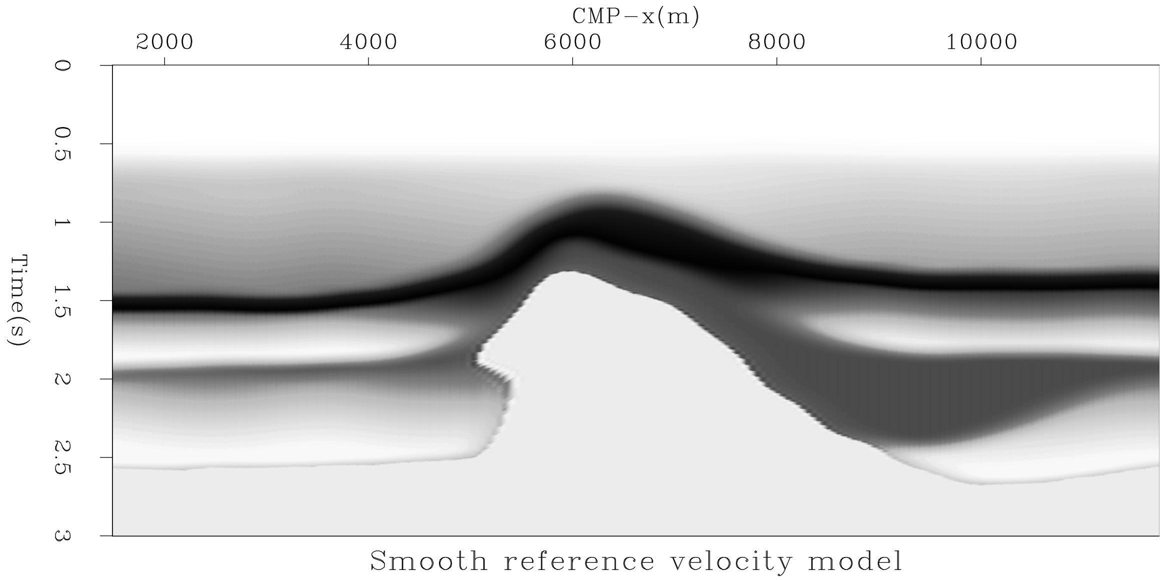


Figure 3: 2-D smooth reference velocity model stoltex-vel-model [ER]

migrated with the Stolt-stretch approach shows almost no difference for flat events. However, a more detailed analysis reveals significant errors for steep events inside and around the salt body. The approximation made by stretching the time axis breaks for recovering steep events.

A way to overcome the difficulties encountered by Stolt's migration is to divide the whole process into a cascade, as suggested by Beasley et al. (1988). The theory of cascaded migration proves that *f-k* migration algorithms with a $v(t)$ velocity model like Stolt-stretch can be performed sequentially as a cascade of n migrations with smaller interval velocities $v_i(t)$, $i = 1,\dots,n$, such as $v^2(t) = \sum_{i=1,n} v_i^2(t)$. At a given vertical traveltime t, all the successive velocity models have to be constant, except the last one (Larner and Beasley, 1987). Typically, the first stage is done with a constant velocity model and can be computed using Stolt's algorithm, which is then accurate for all dips. Figure 4 illustrates such a cascade of velocity models in our particular case, with 3 and 6 stages.

As a consequence of this decomposition, each intermediate velocity model shows not only a smaller velocity but also less vertical heterogeneity. In other words, the Stolt-stretch parameter W estimated for each stage tends to be closer to 1.0, thus reducing the migration errors due to the approximation. Figure 2c shows the migration result using a 6-stage cascaded scheme. All the successive values of W were greater than 0.8. There are almost no differences with the phase-shift result (Figure 2b).

DISCUSSION

Even if the parameter W estimated from equation (9) is optimal in the sense that it minimizes migration errors for the Stolt-stretch method, no single choice of W yields acceptable results for all times and all dips (Beasley et al., 1988): some events are undermigrated, others overmi-

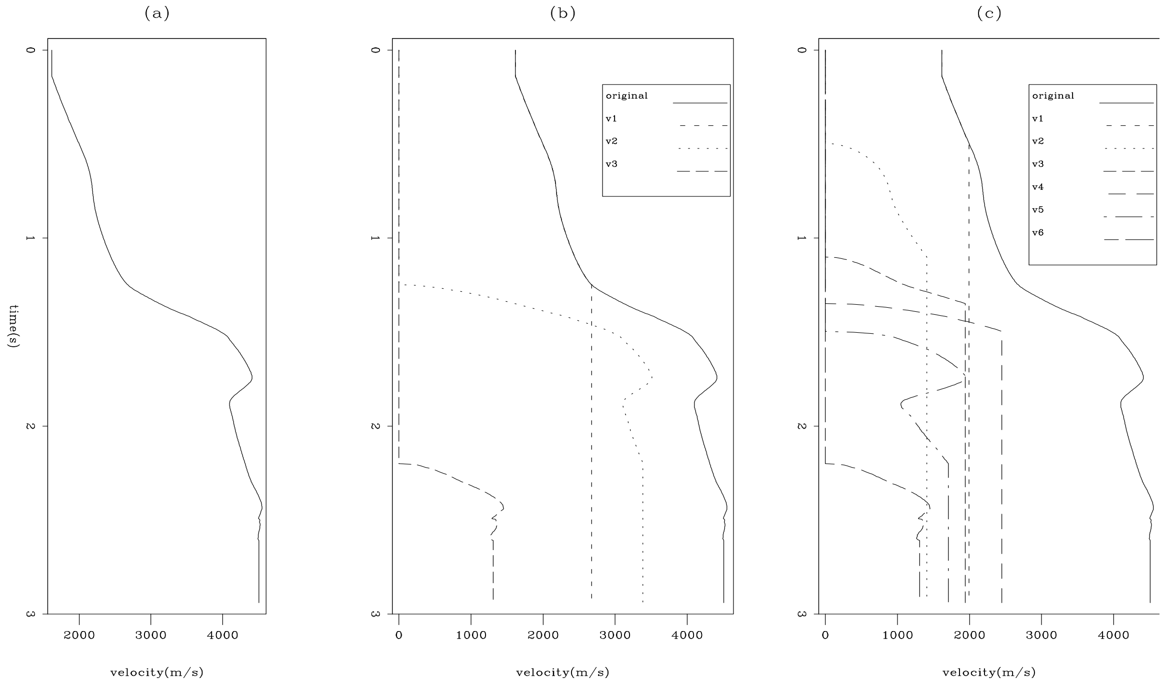


Figure 4: (a) Interval velocity model $v(t)$ estimated from the 2-D reference model. (b) Decomposition in a cascade of 3 models, such as $v^2 = v_1^2 + v_2^2 + v_3^2$. (c) Decomposition in a cascade of 6 models, such as $v^2 = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + v_6^2$ stoltex-velocities [ER]

grated. Instead, the use of cascaded Stolt-stretch migration allows a reduction of the apparent dip perceived in each stage, since the migration velocity used is reduced to a fraction of the original model. A 20-stage cascade of migration with an algorithm accurate for dips up to 15° can yield accurate results for events dipping up to 65° (Larner and Beasley, 1987).

Figure 5 shows a close-up of the salt body region for all migration algorithms. The methods have a different accuracy with respect to steep dips. We notice a gradual improvement of the result from Stolt-stretch to phase-shift as we increase the number of velocities in the cascaded Stolt-stretch scheme. In theory, the migration errors in the cascaded approach can be made as small as desired by increasing the number of stages. At the limit, it corresponds to the velocity continuation concept (Fomel, 1996).

In our case, six stages were enough to obtain a result comparable to phase-shift. In their comparative study on time migration algorithms, Larner et al. (1989) have shown that four-stage cascaded *f-k* migration is accurate for dips up to 85°, which is almost comparable to phase-shift, accurate for all dips. It is worth noting the computational cost difference between the two: on our example, phase-shift migration is about 80 times more expensive than Stolt-stretch!

Another way to look at the problem is to compare the impulse responses of the different algorithms (Figure 6), generated using the same velocity model as before (Figure 4a). There is a kinematic difference in the impulse response of Stolt-stretch compared to phase-shift. While Gazdag's phase-shift honor ray bending in any $v(z)$ model, Stolt-stretch is not that accurate.

Both methods address non-hyperbolic moveout, but Stolt's stretching function is only designed to make the fitting curve look like an hyperbola close to the apex (Levin, 1983), and therefore induces residual migration errors. As seen in Figure 2a, Stolt-stretch result displays residual hyperbolic migration artifacts that are due to this fundamental kinematic difference. Cascading Stolt-stretch makes the impulse response of the migration converge towards the one of phase-shift.

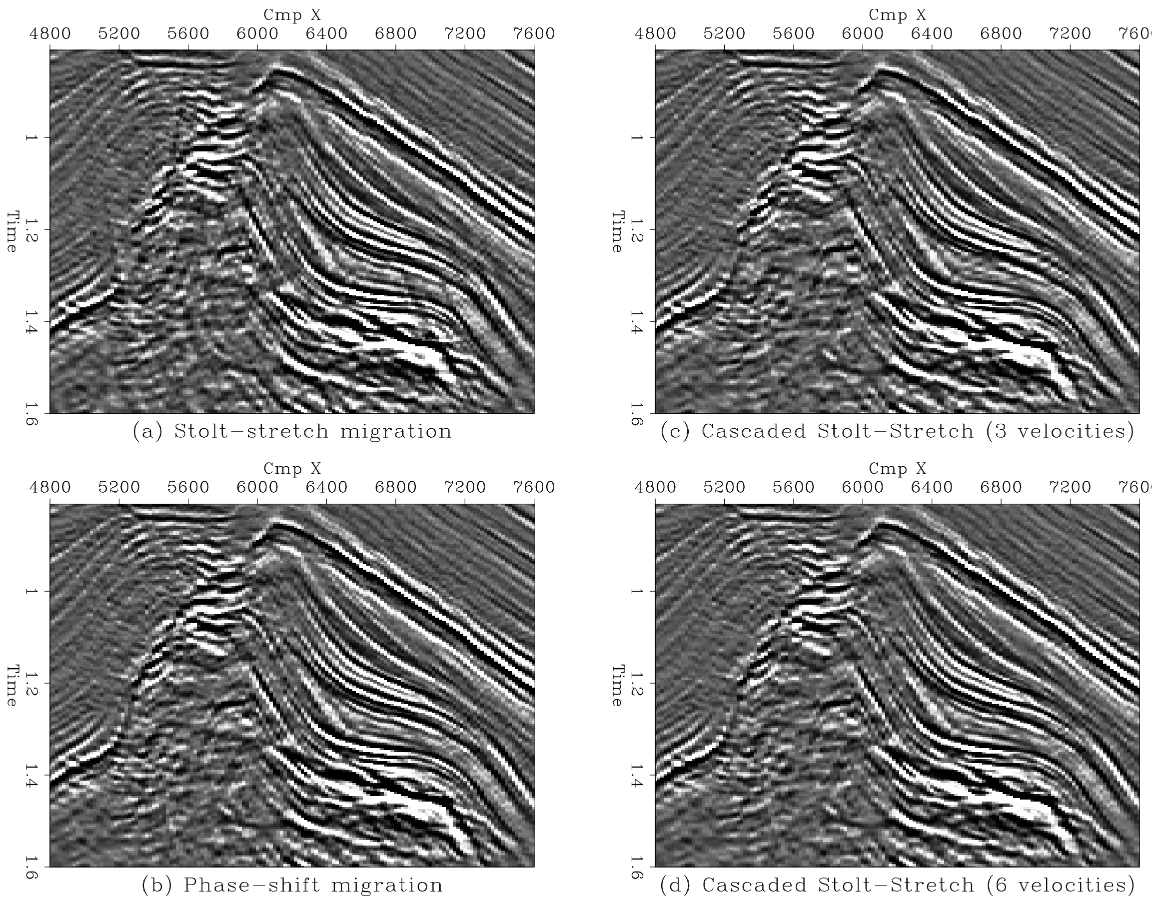


Figure 5: Zoom in the salt body area where steep dips are located. (a) Migration with the Stolt-stretch method. (b) Migration with the phase-shift method. (c) and (d) Migrations with the cascaded Stolt-stretch approach, using, respectively, 3 and 6 velocities. stoltex-dip-zoom [ER]

Now that we are familiar with the role of W in the algorithm, a word should be said about v_0, which is the second arbitrary parameter of the method. As introduced in equation (4), v_0 controls the length of the stretch. In theory, the migration result does not depend on the selected value, since the time stretch is undone after Stolt migration. However, in practice, high values of v_0 can yield an image with interpolation artifacts. In contrast, low values of v_0 yield a significantly stretched time axis, thus the inverse operation may lose information unless the data has been padded with enough zeroes. The cascaded scheme is particularly sensitive to such problems. As a tradeoff between values that are too high or too small, we used the mean of the velocity model extrema for v_0.

Another technical aspect, the division by the Jacobian in equation (2), usually induces high amplitude artifacts for waves close to being evanescent, unless a threshold is introduced. Similarly, evanescent waves need to be scaled down to prevent migration artifacts. We used a simple linear weighting.

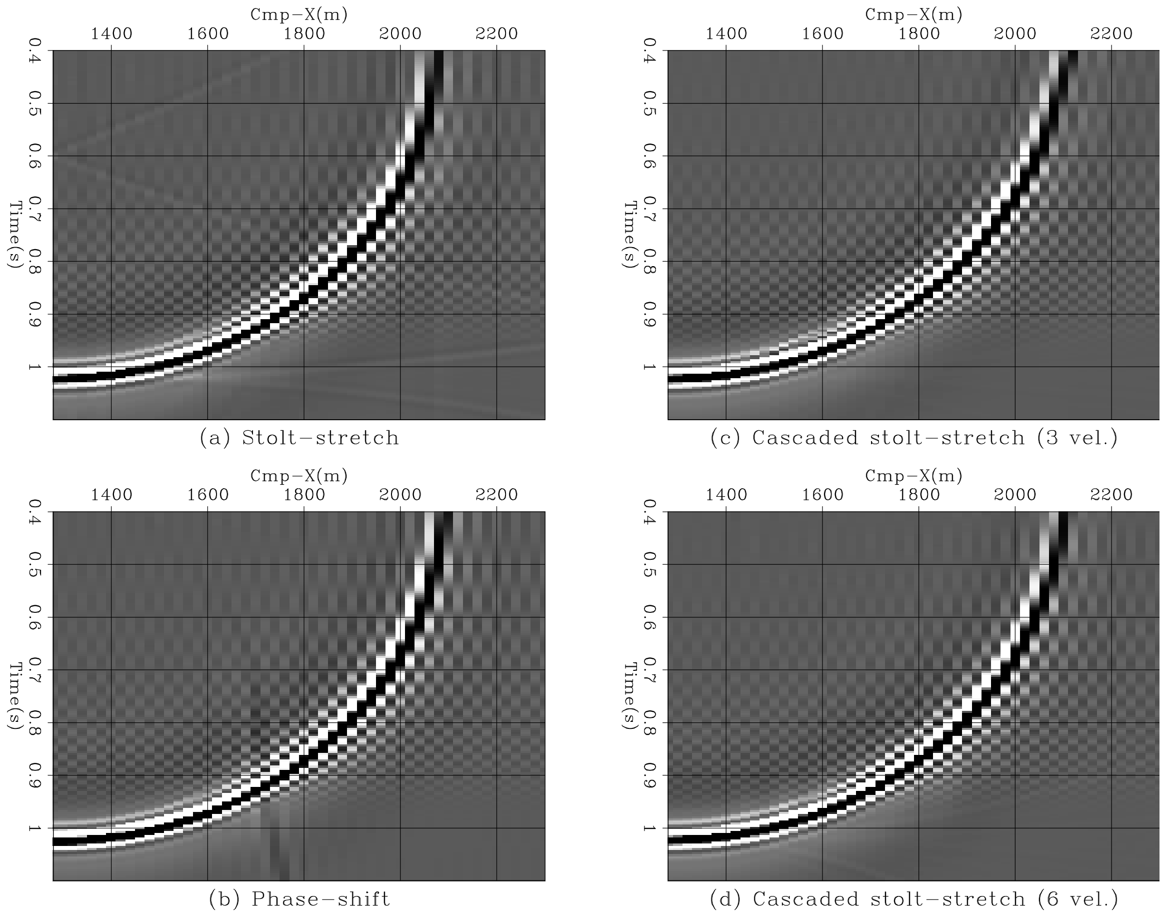


Figure 6: Impulses responses of the different operators. (a) Stolt-stretch. (b) Phase-shift. (c) and (d) Cascaded Stolt-stretch, with 3 and 6 velocities, respectively. stoltex-imp-mig [ER]

CONCLUSION

We show that with an optimal choice for the Stolt-stretch parameter derived analytically and with a cascaded *f-k* migration approach, we manage to obtain time migration results comparable to Gazdag's phase-shift approach. Moreover, the method is considerably more computer-efficient and remains accurate for steeply deeping events.

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