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Iterative resolution estimation in Kirchhoff imaging

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ABSTRACT

We apply iterative resolution estimation to least-squares Kirchhoff migration. Resolution plots reveal low illumination areas on seismic images and provide information about image uncertainties.

INTRODUCTION

Kirchhoff prestack depth migration remains the most widely used method for seismic imaging in complex areas. The method is especially attractive for 3-D imaging because of its ability to handle naturally irregular acquisition geometries. The negative effect of irregular sampling on seismic images can be additionally balanced by applying the least-squares migration approach (Cole and Karrenbach, 1992), which has recently gained a lot of attention in the geophysical literature (Nemeth et al., 1999; Chavent and Plessix, 1999; Duquet and Marfurt, 1999).

According to the least-squares approach, the migration operator is constructed as a least-squares inverse of the forward Kirchhoff modeling (Tarantola, 1987). One can effectively approximate the inverse operator through an application of the conjugate-gradient technique. The conventional migration is then considered as the adjoint of the modeling operator, or, in other words, the first step of a conjugate-gradient iteration (Claerbout, 1992). A more accurate representation (i.e. additional conjugate-gradient steps) can compensate for irregularities and artifacts of irregular acquisition (Nemeth, 1996; Nemeth et al., 1999).

A blind least-squares approach cannot, however, compensate for lack of information in the input data. For example, if a particular area in the subsurface is not illuminated by reflection waves, a proper image of that area cannot be resolved by least-squares migration alone. In this case, part of the image will belong to the null space of the least-squares inverse problem. Spotting low-illumination areas is important both for making acquisition decisions and for evaluating the uncertainty of the existing images. Duquet et al. (1998) have proposed to use the inverse diagonal of the Hessian matrix as a measure of illumination in Kirchhoff imaging. Although this measure does provide useful information about the problem's well-posedness, a more rigorous approach to the solution uncertainty would be to estimate the corresponding model resolution operator (Jackson, 1972).

As shown by Berryman and Fomel (1996), the model resolution matrix can be estimated in an iterative manner. The matrix approximation is constructed from the vectors, already

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appearing in the conjugate-gradient iteration. Therefore, it requires minimal additional computation with respect to an iterative least-squares inversion. The diagonal of the resolution matrix can serve as a rough direct estimate of the model uncertainty. A similar, although less efficient approach, was proposed by Minkoff (1996) and Yao et al. (1999), who applied it in conjunction with the LSQR method (Paige and Saunders, 1982).

In this paper, we apply the iterative technique of Berryman and Fomel (1996) for resolution estimation in Kirchhoff imaging. Synthetic and real data tests show that a resolution estimate can indeed provide valuable information about the uncertainty of Kirchhoff images and reveal image areas with illumination problems.

REVIEW OF RESOLUTION MATRICES

Model resolution operator $\mathbf{R}$ defines the connection between the true model $\mathbf{m}$ and the model estimate from least-squares inversion $\hat{\mathbf{m}}$, as follows:

$$\hat{\mathbf{m}} = \mathbf{R}\mathbf{m}\,. \tag{1}$$

In the case of least-squares Kirchhoff migration, $\mathbf{m}$ corresponds to true reflectivity, $\hat{\mathbf{m}}$ is the output image, and the estimation process amounts to minimizing the least-square norm of the residual $\mathbf{r} = \mathbf{d} - \mathbf{L}\mathbf{m}$, where $\mathbf{d}$ is the observed data, and $\mathbf{L}$ is the Kirchhoff modeling operator. Recalling the well-known formula

$$\hat{\mathbf{m}} = (\mathbf{L}'\mathbf{L})^{\dagger}\mathbf{L}'\mathbf{d}\,, \tag{2}$$

where $\mathbf{L}'$ stands for the adjoint operator (Kirchhoff migration), and the dagger symbol denotes the pseudo-inverse operator, we can deduce from formulas (3) and (2) that

$$\mathbf{R} = (\mathbf{L}'\mathbf{L})^{\dagger}(\mathbf{L}'\mathbf{L})\,. \tag{3}$$

In the ideal case, when all model components are perfectly resolved, the model resolution model matrix is equal to the identity. If the model is not perfectly constrained, the inverted $\mathbf{L}'\mathbf{L}$ matrix will be singular, and the model resolution will depart from being the identity. It means that the model contains some null-space components that are not constrained by the data. The diagonal elements of the resolution matrix will be less than one in the places of unresolved model components. Berryman and Fomel (1996) derive the following remarkably simple formula for the model resolution matrix:

$$\mathbf{R} = \sum_{i=1}^{N} \frac{\mathbf{g_i}\mathbf{g_i}'}{\mathbf{g_i}'\mathbf{g_i}}\,, \tag{4}$$

where N corresponds to the model size, and the $\mathbf{g_i}$'s are the model-space gradient vectors that appear in the conjugate-gradient process (Hestenes and Stiefel, 1952). In large-scale problems, such as a typical Kirchhoff migration, we cannot afford performing all N steps of the conjugate-gradient process, required for the theoretical convergence of the model estimate to the one defined in formula (2). However, formula (4) is still valid in this case, if we replace number N with the actual number of steps. In this case, the matrix R corresponds to the actual resolution of our estimate. To reduce the computational effort, we can use formula (4) only with a few significant gradient vectors $\mathbf{g_i}$ to obtain an effective approximation of the model resolution. The most significant $\mathbf{g_i}$'s will turn out to be those have large components in the direction of eigenvectors having large eigenvalues (or singular vectors have large singular values). The next section exemplifies this approach with synthetic and real data tests.

APPLICATION TO KIRCHHOFF IMAGING

Difficulties in Kirchhoff imaging

When attempting to image complex subsurfaces with Kirchhoff methods, many difficulties may arise. In particular, amplitude behavior of the imaged reflectors can be caused by totally different physical phenomenon. A reflector that appears to fade and disappear along some distance can have several causes, including a real change in reflectivity, an error in the velocity model, or an illumination problem. All of these provide valuable information, but it is important to know which one is causing the effect. By estimating the resolution of the data it is possible to identify areas of low illumination.

Resolution estimation algorithm

To test the resolution matrix estimation we inverted for a single output offset (225m) made from three data offsets (200, 225, and 250m). The Kirchhoff operator was a simple 2-D modeling operator and its adjoint using 2nd order, first-arrival eikonal traveltimes. For the synthetic case a smoothed version of the correct velocity model was used. The real data example uses a smoothed version of the SMART (Jacobs et al., 1992; Ehinger and Lailly, 1995) velocity model provided by Elf Aquitaine.

Results

We began our experiments on the synthetic Elf North Sea dataset. Figure 1 shows the result of conjugate gradient inversion. The deepest reflector seems to disappear as it passes under the edge of the salt body. This behavior is known to be caused by poor illumination (Prucha et al., 1998).

Figures 2 through 5 show the estimated resolution for the synthetic dataset, with increasing numbers of iteration. After only 5 iterations, there is high resolution along the major reflectors (black indicates high resolution, white indicates low resolution). Note that the area of poor illumination has low resolution. As the number of iterations increases, the areas between the reflectors become better resolved. This tells us that conjugate gradient algorithm is spending most of its effort at low iterations resolving model components around the reflector. It moves onto the area between reflectors only at large iterations. This is not surprising behavior, since most of the energy in the model space is found around the reflectors so that is what will be minimized first.

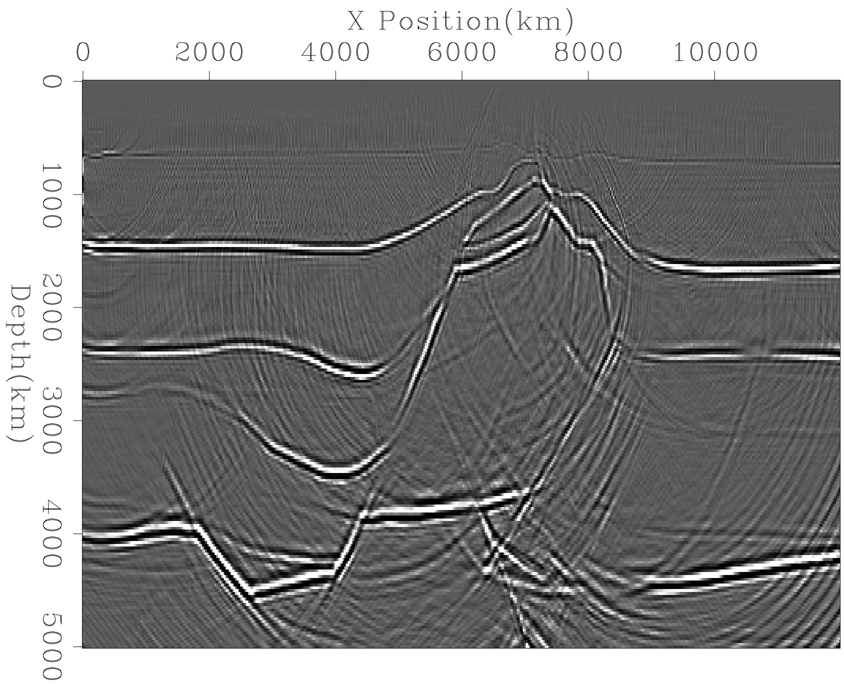

Figure 1: Inversion result on synthetic. illum-synth-cg.5 [NR]

After experimenting with the synthetic dataset, we conducted the same trials on the real Elf North Sea dataset (Figure 6). Note that the x–axis in the real dataset is reversed from that in the synthetic so that the salt structure tilts to the left rather than the right. Figures 8 through 11 show the results of increasing the iterations for estimating the resolution. Once again, there are Kirchhoff-type artifacts in all of the figures. Note that we again see resolution energy beginning around the reflectors, spreading to areas between reflectors at higher iterations. We can see corresponding changes in our image. After 5 iterations the image shows strong energy along the primaries reflectors, but is generally low frequency, Figure 6. After 20 iterations we have an image with more noise, but also a significantly higher frequency image. The later iterations resolved smaller eigenvalues of the model, which corresponded to higher frequency, lower amplitude portions of the model space.

CONCLUSIONS

Iterative estimation of resolution supplies useful information when performing Kirchoff imaging. Areas of low illumination are easily recognizable. In addition, the iterative nature of the algorithm provides useful information on what portion of the image is resolvable at each iteration.

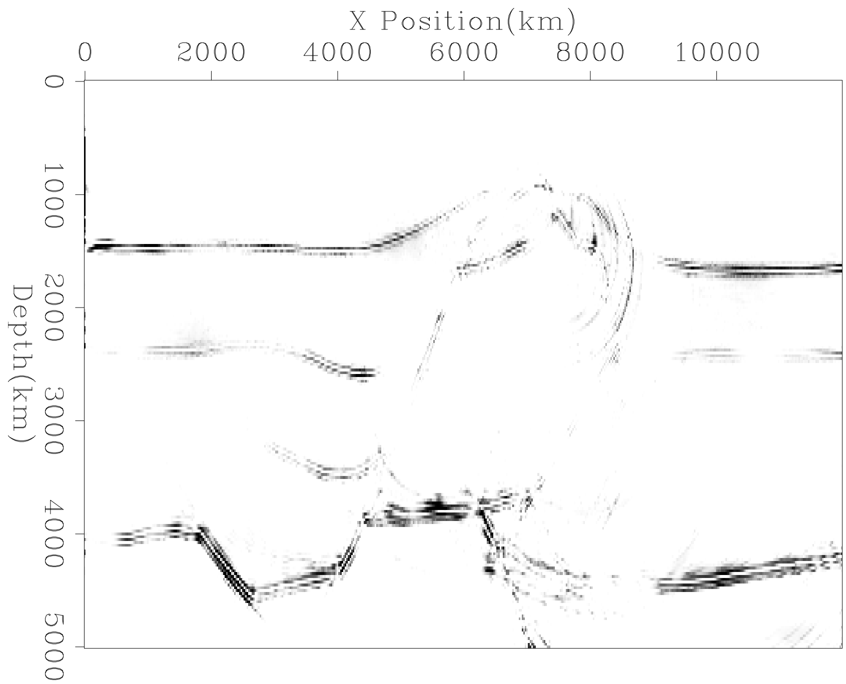

Figure 2: Resolution using conjugate gradient method after 5 iterations. Dark indicates higher resolution. illum-mdiag-synth-cg.5 [NR]

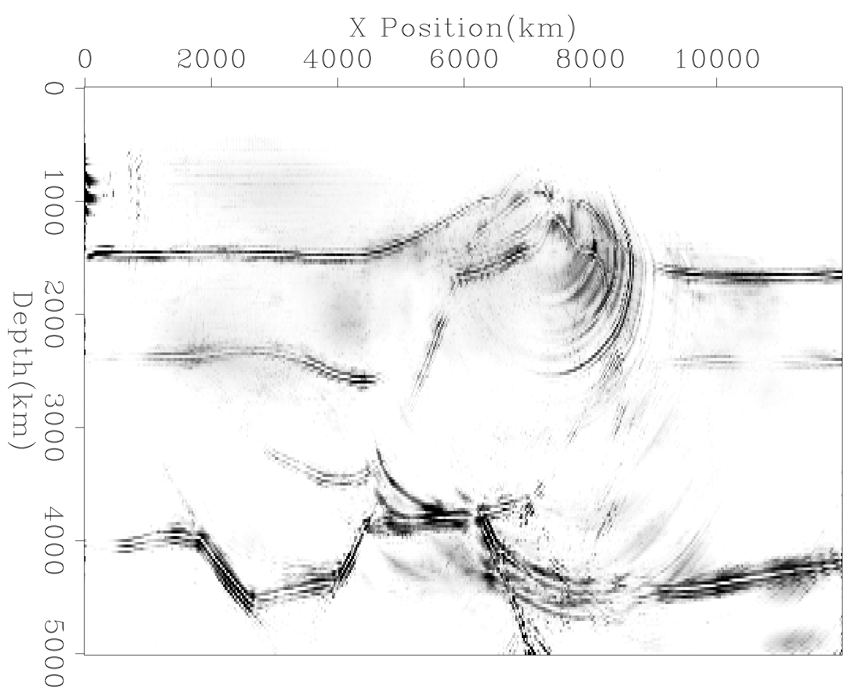

Figure 3: Resolution using conjugate gradient method after 10 iterations. Dark indicates higher resolution. illum-mdiag-synth-cg.10 [NR]

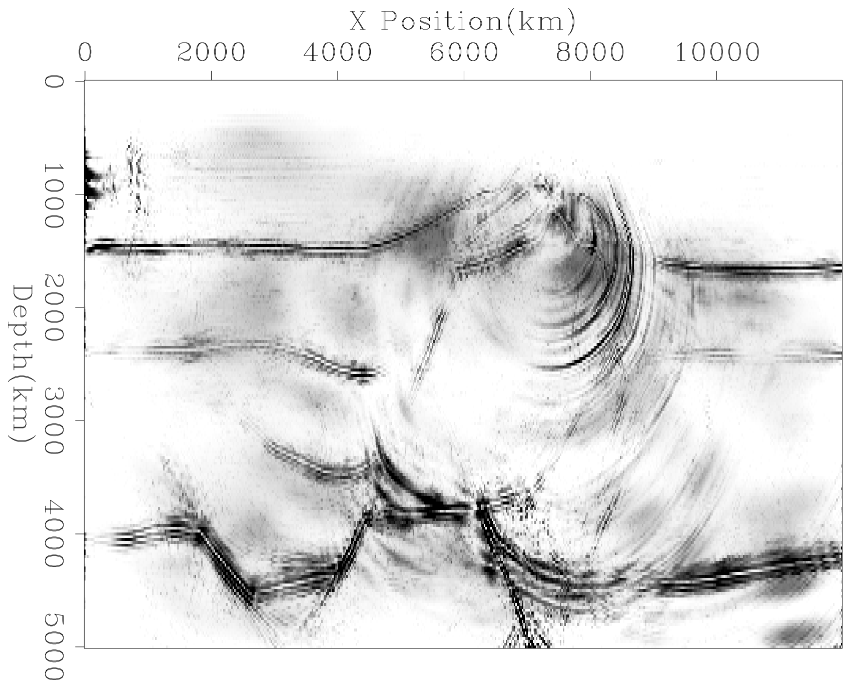

Figure 4: Resolution using conjugate gradient method after 15 iterations. Dark indicates higher resolution. illum-mdiag-synth-cg.15 [NR]

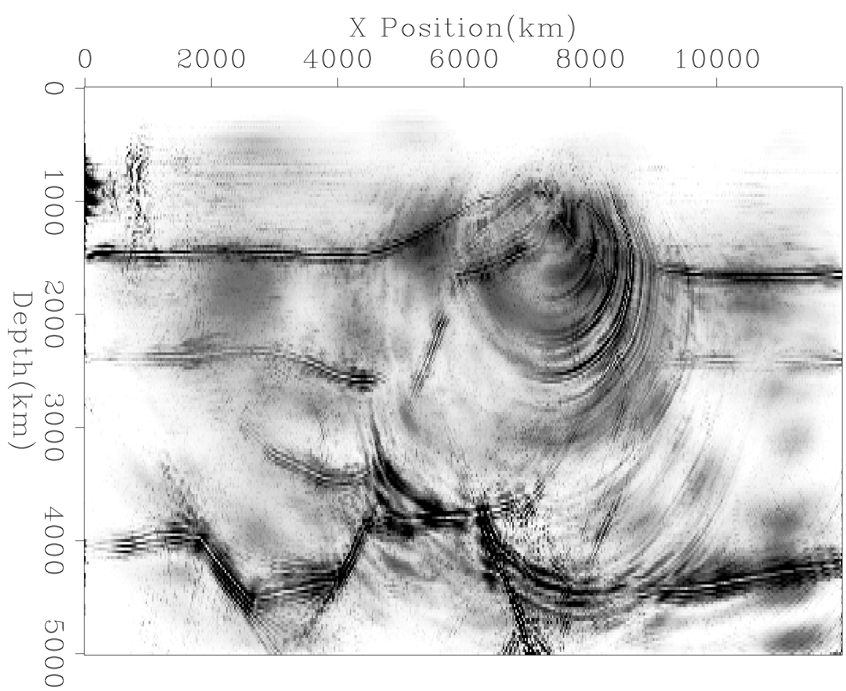

Figure 5: Resolution using conjugate gradient method after 20 iterations. Dark indicates higher resolution. illum-mdiag-synth-cg.20 [NR]

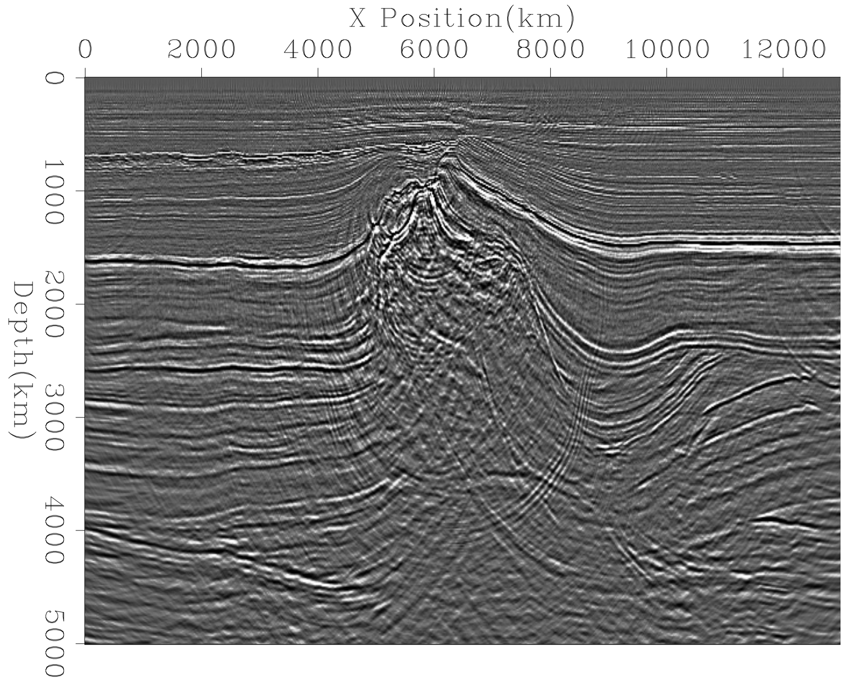

Figure 6: Inversion result on real data after 5 iterations. illum-real-cg.5 [NR]

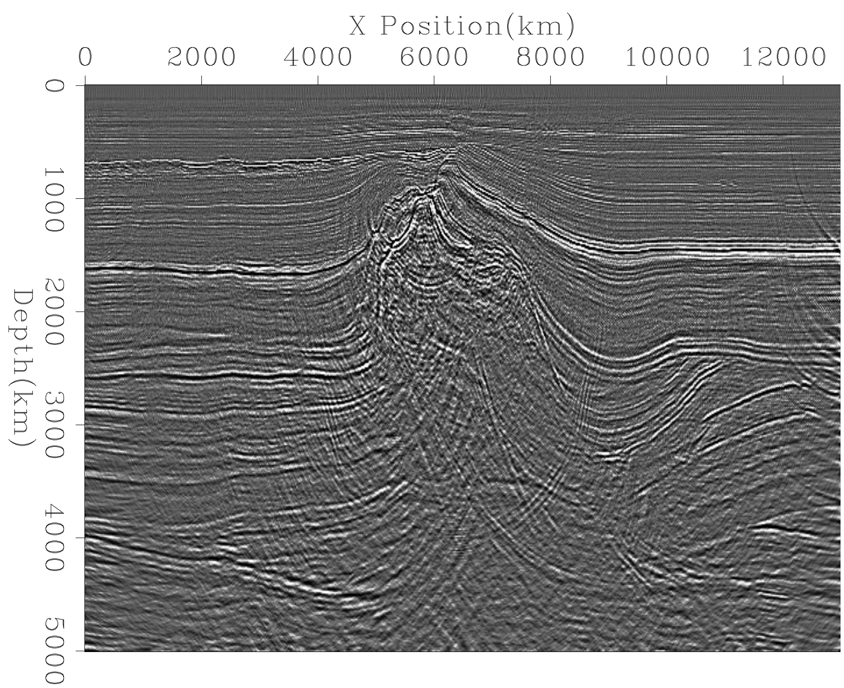

Figure 7: Inversion result on real data after 20 iterations. illum-real-cg.20 [NR]

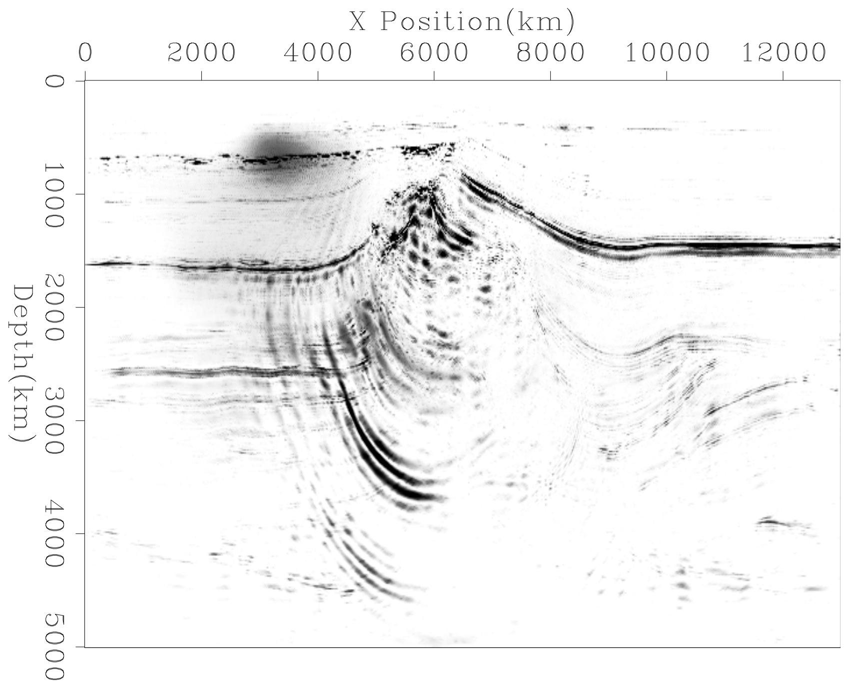

Figure 8: Resolution using conjugate gradient method after 5 iterations of the real data. illum-mdiag-real-cg.5 [NR]

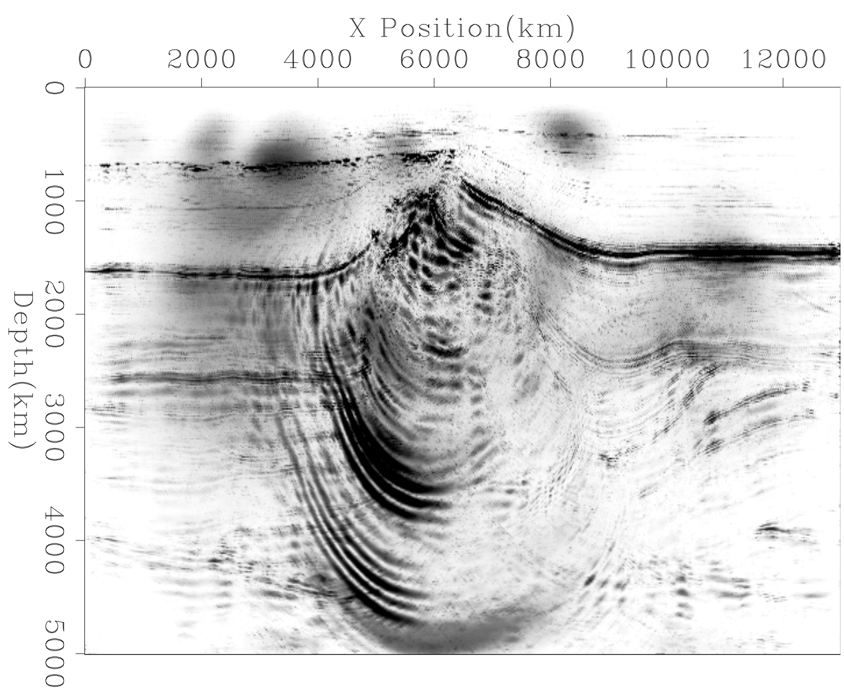

Figure 9: Resolution using conjugate gradient method after 10 iterations of the real data. illum-mdiag-real-cg.10 [NR]

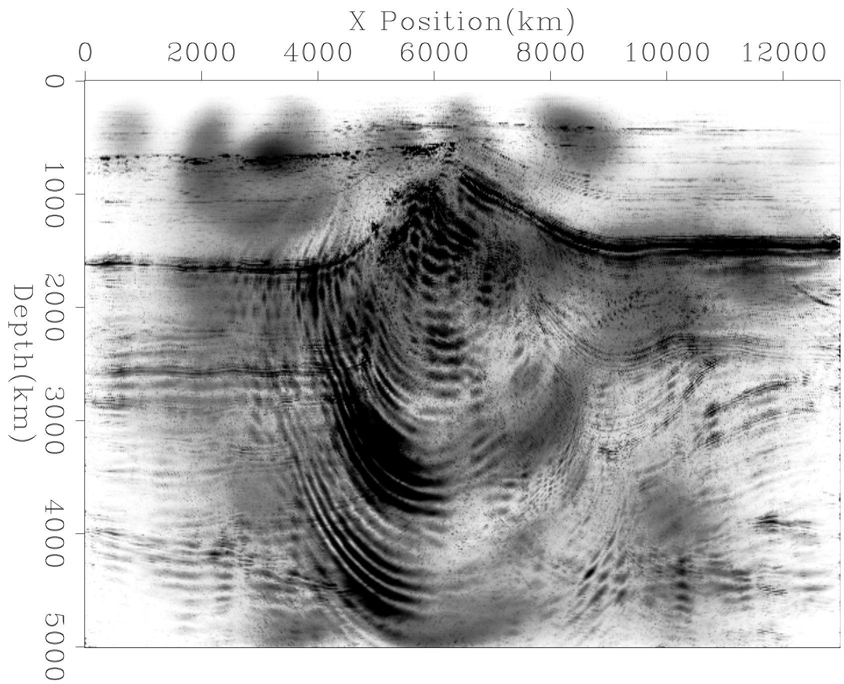

Figure 10: Resolution using conjugate gradient method after 15 iterations of the real data. illum-mdiag-real-cg.15 [NR]

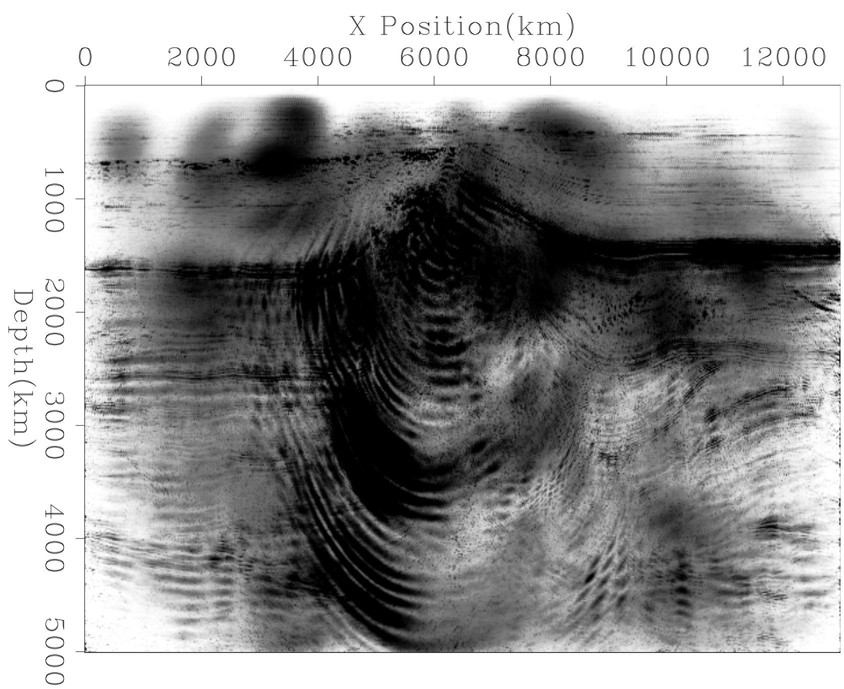

Figure 11: Resolution using conjugate gradient method after 20 iterations of the real data. illum-mdiag-real-cg.20 [NR]

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