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Fundamentals of Geophysical Data Processing

WITH APPLICATIONS TO PETROLEUM PROSPECTING

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PREFACE

This book is based at the level of a bachelor's degree in physical science. Experience at Stanford indicates that a one-semester class in engineering systems theory provides helpful additional background. It will be readable to a general science and engineering audience and should be useful to anyone interested in computer modeling and data analysis in physical sciences.

Inevitably, the book is strongly flavored by my own research interests which are presently mainly in reflection seismology. However, I have taken an interest in a good many of the data processing problems in general geophysics that have arisen in eight years of teaching graduate students and supervising research. This book is intended to be a textbook rather than a research monograph. The exercises are of a reasonable degree of difficulty for first-year graduate students, and most of them have been thoroughly tested. Since its first publication in 1976, and subsequent translation to Russian and Chinese, this book has become the most widely-referenced textbook in journal articles on exploration geophysics. No other book gives so complete an account of the fundamentals of geophysical data processing. Recently Blackwell Scientific Publications offered to reprint the book, so I took advantage of the opportunity to rewrite the preface and introduction, and to update the references.

The subject of discrete-time filters is easier than calculus and provides the needed platform, usually provided by calculus, to explore many concepts from physics and engineering. Concepts expressed in discrete time are already prepared for computers, thus engineering and science professionals are no longer diverted by analytic solutions, but by computer methods. Now with computers in every room, a curriculum that leaves every thought expressed in the continuum is simply out of date. The real "new math" should not be set theory but elementary calculus reexpressed in discrete form.

Fundamentals of Geophysical Data Processing takes the mathematical part of the undergraduate physics and engineering curricula and translates it into a form digestible by computers. So this is not only a book on *geophysical* analysis but also a book for *any* analysis in modern science and engineering. The book is required in our geophysics *graduate* curriculum because the present science and engineering *undergraduate* curriculum has hardly yet entered the computer age.

Geophysical data processing draws from mathematical physics, numerical analysis, and statistics. *Fundamentals of Geophysical Data Processing* develops its theme from a base of the general undergraduate science curriculum and deductions tend to be complete and self sufficient. This book was originally directed to the Bachelor's degree level; however, many students at that level find the book is too terse for self-study, so I have included a new study guide, *Imaging the Earth's Interior* (IEI), which supersedes the last two chapters of *Fundamentals of Geophysical Data Processing* (FGDP). For several years the new book was taught after FGDP. I taught the material in that order because the material was developed in that way and it seemed natural to teach one-dimensional analysis before teaching multidimensional analysis. Subsequently, the order of the courses was reversed and now I teach migration and velocity from IEI before teaching filters and deconvolution from FGDP. The reasons for this are first, multidimensional analysis is more geometrical and less algebraic, so it is naturally somewhat more appealing to most students and second, the theory in IEI provides a realistic guide to migration and velocity analysis in practice, whereas no book, neither FGDP nor the books by Ziolkowski or Treitel provides an equally satisfactory practical guide for deconvolution.

Engineering is about things that work most of the time, like migration and velocity analysis in IEI. Science is about things that are basic, things that should always be true but are often difficult to show convincingly, things that may require judgment in practical cases, like deconvolution and inversion, the major practical applications of FGDP.

I am indebted to a great many friends, associates, and former teachers for much of what I have learned. I have had many fruitful conversations with Steve Simpson, Enders Robinson, and John Burg about time series analysis. Ted Madden taught me much of what is written in this book on stratified media, but most importantly he infected me with the idea that the time had come to go beyond stratified media. John Sherwood and Francis Muir introduced me to reflection seismic prospecting and some unorthodox ways of thinking about it. Several generations of students were a great help in getting many of the "bugs" out of the text and the exercises. Phil Schultz, Don C. Riley and Steve Doherty prepared many of the figures in the final chapters. Mrs. Susana Erlin typed most of the manuscript and finally got the effort all together. My wife, Diane, inspired the continuing effort the project required.

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INTRODUCTION

Geophysical data processing is the use of computers for the analysis of geophysical data. A major task in geophysics is to determine as much as possible about the constitution of the interior of the earth. Where direct penetration is impractical or impossible, seismological, electromagnetic, and gravity measurements are made and the task of making inferences from these measurements is begun. Through systematic application of the laws of physics and the principles of statistics, some of these interpretation tasks can be computerized. When the number of observations is small, it may be satisfactory to match them to the adjustable parameters in known analytic solutions to the equations of classical physics. Today, however, it is common to have massive numbers of observations which contain far more information about the earth than can be modeled by analytic solutions. A typical reflection seismic marine survey ship can collect about a trillion (10^{12}) bits of information per month.

Such massive amounts of data require both statistical reduction and the ability to compute theoretical solutions in many-parameter earth models. Use of digital computers to statistically analyze geophysical data began with the Geophysical Analysis Group (GAG), an industry-supported project at the Massachusetts Institute of Technology which ran from 1953 to 1957 [Ref. 1]. Theoretical geophysical calculations made a great step forward in 1954 when Norman Haskell [Ref. 2] published a famous paper in which he showed how seismic surface waves could be computed for an earth modeled by an arbitrary number of plane parallel layers, each with arbitrarily prescribed physical properties. This enabled utilization of the entire seismic waveform in fitting an arbitrarily stratified earth model. (By "stratified" it is meant that material properties are a function of one coordinate only, usually the depth or radius). Haskell's method has been intensively developed over the last twenty years to the point where we can now readily compute seismic and electromagnetic responses to arbitrary source distributions in any desired stratified model of the earth. Indeed, it seems that the stratified medium has nearly replaced the homogeneous medium as the most popular framework for publication in mathematical geophysics.

Seismograms often consist of hundreds of oscillations, most of which may be inexplicable. Elaborate methodologies have evolved for fitting seismograms to stratified media models with random variations on layer parameters and data. It is astonishing, however, to observe that explosion seismograms with all their complicated, inexplicable details are completely reproducible. Even earthquake seismograms will be reproducible when the source region is small. Thus, the introduction of random variables into data analysis often serves mainly to force fit the data to stratified models.

In contrast to our well developed stratified media tools, most of the questions presently being asked about the earth are really questions about its *departure* from the stratified model. Foremost are the matters of verifying the mechanics of continental drift, understanding earthquakes, and seeking to locate petroleum and minerals. Thus, today, the frontiers in geophysical data processing lie in the reconciliation of field data with two- and three-dimensionally inhomogeneous models of the earth. But before we start we need a good foundation in the traditional material.

Geophysical data processing begins with the study of the sampled time form of filter theory and spectral analysis. The mathematical restraints imposed by the principle of causality are very important. Even arbitrarily complex models of the earth are subject to this principle. Computational stability often hinges on perfectly strict adherence to it. Then basic concepts of resolving power, statistics, and matrices are reviewed preparing the reader for the general theory of least squares along with lots of examples. Least squares has been, of course, the principal vehicle for the reconciliation of data with theoretical models. While it remains in this prominent role, high-resolution techniques (maximum entropy) and robust techniques (the L_1 norm and linear programming) are challenging it.

Following this development of fundamental data processing ideas, the rest of the book is concerned with treating earth models of successively increasing complexity. First, we study multiple reflected plane waves in layered media from a base of only continuity, causality, and energy conservation (no more physics than that). Waves may be calculated from knowledge of the media and the media can be calculated from the waves. Then, the more general theory of mathematical-physical computations in stratified media is introduced and the essential features of finite-difference simulations of partial differential equations are surveyed.

The final chapters are devoted to wave extrapolation and data processing with partial differential equations. I developed this material with my graduate students over the past six years at Stanford University (see Refs. [3] to [8] and [36] and [37]). The basic objective is like that in holography. A wave field is observed on a plane (the surface of the earth) and the goal is to create a two- or three-dimensional model of the scattering objects to one side of (beneath) the plane. The main problems and the techniques used are quite different from holography. Velocity inhomogeneity, diffraction, interference, and multiple reflection are ubiquitous features of seismic propagation though they are rare in common visual experience. The eye is easily deceived in a house of mirrors or when looking into an aquarium.

As I predicted in the original preface to this book, the material on wave-equation data processing developed rapidly. In 1985 it became the subject of a new book, IMAGING THE EARTH'S INTERIOR also published by Blackwell Scientific Publications.