# INTERVAL VELOCITY AND NULL SPACE

A central problem in reflection seismology is building the velocity as a function of depth (or vertical travel time) starting from certain measurements. The measurements are described elsewhere (BEI for example). They amount to measuring the integral of the velocity squared from the surface down to the reflector. The integral gives the RMS (root-mean-square) velocity. Although good quality echos may arrive often, they rarely arrive continuously for all depths. Good information is interspersed unpredictably with poor information. Luckily we can also estimate the data quality by the "coherency" or the "stack energy". In summary, what we get from observations and preprocessing are two functions of travel-time depth: (1) the velocity (squared) and integrated from the surface, and (2) a measure of the quality of the integrated velocity estimate. Some definitions:

**d** is the data vector given to us by the preprocessing. Its components range over the vertical traveltime depth $\tau$, and its component values contain the scaled RMS velocity squared $\tau v_{\mathrm{RMS}}^2/\Delta\tau$ where $\tau/\Delta\tau$ is the index on the time axis.

**W** is a diagonal matrix along which we lay the given measure of data quality. We will use it as a weighting function.

**u** is the vector we seek. Its components range over the vertical traveltime depth $\tau$, and its component values contain the local velocity (called the interval velocity) squared $v_{\mathrm{interval}}^2$.

**C** is the matrix of causal integration, a lower triangular matrix of ones.

**D** is the matrix of causal differentiation, namely, $\mathbf{D} = \mathbf{C}^{-1}$.

From these definitions, under the assumption of a stratified earth with horizontal reflectors (and no multiple reflections) the theoretical interval velocity enables us to define the theoretical RMS velocity by

$$\mathbf{Cu} \quad = \quad \mathbf{d} \tag{1}$$

Our data-fitting goal is to minimize the residual

$$\mathbf{0} \quad \approx \quad \mathbf{W}[\mathbf{Cu} - \mathbf{d}] \tag{2}$$

To find the interval velocity where there is no data (where the stack power vanishes) we have the "model damping" goal to minimize the wiggliness **x** of the (squared) interval velocity **u**.

$$\mathbf{0} \quad \approx \quad \mathbf{Du} \quad = \quad \mathbf{x} \tag{3}$$

We "precondition" these two goals by changing the optimization variable from interval velocity squared **u** to its wiggliness **x**. Substituting $\mathbf{u} = \mathbf{Cx}$ gives the two goals expressed as a function of wiggliness $\mathbf{x}$.

$$\mathbf{0} \approx \mathbf{W}\left[\mathbf{C}^2\mathbf{x} - \mathbf{d}\right] \tag{4}$$
$$\mathbf{0} \approx \epsilon\,\mathbf{x} \tag{5}$$

### Balancing good data with bad

Roughly, choosing the size of $\epsilon$ chooses the stiffness of the curve that connects regions of good data. Our first test cases gave solutions that we interpreted to be too stiff at early times and too flexible at later times. This led to the idea of modifying the goal $\mathbf{0} \approx \mathbf{D}\mathbf{u}$ to include a depth-dependent scaling. The first try would be proportional to the travel-time-depth $\tau$. Thus we define a diagonal matrix $\mathbf{\Lambda} = \mathbf{diag}(\tau)$ and revise our model-damping goal to be $\mathbf{0} \approx \mathbf{\Lambda D u}$.

### Lateral variations

Above we built interval velocity models as a function of depth only. Now we seek to build them as a function of depth and lateral location. A conventional method builds a velocity-depth model independently at each lateral location. To combine the estimation of a depth function at one location with neighboring locations, we enlarge the fitting goals. Instead of merely minimizing the vertical gradient of velocity $\mathbf{u}$, we minimize its vector gradient. We change the regularization to something like $\mathbf{0} \approx \mathbf{\Lambda}\nabla\mathbf{u}$. Luckily we have preconditioning and the helix to speed the solution. Bob Clapp has gotten some good quality preliminary results.