Estimating an image of Galilee

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Estimating an image of Galilee

- Inverse theory says data is imperfect and should be understood to have additive noise.
- In practice I find the modeling less perfect than the data.
- The Galilee data set illustrates the usual case, that the data requires a more complicated model.
Image estimation is nontrivial inversion

- Largest easily invertible matrix has 10,000 columns.
- A small image (say 100x100) has 10,000 values.

Conclusion:

- Image estimation requires iterative methods.
- Speed of convergence is a significant issue.

These complicated issues mostly ignored here today.
Estimating an image of Galilee

Next three slides

1. Model is depth $h(x,y)$.

2. Data is $(x,y,z)$ at 132,044 locations.

3. Operator (sparse matrix) is the transpose of binning.
Fitting drift
module bin2 {
# Data-push binning in 2-D.
integer :: m1, m2
real :: o1,d1,o2,d2
real, dimension (::,:), pointer :: xy

#_init( m1,m2, o1,d1,o2,d2,xy)
#_lop ( mm (m1,m2), dd (:))
integer i1,i2, id

do id = 1, size(dd) {
    i1 = 1.5 + (xy(id,1)-o1)/d1
    i2 = 1.5 + (xy(id,2)-o2)/d2
    if( 1<=i1 && i1<=m1 &&
        1<=i2 && i2<=m2 )
        if( transpose )
            mm(i1,i2) = mm(i1,i2) + dd( id)
        else
            dd( id) = dd( id) + mm(i1,i2)
    }
}
Data binned, coarse and fine

Coarse Binning

Fine Binning
Roughen with east-west derivative
Regularize the empty bins

- Binned
- Missing filled
Depth $h(x, y)$ is a poor variable.

- It is too smooth for viewing convenience.
- Iterative convergence prefers an IID variable.

Define a “preconditioned” variable

$$p(x, y) = \text{roughened } h(x, y)$$

$$\text{roughening filter} = FT \sqrt{k_x^2 + k_y^2}$$

We call it a “helix derivative.”

Equivalent to regularizing with the Laplacian.
Model the drift along the track

The “track axis” is an integer \( s \) going with the boat.

Drift (on \( s \) axis) is output of random numbers (unknowns) into a low-pass filter.

Adjust \( h(x, y) \) and drift, to minimize the residual

\[
0 \approx \text{bin}^T h(x, y) + \text{drift}(s) - \text{data}(s)
\]

The free variables in the conjugate gradient iterations are \( p(x, y) \) and the random numbers (into the low-pass filter on \( s \)).

Both free variable sets require regularization.
Modeling drift on track

Ignoring acquisition drift

Fitting drift
I could stop here and take questions,

or examine the residuals

and see some failed ideas.
Data subset: raw, modeled, drift, residual
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Data drift in model space

Data residual in model space
L2 and L1 norms
L2 and L1 norms

L2 norm

L1 norm
Median stack in each bin
Failure: Minimize \( \frac{d}{ds} \) residual

Minimum \( \frac{d}{ds} \) residual

Fitting drift
That’s all folks!

More details on-line.

Google for “Claerbout” to find free book.