3-D prestack migration of common-azimuth data

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ABSTRACT

In principle, downward continuation of 3-D prestack data should be carried out in the 5-D space of full 3-D prestack geometry (recording time, source surface location, and receiver surface location), even when the data sets to be migrated have fewer dimensions, as in the case of common-azimuth data sets that are only four dimensional. This increase in dimensionality of the computational space causes a severe increase in the amount of computations required for migrating the data. Unless this computational efficiency issue is solved, 3-D prestack migration methods based on downward continuation cannot compete with Kirchhoff methods. We address this problem by presenting a method for downward continuing common-azimuth data in the original 4-D space of the common-azimuth data geometry. The method is based on a new common-azimuth downward-continuation operator derived by a stationary-phase approximation of the full 3-D prestack downward-continuation operator expressed in the frequency-wavenumber domain. Although the new common-azimuth operator is exact only for constant velocity, a ray-theoretical interpretation of the stationary-phase approximation enables us to derive an accurate generalization of the method to media with both vertical and lateral velocity variations. The proposed migration method successfully imaged a synthetic data set that was generated assuming strong lateral and vertical velocity gradients.

The common-azimuth downward-continuation theory also can be applied to the derivation of a computationally efficient constant-velocity Stolt migration of common-azimuth data. The Stolt migration formulation leads to the important theoretical result that constant-velocity common-azimuth migration can be split into two exact sequential migration processes: 2-D prestack migration along the inline direction, followed by 2-D zero-offset migration along the cross-line direction.

INTRODUCTION

To accurately image complex 3-D structures, it is often necessary to migrate 3-D seismic data by prestack depth migration. However, at the present, 3-D prestack depth migration is not employed routinely for imaging 3-D data sets because of its computational cost and the uncertainty on the quality of results because of inadequate knowledge of the velocity model. In particular, for large 3-D marine surveys the huge computational cost of 3-D prestack migration by conventional Kirchhoff methods makes large-scale prestack imaging a very expensive effort. Kirchhoff methods are usually preferred over methods based on recursive downward continuation because of their flexibility in handling 3-D prestack data geometries (Western and Ball, 1991; Ratcliff et al., 1994). Kirchhoff methods can be employed to efficiently migrate data sets with uneven spatial sampling and data sets that are subsets of the complete prestack data, such as common-offset cubes and common-azimuth cubes. For recursive methods, the irregular sampling problem can be solved with an interpolation preprocessing step, though in practice such a task can be challenging. However, the computational problem of having to carry out the downward continuation in the full 5-D time-source-receiver space needs also to be addressed. A 5-D computational space is required because after downward continuation the wavefield expands into the full 5-D space, even when the wavefield at the surface is only 3-D (common offset) or 4-D (common azimuth). As a result of these constraints on the dimensionality of the computational domain, most of the computations are wasted on propagating components of the wavefield that either are equal to zero or do not contribute to the final image. These potential limitations of recursive methods have led the industry to adopt almost exclusively Kirchhoff methods for 3-D prestack migration though recursive methods

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have some intrinsic advantages over Kirchhoff methods. First, they are potentially more accurate and robust because they are derived from the full wave-equation and not an asymptotic solution based on a high-frequency assumption. Second, when downward-continuation techniques can be used to extrapolate the recorded data without increasing their dimensionality (e.g., zero-offset data), they can also be implemented more efficiently that the corresponding Kirchhoff methods.

This paper introduces a method for depth migrating common-azimuth data efficiently by recursive downward continuation (see Figure 1 for a schematic representation of common-azimuth acquisition geometry). A n efficient algorithm for migrating common-azimuth subsets has useful practical applications because common-azimuth data are either the result of a collection of actual physical experiments (e.g., a single-streamer marine survey with negligible cable feather) or they may be synthesized by preprocessing (Canning and Gardner, 1992a; Biondi and Chemingui, 1994) from marine data recorded using multiple streamers. Our method is based on a new downward-continuation operator that can be applied directly to common-azimuth data. The common-azimuth downward-continuation operator evaluates common-azimuth data at a new depth level, starting from common-azimuth data at the preceding level. It thus allows the computations to be carried out in the original 4-D space, instead of the 5-D space that the application of the conventional full 3-D prestack downward-continuation operator would require.

Common-azimuth downward continuation greatly reduces the amount of computations by selectively propagating only a portion of the whole prestack wavefield. From an imaging prospective, the propagated portion of the wavefield is the main component of the wavefield. In absence of ray bending it yields an exact image of the reflector at depth. Although errors are in-
results to a medium in which the velocity function varies laterally, and to analyze the errors caused by the use of common-azimuth continuation in the presence of lateral velocity variations.

The acquisition geometry of common-azimuth data is shown in Figure 1. Without loss of generality, we assume that the common offset-azimuth is aligned with the x-axis (in-line axis). Since the offset vectors between source and receivers are constrained to have the same azimuth, common-azimuth data have only four dimensions. The four axes are the recording time, the two midpoints, and the offset along the azimuthal direction. A common azimuth data set can be downward continued by applying the full 3-D prestack downward-continuation operator. In the frequency-wavenumber domain this continuation can be expressed as a phase-shift operator, with the phase given by the 3-D double square root equation (DSR) (Claerbout, 1985). In three-dimensions the DSR equation is a function of five scalar variables: the temporal frequency \(\omega\), the two components of the midpoint wavenumber vector \(\mathbf{k}_m = k_m x + k_my\), and the two components of the offset wavenumber \(\mathbf{k}_h = k_h x + k_hy\). The downward continuation of common-azimuth \(\text{CA}(\omega, \mathbf{k}_m, k_h)\) data from depth \(z\) to depth \(z + dz\) by full 3-D prestack continuation can be expressed as

\[
D_{z+dz}(\omega, \mathbf{k}_m, k_h) = \text{CA}_{z+dz}(\omega, \mathbf{k}_m, k_h) e^{-ik_\xi dz},
\]

where the vertical wavenumber is given by the double square root equation

\[
k_\xi = \text{DSR}(\omega, \mathbf{k}_m, k_h, z) = \omega \sqrt{\frac{1}{v(r,z)^2} - \frac{1}{4\omega^2} \left[ (k_{mx} + k_{hx})^2 + (k_{my} + k_{hy})^2 \right]} + \sqrt{\frac{1}{v(s,z)^2} - \frac{1}{4\omega^2} \left[ (k_{mx} - k_{hx})^2 + (k_{my} - k_{hy})^2 \right]},
\]

(2)

where \(v(s,z)\) and \(v(r,z)\) are the velocities evaluated respectively at the source and receiver locations at depth \(z\). The common-azimuth geometry is only four-dimensional. This increase in the dimensionality of the computational domain causes a drastic increase in the computational and memory requirements of 3-D prestack downward continuation, making it unattractive compared with other migration methods, such as those based on the Kirchhoff integral. However, the computational cost can be greatly reduced by applying a new downward-continuation operator that evaluates the wavefield at the new depth level only along the offset-azimuth of the original data. We derive this operator by evaluating the data at the new depth level \((z + dz)\) at the origin of the cross-line offset axis \((k_hy = 0)\) by integration over the cross-line offset wavenumber

\[
\text{CA}_{z+dz}(\omega, \mathbf{k}_m, k_h) = \int_{-\infty}^{+\infty} dk_{hy} \text{CA}_{z}(\omega, \mathbf{k}_m, k_{hx}) e^{-ik_\xi dz} = \text{CA}_{z}(\omega, \mathbf{k}_m, k_{hx}) \left\{ \int_{-\infty}^{+\infty} dk_{hy} e^{-ik_\xi dz} \right\}.
\]

(3)

and by recognizing that since common-azimuth data is independent of \(k_{hx}\), the integral can be pulled inside and analytically approximated by the stationary phase method (Bleistein, 1984). The application of the stationary-phase method is based on a high-frequency approximation; that is, it assumes that \(\omega\) tends to infinity.

The common-azimuth downward-continuation operator can thus be written as

\[
\text{Down}(\omega, \mathbf{k}_m, k_{hx}, z) = \frac{2\pi}{\sqrt{k_{hx}^2 dz}} e^{-ik_\xi dz + i \frac{\pi}{4}}.
\]

(4)

The new vertical wavenumber \(\hat{k}_\xi\) is equal to the wavenumber in equation (2) evaluated along the stationary path, and is equal to

\[
\hat{k}_\xi(\omega, \mathbf{k}_m, k_{hx}, z) = \text{DSR} \left[ \omega, \mathbf{k}_m, k_{hx}, \hat{k}_{hx}(z) \right],
\]

(5)

where \(\hat{k}_{hx}(z)\) is the stationary path for the double square root equation, as a function of \(k_{hx}\). There are two distinct solutions for the stationary path of the double square root equation; they are

\[
\hat{k}_{hx}(z) = \frac{\sqrt{\frac{1}{v(r,z)^2} - \frac{1}{4\omega^2} (k_{mx} + k_{hx})^2} \mp \sqrt{\frac{1}{v(s,z)^2} - \frac{1}{4\omega^2} (k_{mx} - k_{hx})^2}}{\sqrt{\frac{1}{v(r,z)^2} - \frac{1}{4\omega^2} (k_{mx} + k_{hx})^2} \pm \sqrt{\frac{1}{v(s,z)^2} - \frac{1}{4\omega^2} (k_{mx} - k_{hx})^2}}.
\]

(6)

In choosing between these two solutions, we consider the limiting case of the in-line offset wavenumber \((k_{hx})\) equal to zero. In this case, one solution diverges while the other, which has the minus sign at the numerator, is equal to zero. We accept this second solution because it is consistent with the notion that when both \(k_{hx}\) and \(k_{hy}\) vanish, the double square root equation reduces to the single square root equation commonly used for migrating zero-offset data (Claerbout, 1985).

The implementation of common-azimuth downward continuation as described by equations (2) through (6) yields an
algorithm for efficient migration of common-azimuth data. When the velocity field is only a function of depth, the method can be implemented efficiently with a simple phase-shift algorithm. When the velocity field is also a function of the lateral coordinates, and accurate depth-migration is required, the method should be implemented using a mixed space-wavenumber domain migration scheme, such as phase-shift plus interpolation (PSPI) (Gazdag and Squazzero, 1984) or split step (Stoffa et al., 1990) migrations. We implemented the method using a straightforward generalization of PSPI to common-azimuth migration because of its simplicity and computational efficiency; other choices would have probably been just as effective.

In the next two subsections, we analyze common-azimuth downward continuation by using a ray-theoretical interpretation of the method. The goal is to expose the underlying assumptions and to evaluate the limitations of the method’s accuracy when velocity variations cause ray bending.

**GEOMETRIC INTERPRETATION OF COMMON-AZIMUTH DOWNWARD CONTINUATION**

The common-azimuth downward continuation operator derived by the stationary phase method has a straightforward geometric interpretation that relates the propagation directions of the rays of the continued wavefield. In the Appendix, we show that the expression for the stationary path of equation (6) is equivalent to the relationship

$$\frac{p_{sy}}{p_{sz}} = \frac{p_{ry}}{p_{rz}}$$

(7)

among the ray parameters for the rays downward propagating the sources \((p_{sx}, p_{sy}, p_{sz})\) and the ray parameters for the rays downward propagating the receivers \((p_{rx}, p_{ry}, p_{rz})\). This relationship between the ray parameters constrains the direction of propagation of the source and receiver rays, for each possible pair of rays. In particular, the source ray and the receiver ray must lie on the same plane, with all the possible propagation planes sharing the line that connects the source and receiver location at each depth level. This geometric relationship constrains the sources and receivers at the new depth level to be aligned along the same azimuth as the source and receivers at the preceding depth level, consistently with the condition that we imposed for the stationary phase derivation of the common-azimuth downward-continuation operator [equation (3)]. Figure 2 is a graphic representation of the geometric interpretation of common-azimuth downward continuation. The source ray and the receiver ray must lie on any of the slanted planes that share the line connecting the source and receiver locations.

The Appendix shows that from equation (7) it is possible to derive the ray parameter equivalent of the stationary-path expression of equation (6); that is,

$$(p_{ry} - p_{sy}) = (p_{ry} + p_{sy}) \times \sqrt{\frac{1}{v(r, z)^2} - \frac{1}{p_{rx}} - \frac{1}{p_{sx}}} - \sqrt{\frac{1}{v(s, z)^2} - \frac{1}{p_{sz}}}$$

(8)

In this expression, the distinction between the velocity at the source location and the velocity at the receiver location, introduced in equation (2), and formally carried through the stationary phase approximation, is now physically meaningful. It implies that the source rays and receiver rays must lie on the same plane, notwithstanding different local velocities and ray bending.

**Common-azimuth downward continuation and ray bending**

When the propagation velocity is constant, common-azimuth downward continuation is exact, within the limits of the stationary-phase approximation. In this case, there is no ray bending and the source and receiver rays propagate straight along the slanted planes shown in Figure 2 at every depth level, until they meet to image a diffractor at depth. On the other hand, when ray bending occurs, common-azimuth downward continuation introduces an error. However, the accurate results obtained by the application of the method indicate that the magnitude of this error is small. In the following, we present an intuitive reasoning that supports the conclusion that the error introduced by common-azimuth downward continuation is small also in the presence of ray-bending. We first discuss the simpler case of velocity varying with depth and then the general case of velocity varying laterally.

Inspection of the stationary-phase results [equations (6) and (8)] shows that in a horizontally stratified medium the cross-line ray parameters for the source and receiver rays \((p_{rx}, p_{ry})\) change across boundaries between layers with different velocities. This result contradicts Snell’s law that states that the horizontal ray parameters must be constant when the velocity varies only vertically. In common-azimuth downward continuation, by imposing the constraint that the source and receiver ray must lie on the same plane, we force the source and receiver rays to bend across interfaces in a way that may not be
consistently with Snell’s law. In particular, the ray bending determined by common-azimuth continuation is incorrect when the velocity variations would make the source ray bend differently than the receiver ray along the cross-line direction.

The error in the ray bending can be analyzed by evaluating the perturbations in the values of \( p_{ry} \) and \( p_{sy} \) induced by the common-azimuth constraints when the rays cross an horizontal interface. According to Snell’s law these perturbations should be zero, since all horizontal ray parameters should remain constant across the interface. By direct application of equation (8), and assuming that the velocities above and below the interface are \( V_1 \) and \( V_2 \) the perturbations \( \Delta(p_{ry} - p_{sy}) \) are equal to,

\[
\Delta(p_{ry} - p_{sy}) = \Delta \left( \frac{\hat{k}_{by}(z)}{\omega} \right) = 2(p_{ry} + p_{sy}) \sqrt{\frac{1}{V_1^2} - p_{rx}^2} \sqrt{\frac{1}{V_2^2} - p_{rx}^2} - \sqrt{\frac{1}{V_1^2} - p_{sx}^2} \sqrt{\frac{1}{V_2^2} - p_{sx}^2}.
\]

The expression for the error in equation (9) is equal to zero, if and only if, one of the following conditions are fulfilled:

\[
|p_{rx}| = |p_{sx}|
\]

(10)

\[
p_{ry} = -p_{sy}
\]

(11)

\[
V_1 = V_2.
\]

(12)

The first condition is fulfilled when either the source ray is parallel to the receiver ray (as with zero offset data) or the two rays converge and form opposite angles with the in-line axis \( x \) (as with flat reflections along the in-line midpoint axis). The second condition is fulfilled in the case of vertical propagation (2-D), while the third condition confirms that when the velocity is constant, common-azimuth downward continuation is exact kinematically.

The preceding analysis shows that common-azimuth downward continuation introduces an error in the ray bending across velocity interfaces. However, this consideration does not necessarily lead to the conclusion that common-azimuth continuation is inaccurate. On the contrary, we argue that the error in the ray bending causes only second-order errors in the continuation results. This claim can be simply verified by recognizing that for downward continuing common-azimuth data we evaluate the vertical wavenumber \( k_z \) [equation (2)] at its stationary point \( \hat{k}_{by} \) [equation (6)] with respect to the cross-line offset wavenumber \( k_{by} \). Since the phase function is stationary at \( \hat{k}_{by} \), the first-order term of its Taylor expansion as a function of \( k_{by} \) around \( \hat{k}_{by} \) is equal to zero. Therefore, an error in \( \hat{k}_{by} \) has only a second-order effect on the evaluation of the vertical wavenumber \( k_z \) [equation (5)]. In other words, the error introduced by the incorrect ray bending has second-order effects on the continuation results, and consequently on the migration results. This conclusion is supported by the accuracy of the migrated images shown at the end of this section.

When the velocity field varies laterally, the analysis becomes more complex because the incorrect ray bending causes errors in the evaluation of the phase function not only through errors in the cross-line offset wavenumber \( k_{by} \), but also through errors in the horizontal locations where the velocity function is evaluated. The arguments that the errors in the phase function caused by an error in \( k_{by} \) are of the second order also can be qualitatively applied to the general case of lateral velocity variations. However, the magnitude of the error introduced by the mispositioning of sources and receivers at depth when evaluating the velocity function cannot be neglected in principle, and it cannot be readily analysed analytically because it is dependent on the spatial variability of the velocity function. In the next subsection, we show accurate migration results obtained over a velocity function varying both laterally and vertically. The quality of these migration results suggests that the range of application of common-azimuth migration to depth migration problems is fairly wide, although the method needs to be applied to several recorded data sets for a more thorough analysis of its range of applicability.

Results of common-azimuth depth migration

We tested common-azimuth depth migration in imaging a synthetic data set generated by using a modeling program based on the Kirchhoff integral. The Green’s function is computed analytically assuming a velocity function with a constant spatial gradient. The velocity at the origin of the spatial coordinates is equal to 1.5 km/s. Both the horizontal component and the vertical component of the velocity gradient are equal to 0.5 s\(^{-1}\). The horizontal component of the gradient is oriented at an angle of 45° with respect to the offset azimuth (in-line direction) of the acquisition geometry. The reflectivity model is composed by a half-spherical dome and a fault block superimposed onto a horizontal planar reflector. The fault reflection is dipping 45°. Figure 3 is a graphic representation of the reflectivity model and the direction of the velocity gradient vector with respect to the reflectors.

The acquisition geometry has 128 midpoints along both the in-line and cross-line directions, with a midpoint spacing of 30 m in both directions. Each midpoint gather has 64 offsets, spaced every 40 m; the nearest offset traces are actually recorded at zero offset. The offset-azimuth of the data is aligned with the in-line direction. Figure 4 shows an in-line zero-offset section extracted from the data set. The effects of the lateral component of the velocity gradient are evident both in the tilting of the horizontal reflector and in the asymmetry of the quasi-hyperbolic reflections from the dome.
Figures 5 and 6 show the result of migrating the data set using our depth migration method. Four reference velocities were used for downward continuing the data using a PSPI algorithm. When fewer reference velocities were used for the downward continuation, the quality of the results was degraded. This observation indirectly confirms the fact that the lateral velocity variations in the data presented a challenging depth migration problem. Notwithstanding the ray bending caused by the strong velocity gradient, common-azimuth migration has accurately imaged the data; both the in-line section and the depth slice show an excellent focusing of the reflectors. The planar reflector has been flattened, and both the spherical dome and the dipping reflector have been focused properly. Few artifacts are visible in the results; these artifacts are caused mostly by limited spatial aperture of the data set and by some aliasing of the high-frequency components of the data.

**COMMON-AZIMUTH STOLT MIGRATION**

The preceding section introduced an efficient phase-shift migration algorithm for common-azimuth data sets. Because for constant velocity, Stolt migration algorithms are more efficient than phase-shift ones. It is useful to derive an equivalent common-azimuth Stolt migration method. Probably the most attractive application of this new 3-D Stolt migration is the computation of migration velocity scans, which are routinely computed by using 2-D prestack Stolt migration. These multiple-velocity migrations can be used for extracting a time-migrated image and for estimating the velocity function in a dip-independent fashion. One of the most important results presented in this section is the rewriting of the dispersion relation used for common-azimuth downward continuation as a cascade of the two well-known dispersion relations used for prestack and zero-offset downward continuation of 2-D data. This result not only allows a simpler expression for the Stolt change of variable, but also leads to a meaningful interpretation of the common-azimuth migration process.

Common-azimuth Stolt migration can be derived directly by applying the usual Stolt change of variable to the expression of common-azimuth migration. The migrated volume evaluated by common-azimuth downward continuation can be expressed as

\[
    \hat{M}(k_m, z) = \int_{-\infty}^{+\infty} dk_h \int_{-\infty}^{+\infty} d\omega \\
    \times CA(\omega, k_m, k_h) e^{-i k_z(\omega, k_m, k_h, z)}z,
\]

(13)
Fig. 5. Cross-line section of the common-azimuth prestack depth migration results of the synthetic data set shown in Figure 4. This section passes through the middle of the dome.

Fig. 6. Depth slice of the common-azimuth prestack depth migration results of the synthetic data set shown in Figure 4. This slice passes through the base of the dome.
where the vertical wavenumber \( \hat{k}_z \) is given by the common-
azimuth dispersion relation derived in the previous section [equation (5)]. When the velocity is constant, the integration variable of the innermost integral can be changed from \( \omega \) to \( \hat{k}_z \), transforming the integral into an inverse Fourier transform along the depth axis; that is,

\[
\hat{M}(k_m, z) = \int_{-\infty}^{+\infty} dk_h \int_{-\infty}^{+\infty} d\hat{k}_z \left[ \frac{d\omega}{d\hat{k}_z} \right] \times CA[\omega(\hat{k}_z, k_m, k_{hx}), k_m, k_{hx}]e^{-i\hat{k}_z z}.
\]

(14)

Stolt migration is thus accomplished by applying, to the common-azimuth data, the transformation of variable defined by the inverse of the dispersion relation expressed in equation (5). The derivation of this inverse is greatly facilitated by recognizing that when the propagation velocity is constant, equation (5) can be rewritten as the cascade of two dispersion relations. The first dispersion relation is that for 2-D prestack downward continuation along the in-line direction,

\[
k_{zx} = \omega \left[ \sqrt{\frac{1}{V^2} - \frac{1}{4\omega^2}(k_{mx} + k_{hx})^2} + \sqrt{\frac{1}{V^2} - \frac{1}{4\omega^2}(k_{mx} - k_{hx})^2} \right],
\]

and the second is the one for 2-D zero-offset downward continuation along the crossline axis,

\[
k_z = k_{zx} \sqrt{1 - \frac{k_{mx}^2}{k_{zx}^2}}.
\]

(15)

and (16)

The immediate interpretation of this recasting of the common-azimuth dispersion relation is that Stolt 3-D migration of common-azimuth data can be split into two sequential steps: 2-D prestack Stolt migration along the in-line axis, followed by 2-D zero-offset Stolt migration along the cross-line axis. However, although this result was derived for Stolt migration, it must be kinematically correct also for other constant-velocity migration methods (e.g., Kirchhoff). Therefore, we can draw the general conclusion that constant-velocity migration of common-azimuth data can be split exactly as 2-D prestack migration along the in-line axis followed by 2-D zero-offset migration along the cross-line axis. Our result can be viewed as the generalization to common-azimuth migration of the results presented in Jakubowicz and Levin (1983) in their paper on splitting 3-D zero-offset migration. As for the zero-offset case, also in common-azimuth migration, splitting is only exact for the full migration process; that is, downward continuation and imaging. In contrast, splitting applied to the downward-continuation operator alone is not exact because the correct dispersion relation for downward continuing common-azimuth data is given by the cascade of equations (15) and (16), not by their sum. Canning and Gardner (1992b) have recently presented a similar decomposition of 3-D prestack migration. However, in their two-pass migration, the post-stack migration along the cross-azimuth direction comes before the 2-D prestack migration.

The exact splitting of common-azimuth migration has a simple, intuitive explanation in terms of what is known about the kinematics of two-pass 3-D post-stack migration (Jakubowicz and Levin, 1983) and 3-D DMO (Hale, 1983). The outline of the reasoning is the following: first we consider that constant velocity 3-D poststack migration can be split along two perpendicular directions (azimuth and cross-azimuth in our case) without loss of accuracy. Second, we consider that for constant velocity, 3-D DMO is kinematically exact and reduces to 2-D DMO along the azimuthal direction. Consequently, constant velocity prestack migration of common-azimuth data can be achieved by cascading NMO+DMO with poststack migration along the azimuth direction, followed by poststack migration along the cross-azimuth direction. Finally, we recognize that the first two steps of this procedure are equivalent to 2-D prestack migration along the azimuthal direction.

A purely geometric interpretation of the splitting of common-azimuth migration is shown in Figure 7. The figure shows how the ellipsoidal impulse response of 3-D prestack migration can be constructed by revolution of the vertical elliptical impulse response of 2-D poststack migration. This revolution is correctly accomplished by the cross-line zero-offset migration operator.

Finally, we notice that although the transformation of variable for common-azimuth prestack migration can be more simply and efficiently evaluated as the cascade of two transformations, the actual application of the change of variable to the data can be accomplished in a single interpolation step. Consequently, if the additional costs of Fourier transforming the cross-azimuth midpoint axis and of handling the large amount of data are neglected, the proposed 3-D prestack Stolt migration of a set of parallel lines with a common azimuth is comparable to the cost of migrating the same lines with a conventional 2-D prestack Stolt migration algorithm.

### Results of common-azimuth Stolt migration

We tested the common-azimuth Stolt migration by imaging a synthetic data set generated assuming the same reflectivity function as the one used for the previous example (Figure 3).

![Figure 7. Schematic showing the geometric interpretation of splitting constant-velocity common-azimuth migration. The ellipsoidal impulse response of 3-D prestack migration can be constructed by revolution of the vertical elliptical impulse response of 2-D poststack migration. This revolution is accomplished by convolution with the zero-offset migration response along the cross-line direction.](image-url)
but with a constant propagation velocity of 3.2 km/s. The acquisition geometry has 128 midpoint along both the in-line and cross-line directions, with midpoint spacing of 30 m in both directions. Each midpoint gather has 32 offsets, spaced every 80 m. The nearest offset traces are actually recorded at zero offset. The offset-azimuth of the data is aligned with the in-line direction.

Figures 8 and 9 show the results of migrating the data set using the proposed Stolt migration method. As expected, common-azimuth migration has imaged the data perfectly; both the in-line section and the depth slice show an excellent focusing of the reflectors. These results are similar to the prestack depth migration results shown in the previous section (Figures 5 and 6). The most noticeable differences are a slightly different

Fig. 8. Cross-line section of the common-azimuth prestack Stolt migration results of the constant-velocity synthetic data. This section passes through the middle of the dome.

Fig. 9. Depth slice of the common-azimuth prestack Stolt migration results of the constant-velocity synthetic data. This slice passes through the base of the dome.
Three-dimensional prestack depth migration of common-azimuth data sets can be accomplished efficiently by applying the proposed common-azimuth downward continuation operator. The common-azimuth continuation operator is more efficient than the full 3-D prestack continuation from which it is derived by a stationary phase approximation, because it reduces the dimensionality of the computational space from five to the original four of the common-azimuth geometry. The application of common-azimuth migration to a synthetic data set with strong lateral variations in propagation velocity demonstrates its applicability to challenging prestack depth migration problems. However, further tests are required to show that applicability of the method to the cases where the velocity function is blocky and laterally discontinuous.

A fast constant-velocity migration algorithm also can be derived by using common-azimuth downward continuation in a Stolt migration method. This Stolt formulation leads directly to the important theoretical conclusion that constant-velocity migration of common-azimuth data can be split into two exact sequential migrations: 2-D prestack migration along the in-line direction, followed by 2-D zero-offset migration along the cross-line direction.

ACKNOWLEDGMENTS

The research presented in this paper was supported by the sponsors of the Stanford Exploration Project. The computations required for the development and testing of the methods were executed on a Connection Machine 5, acquired with the financial support of the companies supporting the Super Tier of the Stanford Exploration Project.

REFERENCES


APPENDIX

RELATIONSHIP BETWEEN THE STATIONARY PATH AND THE CONSTRAINT ON RAY DIRECTIONS

The purpose of this appendix is to demonstrate the equivalence of the stationary phase derivation of the common-azimuth downward-continuation operator [equation (6) in the main text] and the constraint on the propagation directions of the source rays $(p_{sx}, p_{sy}, p_{sz})$ and receiver rays $(p_{rx}, p_{ry}, p_{rz})$ expressed in equation (7).

We start by showing that equation (7) is derived directly by imposing the condition that the source ray and the receiver ray lie on the same plane. For this condition to be fulfilled, the components of the two rays in the direction of the cross-line $y$ axis must be equal. From elementary geometry, these components are

\[ d_{ys} = v(s, z) p_{sy} d\ell_s = \frac{p_{sy}}{p_{sz}} dz \tag{A-1} \]

\[ d_{yr} = v(r, z) p_{ry} d\ell_r = \frac{p_{ry}}{p_{rz}} dz, \]

where $d\ell_s$, $d\ell_r$ are the differential raypath lengths for the source and receiver rays, $d_{ys}$, $d_{yr}$ are their components along the $y$-axis, and $dz$ is the component along the depth axis constrained to be the same for the source and receiver rays. By equating the two equations in (A-1), we immediately derive equation (7) as

\[ \frac{p_{sy}}{p_{sz}} = \frac{p_{ry}}{p_{rz}}. \tag{A-2} \]

The second step is to eliminate $p_{sz}$ and $p_{rz}$ from equation (A-2) by using the following relationships among the ray parameters:

\[ p_{sx}^2 + p_{sy}^2 + p_{sz}^2 = \frac{1}{v(s, z)^2} \tag{A-3} \]

\[ p_{rx}^2 + p_{ry}^2 + p_{rz}^2 = \frac{1}{v(r, z)^2}. \]
A few simplifications or application of the rules of proportions, we get the equivalent of the stationary path relationship [equation (6)] but expressed in terms of ray parameters

\[
(p_{ry} - p_{sy}) = (p_{ry} + p_{sy})
\]

\[
\times \sqrt{\frac{1}{v(r, z)^2} - p_{rx}^2} - \sqrt{\frac{1}{v(s, z)^2} - p_{sx}^2}
\]

\[
\sqrt{\frac{1}{v(r, z)^2} - p_{rx}^2} + \sqrt{\frac{1}{v(s, z)^2} - p_{sx}^2}
\]

\( \text{(A-4)} \)

To derive equation (6) from equation (A-4) it is sufficient to substitute the wavenumbers for the corresponding ray parameters by applying the relationships

\[ p_{sx} = \frac{k_{mx} - k_{hx}}{2\omega}, \quad (A-5) \]
\[ p_{sy} = \frac{k_{my} - k_{hy}}{2\omega}, \quad (A-6) \]
\[ p_{rx} = \frac{k_{mx} + k_{hx}}{2\omega}, \quad (A-7) \]
\[ p_{ry} = \frac{k_{my} + k_{hy}}{2\omega}. \quad (A-8) \]