Dip moveout in shot profiles

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ABSTRACT

All the known dip-moveout (DMO) algorithms that are not integral methods require the seismic data to be sorted in regularly sampled constant-offset sections. In contrast, the dip-moveout method proposed here can be applied directly to recorded shot profiles and thus can handle data that cannot be sorted in regular constant-offset sections.

The definition of the shot-DMO operator is analogous to that of the dip-moveout operator for constant-offset sections. The two operators have impulse responses with the same projection on the zero-offset plane, i.e., the stacking plane; therefore, the application of dip moveout in constant-offset sections or in shot profiles gives the same stacked section. Dip moveout transforms shot profiles to zero-offset data, to which any poststack migration can be applied.

The shot-DMO operator is space-variant and time-variant; thus direct application of the operator would be computationally expensive. Fortunately, after a logarithmic transformation of both the time and the space coordinates, the operator becomes time-invariant and space-invariant; then dip moveout can be performed efficiently as a multiplication in the Fourier domain. Shot dip moveout is also a useful tool for improving the accuracy of residual velocity analysis performed after the DMO process.

Field-data examples show that the shot-profile dip-moveout method yields stacked sections similar to those from Hale's (1984) dip moveout for constant-offset sections.

Ties arise in the cases of irregular geometries and in three dimensions. For example, a marine survey with strong cable feathering will have well-sampled shot profiles, but cannot be sorted to regular constant-offset sections. In these cases the only algorithms that could be used to perform dip moveout, aside from the technique presented here, are the costly summation methods (Deregowski, 1985).

Although the proposed algorithm is applied in shot profiles instead of in constant-offset sections, it must produce the same results as the conventional algorithms; thus the projections of the impulse response on the stacking plane of the new operator in shot profiles and of the conventional operators in constant-offset sections must be equal. This argument is illustrated in Figures 1 and 2. In Figure 1 is shown the conventional DMO "smile" (Deregowski and Rocca, 1981) in a constant-offset section and its projection on the stacking plane. In Figure 2 is shown the impulse response of the new operator that satisfies the constraint of having the same projection on the stacking plane. With this specification we have defined the shot-DMO operator. In the next section an analytical derivation of the operator based on ray theory proves the soundness of these geometric considerations and defines the new operator more precisely.

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The shot-DMO process produces the same stack as conventional algorithms, but the prestack result is different. The different result is relevant when other prestack processes, such as residual velocity analysis, are applied after dip moveout.

Conventional DMO methods are computationally expensive; the procedure proposed in this paper for a fast shot-DMO algorithm is, on the other hand, efficiently implemented as invariant convolution by multiplication in the frequency-wavenumber domain. The implementation is possible after a transformation of coordinates that makes the DMO operator approximately time-invariant and space-invariant. An advantage of implementing the procedure as a fast convolution in the frequency-wavenumber domain in shot profiles instead of in constant-offset sections is that the DMO operator can be computed only once for the whole line.

**THE DMO OPERATOR IN SHOT PROFILES**

Normal moveout should transform nonzero-offset data to equivalent zero-offset data. When there are dipping reflectors, normal moveout fails because of the dependence of NMO velocity on the reflector dip and because of the phenomenon of reflector point dispersal (Levin, 1971). Dip moveout is defined as the process which corrects the effect of dip in the data after normal moveout and transforms it to zero-offset data (Deregowski, 1982).

We now derive the DMO operator in shot profiles by considering a seismic experiment conducted on a constant-velocity medium with a dipping reflector, as illustrated in Figure 3. Using the rule of cosine, for the shaded triangle in the figure, the traveltime $t$ of the reflection from the dipping bed is given, in field coordinates, by

$$t = \frac{1}{v} (s^2 + g^2 - 2sg \cos 2\alpha)^{1/2},$$  \hspace{1cm} (1)

where $s$ and $g$ are the horizontal coordinates of the shot and the receiver, the origin is the point where the dipping reflector meets the surface, $\alpha$ is the dip of the reflector, and $v$ is the medium velocity. Changing variables $g = s + f$, where $f$ is the full source-receiver offset, and performing some algebraic manipulations, equation (1) becomes

$$t = \frac{1}{v} (4s^2 \sin^2 \alpha + 4sf \sin \alpha f + f^2)^{1/2}. \hspace{1cm} (2)$$

If we apply the dip-corrected NMO transformation to equation (2), using $v/cos \alpha$ as the NMO velocity,

$$t = \left( t_0^2 + \frac{f^2 \cos^2 \alpha}{v^2} \right)^{1/2},$$  \hspace{1cm} (3)

we get the following relation in zero-offset time $t_0$ and midpoint coordinate $y$:

$$t_0 = \frac{2(s + f/2) \sin \alpha}{v} = \frac{2y \sin \alpha}{v}. \hspace{1cm} (4)$$

The dip-corrected NMO transformation can be performed as two cascaded processes: the first is the conventional normal moveout

$$t = \left( t_0^2 + \frac{f^2}{v^2} \right)^{1/2}, \hspace{1cm} (5)$$

where $t_0$ is the NMO time not corrected for dip; the second is
If \( p(t, f) \) is the recorded shot profile, then the NMO-corrected profile \( p_{\text{NMO}}(t, f) \) is given by

\[
p_{\text{NMO}}(t, f) = p(t, f) + f^2/v^2, \quad \text{with } f = \omega/k.
\]

(7)

and the shot profile transformed into the zero-offset seismogram is

\[
P_{\text{Z}}(t_0, f) = p_{\text{NMO}}(t_0, f).
\]

(8)

Problems arise when equation (8) is applied to data with conflicting dips at the same traveltime and offset; then the dip of the reflector \( \alpha \) is multivalued. This difficulty can be overcome by doing the computation in the frequency-wavenumber domain.

From equation (4), the dip of the reflector, expressed in terms of the slope \( \Delta t_0/\Delta f \) in the zero-offset profile, is

\[
\sin \alpha = \frac{\Delta t_0}{v} \Delta f
\]

(9)

and, from Fourier theory, the slope in the zero-offset profile is

\[
\sin \alpha = \frac{\Delta t_0}{\Delta f} = \frac{k_f}{\omega_0},
\]

(10)

where \( \omega_0 \) is the angular frequency corresponding to \( t_0 \), and \( k_f \) is the wavenumber that corresponds to the full offset \( f \). Substituting equation (10) into equation (6), we find the change of variables that performs dip moveout in shot profiles:

\[
t_n = \left( t_n^2 \omega_0^2 - \frac{f^2 k_f^2}{\omega_0^2} \right)^{1/2}.
\]

(11)

The transformation (11) can be performed in the Fourier domain in a way similar to Hale's DMO (1984); the Fourier transform \( P_{\text{NMO}}(\omega_0, k_f) \) of the zero-offset profile \( p_{\text{NMO}}(t_0, f) \) is equal to

\[
P_{\text{NMO}}(\omega_0, k_f) = \int dt_0 df e^{i\omega_0 t_0} e^{-ik_f f} p_{\text{NMO}}(t_0, f);
\]

(12)

changing the integration variable \( t_0 = t_0(t_n, f) = \sqrt{t_n^2 + f^2 k_f^2/\omega_0^2} \), from equation (11), the Fourier transform becomes

\[
P_{\text{NMO}}(\omega_0, k_f) = \int dt_n df \frac{\partial t_0}{\partial t_n} e^{i\omega_0 t_0} e^{-ik_f f} p_{\text{NMO}}(t_0(t_n, f), f)
\]

\[
= \int dt_n df \frac{\omega_0}{f} e^{i\omega_0 t_0} e^{-ik_f f} p_{\text{NMO}}(t_0(t_n, f), f).
\]

(13)

Letting

\[
A = \sqrt{1 + \frac{f^2 k_f^2}{t_n^2 \omega_0^2}}
\]

(14)

the transformation of variables becomes \( t_0 = t_n A \) and the Jacobian \( \partial t_0/\partial t_n = A^{-1} \). The final expression of the double integral is

\[
P_{\text{NMO}}(\omega_0, k_f) = \int dt_n df A^{-1} e^{i\omega_0 t_n} e^{-ik_f f} p_{\text{NMO}}(t_n, f).
\]

(15)

The zero-offset shot profile is given by the inverse Fourier transform of \( P_{\text{NMO}}(\omega_0, k_f) \):

\[
p_{\text{NMO}}(t_0, f) = \int d\omega_0 dk_f e^{i\omega_0 t_0} e^{ik_f f} P_{\text{NMO}}(\omega_0, k_f).
\]

(16)

The operator defined by equation (15) requires the numerical computation of the double integral without the benefit of the speed provided by the fast Fourier transform; its implementation is thus computationally expensive. In the next section we present a method to eliminate this problem and to implement the operator efficiently.

An example of the direct application of equation (15) is shown in Figure 4, where it is applied to four impulses. As seen in Figure 4, and as derived in the Appendix by a stationary-phase approximation of the inverse Fourier transform in equation (16), the impulse response of the shot-DMO operator is the ellipse

\[
\left( \frac{t - t_i}{t_i} \right)^2 + \left( \frac{f - f_i}{h_i} \right)^2 = 1,
\]

(17)

where \( t_i \) is the impulse time and \( f_i \) is the impulse full offset. For different \( t_i \) and \( f_i \), the axes of the ellipse change; consequently, the shape of the impulse response also changes, but the curve always passes through the origin. The projection of this ellipse on the stacking plane is exactly the conventional smile

\[
\left( \frac{x - x_i}{h_i} \right)^2 + \left( \frac{y - y_i}{h_i/2} \right)^2 = 1,
\]

where the midpoint of the impulse is \( x_i = x_i + f_i/2 \), the half-offset of the impulse is \( h_i = f_i/2 \), and the projected midpoint coordinate on the stacking plane is \( y = x_i + f_i/2 \). The geometric specification described in the Introduction and illustrated in Figure 4. Shot-DMO impulse response to four spikes. This figure was obtained by numerically computing the double integral in equation (15). The curves are ellipses of equation (17) and pass through the origin.
Figures 1 and 2 is satisfied by the shot-DMO operator derived kinematically.

The shot-DMO operator defined by equation (15) is independent of the velocity of the medium, but it was derived assuming a constant-velocity medium. In the case of velocity variation, our DMO method is only approximate, although the approximation is often satisfactory in practice. Constant-offset DMO has been generalized to handle velocity variation with depth with a better approximation (Hale, 1983; Bolondi et al., 1984). We think that shot-DMO can be generalized in a similar way.

In zero-offset data, events steeper than a maximum time dip which is velocity-dependent are evanescent energy. In the practical application of the shot-DMO operator it is possible to truncate the impulse response at a maximum time dip to avoid enhancing the evanescent energy in the final result. This truncation is easily achieved by dip filtering in the frequency-wavenumber domain. Alternatively it is possible, as was done for the examples shown in this paper, to keep the DMO operator completely independent of velocity and leave the suppression of evanescent energy to poststack migration.

A FAST ALGORITHM FOR SHOT-PROFILE DMO

The direct implementation of the shot-DMO operator derived in the previous section requires numerical integration of the double integral in equation (16); therefore its computational cost is proportional to both the square of the number of time samples in the data and the square of the number of offsets in the shot profile.

Instead of computing the integrals, we would prefer to exploit the speed of fast Fourier transforms to perform efficient convolution. However, this is impossible because the elliptical impulse response expressed in equation (17) and illustrated in Figure 4 is time-variant and space-variant. Convolution in the frequency-wavenumber domain becomes possible, however, after an appropriate change of variables of both the time and the offset coordinates. That change of variables is of the logarithmic type, as used by Bolondi et al. (1982), Wang et al. (1985) and Ronen (1985) for the time axis in constant-offset DMO. We extend this change of variables to the offset axis because shot-DMO is also space-variant. Let

\[ \tau = \log (t_o) \]

and

\[ \phi = \log (f) \]

in expression (17) of the elliptical impulse response. The ellipses in Figure 4 will become the curves shown in Figure 5, whose analytical expression is

\[ \left[ \exp (\tau - \tau_o) \right]^2 + \left[ \exp (\phi - \phi_o) - 1 \right]^2 = 1. \]

This curve depends only on the differences \((\tau - \tau_o)\) and \((\phi - \phi_o)\); therefore in the new coordinates the shape of the shot-DMO impulse response is time-invariant and space-invariant. Consequently, in the new coordinates we can efficiently perform convolution by a multiplication in the Fourier domain, even if the operator itself is only approximately invariant. This
Dip Moveout in Shot Profiles

Stack with Shot-DMO

midpoint (m)

0 500 1000 1500 2000 2500

Stack with Constant-Offset DMO

midpoint (m)

0 500 1000 1500 2000 2500

Dip Moveout in Shot Profiles

approximation is consistent with our proof of shot-DMO in
the previous section, which was founded on a geometrical
analysis of the raypath.

The proposed algorithm for shot-DMO is thus composed
of three basic steps. The first step is stretching the data according
to transformations (19); the second is convolving the stretched
data with the invariant operator in the Fourier domain. The
final step is transforming the data back to the original time
and offset space.

The 2-D Fourier transform is the most computationally in-
tensive operation included in the algorithm; therefore the cost
of the whole algorithm is now only proportional to the
number of time samples times its logarithm and the number of
offsets in the shot profile times its logarithm. A logarithmic
change of the variables of the time axis would also speed up a
constant-offset DMO, but working in shot profiles has the
additional computational advantage of precomputing the op-
erator and then using it for all the shot profiles in the survey.

One theoretical problem that arises when applying our al-
gorithm is that the logarithmic transformation of variables is
not defined at zero time and at zero offset. In practice, first
time samples are seldom interesting, and the data at zero
offset (if they exist) do not need dip moveout. Furthermore,
the shot-DMO impulse response extends for the full offset on
either side of its center; to preserve all dips, we need to pad
out the shot record to twice the original far offset.

Another problem that must be faced when this algorithm is
applied is that we do not have an analytical expression for the
operator in the new coordinates. The solution we adopted in
the following examples was to compute the operator numeri-
cally once at the beginning, then store it and use it for all the
shots. The operator in the frequency-wavenumber domain was
obtained by a numerical Fourier transformation of the
stretched ellipse of equation (20).

FIELD-DATA RESULT

The performance of the DMO algorithms was tested by
using a data set from the Gulf of Mexico. In these examples
our goal was not to show the importance of applying DMO to
seismic data but rather to check the equivalence of the results
from processing the data using our shot-DMO and the results
from processing the data using constant-offset DMO. Both
DMO methods were applied to the data after velocity analysis
and NMO correction; neither residual velocity analysis nor
residual NMO was used. The shot-profile algorithm used in
the examples is the one based on the logarithmic transforms,
as described in the previous section. The constant-offset pro-
cessing is Hale's DMO by Fourier transform.

Figures 6 and 7 show the stacked sections obtained without
and with the application of shot-DMO. Figure 8 shows the
result of constant-offset DMO processing, for comparison
with the shot-DMO result. The most evident differences in the
results with and without DMO are in the steeply dipping
events. The DMO processes preserve the fault-plane reflection
and the tails of the diffraction hyperbolas, which are smeared
by the conventional stack.

Dip-moveout correction is greater at earlier times, and thus
its most evident effects are in the shallower part of the stack.
Therefore the different results produced by the different pro-
cesses are better compared on windows of the upper-right part

Fig. 7. Stacked section of the same data set as in Figure 6.
This stack was obtained after shot DMO; the dipping events
are properly imaged.

Fig. 8. Stacked section of the same data set as in Figures 6
and 7. This stack was obtained after constant-offset DMO; it
is similar to the stack after shot DMO shown in Figure 7.
Stack without DMO

midpoint (m)
1000 1500 2000 2500

FIG. 9. Upright window of the stack in Figure 6. The stack without DMO has smeared the fault-plane reflection and the diffraction hyperbolas.

Stack with Shot-DMO

midpoint (m)
1000 1500 2000 2500

FIG. 10. Upright window of the stack in Figure 7. The shot DMO has restored the dipping events such as the fault-plane reflection and the diffraction hyperbolas.

Stack with Constant-Offset DMO

midpoint (m)
1000 1500 2000 2500

FIG. 11. Same window of the stack as in Figures 9 and 10, but the data were stacked after applying conventional DMO in constant-offset sections. There are some slight differences between this result and Figure 10 since the DMO operator in constant-offset sections was slightly aliased.

FIG. 12. Synthetic CMP gather and correspondent semblance function for velocity analysis. The upper hyperbola is the reflection of a 60-degree dipping bed and the lower one of a flat reflector. The velocity of the medium was supposed constant and equal to 2000 m/s (slowness equal to .0005 s/m).
Fig. 13. The same CMP gather as in Figure 12, after NMO, DMO, and inverse NMO: (a) with shot DMO, and (b) with constant-offset DMO. The velocity used for NMO is the correct velocity: (2000 m/s). The correspondent velocity analyses are shown next to the gathers. Since the NMO velocity is the medium velocity, both methods correct the dip effect perfectly. In the gather after constant-offset DMO there is some aliasing noise that will disappear in the stack.

Fig. 14. The same CMP gathers as in the previous figures but now with NMO velocity equal to 1800 m/s, that is, 10 percent lower than the correct velocity: (a) with shot DMO, and (b) with constant-offset DMO. Conventional DMO is less sensitive to errors in NMO velocity than shot DMO. The error in the residual velocity analysis after shot DMO is about −5 percent.
Fig. 15. The same CMP gathers but now with NMO velocity equal to 2200 m/s, that is, 10 percent higher than the correct velocity: (a) with shot DMO, and (b) with constant-offset DMO. The error in the residual velocity analysis after shot DMO is about +4 percent.

Fig. 16. Three stacks of the synthetic data set using shot DMO. The stacks were obtained using different NMO velocities: (a) the correct velocity, (b) a 10 percent lower velocity, and (c) a 10 percent higher velocity. The result degrades slightly with the incorrect NMO velocities.
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FIG. 17. Three stacks of the synthetic data set using constant-offset DMO. The stacks were obtained using different NMO velocities: (a) the correct velocity, (b) a 10 percent lower velocity, and (c) a 10 percent higher velocity. Note that the aliasing noise visible in the gathers has almost disappeared in the stacks.

of the stacks. In Figure 9, a conventional stack is shown. The fault reflection has completely disappeared, and the diffraction hyperbolas are weak. After applying shot-DMO (Figure 10), the fault reflection is present in the stack, and the diffractions are easily noticed. Figure 11 shows (for control) the stacked section after applying constant-offset DMO. The two results in Figures 10 and 11 are almost identical; there is only a slight difference in the shallow part, where shot-DMO has done a better job of imaging some diffraction hyperbolas. Our explanation of this difference is that the DMO operator in constant-offset sections is slightly aliased in space. We could have avoided the aliasing problem by, for example, filling in dead traces in the constant-offset section to reduce the common-midpoint spacing (Bolondi et al., 1982).

SHOT-DMO AND RESIDUAL VELOCITY ANALYSIS

One would like to perform a residual velocity analysis (RNMO) after DMO, because velocity analysis is more accurate after the DMO process has removed the effects of the dips on the stacking velocity. On the other hand, this procedure is not exact; in the definition of shot DMO, found in equations (5) and (6), the dip-dependent transformation follows normal moveout, and the two operators do not commute. Therefore, the application of residual velocity analysis after dip moveout introduces some error. To assess the error due to the application of residual normal moveout after dip moveout, we show below for some synthetic data a common-midpoint gather after dip moveout, the results of residual velocity analysis, and the stack after residual normal moveout.

The synthetic data set was generated assuming two reflectors, one flat and one dipping at 60 degrees, in a medium of constant velocity equal to 2000 m/s. The shot interval was assumed to be 25 m and the group interval was 12.5 m; the resulting offset interval in the CMP gathers is 50 m. In Figure 12 a CMP gather and the corresponding contour plot of the semblance function of the stacking slowness and time are shown. The upper hyperbola is the dipping-bed reflection, and the lower hyperbola is the flat-bed reflection. Figure 13 shows the same CMP gather after NMO with the correct velocity, followed by DMO and inverse NMO. The plots on the left are obtained after the application of shot-DMO and the ones on the right, after constant-offset DMO. The corresponding semblance plots for velocity analysis are also shown. The results after a correct NMO are equivalent. Note that the constant-offset section DMO operator was aliased, and thus in the CMP gather after constant-offset DMO there is some aliasing noise that will disappear in the stack.

Sensitivity to an inaccurate NMO velocity is analyzed in Figures 14 and 15. These figures show the same CMP as Figure 13, but with different NMO velocities. In Figure 14 the NMO velocity is 1800 m/s, 10 percent lower than the correct velocity. In Figure 15 the velocity is 2200 m/s, 10 percent higher than the correct one. Residual velocity analysis after shot-DMO is off by about 5 percent in Figure 14 and by about 4 percent in Figure 15; by contrast, residual velocity analysis after DMO in constant-offset sections is off by only about 2 percent in both cases.

The stacks corresponding to these CMP gathers (Figures 12–15) are shown in Figures 16 and 17. We always choose as stacking velocity the one that was correct for the flat reflector, that is, the medium velocity. The stack after shot-DMO is somewhat more sensitive to errors in NMO velocity.

The synthetic examples show that residual velocity analysis and residual NMO after shot-DMO are feasible, and further that these steps help to obtain a more accurate velocity profile and to image the dipping reflectors better. On the other hand,
shot-DMO seems more sensitive than constant-offset DMO to errors in NMO velocity; this difference is not surprising since the prestack results of the two processes are different.

**CONCLUSIONS**

The algorithm proposed here for shot dip moveout (DMO) has all the advantages of processing data in field profiles, and it produces stacked sections equivalent to those produced by DMO in constant-offset sections.

We have also shown that residual velocity analysis and residual normal moveout can be performed after shot DMO. We can conclude that all prestack processing can be performed in shot profiles without resorting the data in midpoint and offset coordinates; only some common-midpoint gathers used for velocity analysis need to be sorted out.

The shot-DMO algorithm, because of a logarithmic change of variables, performs convolution as a multiplication in the Fourier domain and is therefore computationally efficient.

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**REFERENCES**


**APPENDIX**

The purpose of this appendix is to derive the impulse response of the shot-DMO operator [equation (17)] using a stationary-phase approximation of the inverse Fourier transform of equation (16).

If the shot profile after NMO is an impulse at time $t_i$ and full offset $f_i$, that is, 

$$p(t, f) = \delta(t_n - t_i) \delta(f - f_i), \quad (A-1)$$

the Fourier transform of the profile after DMO is 

$$P(\omega, k) = \int dt \int df A^{-1} e^{i\omega t + ikf} \delta(t_n - t_i) \delta(f - f_i) = A_i \int df e^{i\omega t + ikf} \delta(f - f_i), \quad (A-2)$$

where 

$$A_i = \sqrt{1 + \frac{f_i^2}{t_i^2 \omega_0^2}}. \quad (A-3)$$

Therefore, the inverse Fourier transform $p(t, f)$ of the shot-DMO impulse response is 

$$p(t, f) = \int d\omega \int df k A_i^{-1} e^{-i\omega t - k f_i} \delta(f - f_i). \quad (A-4)$$

The phase function $\Phi(\omega_0, k_f, t, f; t_i, f_i)$ of the integrand in equation (A-4) is 

$$\Phi(\omega_0, k_f, t, f; t_i, f_i) = \omega_0 (t - t_i A_i) + k_f (f - f_i) \quad (A-5)$$

The impulse response is different from zero where the phase function is stationary with respect to $\omega_0$ and $k_f$, that is, in the points solution of the system of equations 

$$\begin{align*}
\frac{\partial \Phi}{\partial \omega_0} &= 0 \\ \frac{\partial \Phi}{\partial k_f} &= 0.
\end{align*} \quad (A-6)$$

Computing the derivatives leads to the system 

$$\begin{align*}
\omega_0 t_i^2 - t &= 0 \\ k_f f_i^2 - f + f_i &= 0.
\end{align*} \quad (A-7)$$

and, after some algebraic manipulation, leads to 

$$\begin{align*}
\frac{(\omega_0 t_i^2 - k_f f_i^2)}{\omega_0 t_i^2 + (k_f f_i)^2} &= \left(\frac{t}{t_i}\right)^2 \\
\frac{(k_f f_i^2)}{\omega_0 t_i^2 + (k_f f_i)^2} &= \left(\frac{f - f_i}{f_i}\right)^2.
\end{align*} \quad (A-8)$$

Summing the two equations in system (A-8) gives equation (17), that is, the stationary-phase approximation of the shot-DMO impulse response.