Numerical analysis of the azimuth moveout operator for vertically inhomogeneous media

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ABSTRACT

The azimuth-moveout (AMO) operator, unlike the dip-moveout (DMO) operator, has a 3D structure in homogeneous isotropic media, with an out-of-plane (crossline) component. In general, this component is concave downward, giving the operator an overall skewed-saddle shape. The AMO operator, necessary for azimuth correction only, is typically smaller in size than conventional DMO operators, which corresponds to purely offset correction to zero offset. When velocity varies vertically, the operator shape changes depending on the degree of velocity variation. The general shape of the operator, however, remains saddlelike. In fact, for smooth velocity increases with depth, similar to those found in the Gulf of Mexico, the $v(z)$ AMO operator does not differ much from its homogeneous counterpart. In this case, the residual AMO operator, constructed by cascading a forward homogeneous AMO operator with an inverse $v(z)$ one, is extremely small, which suggests that the impact of such $v(z)$ variations on the AMO operator is generally small. Complex vertical velocity variations, on the other hand, result in more complicated AMO operators that include, among other things, triplets at moderate angles. Regardless of the complexity of the model, the $v(z)$ operator has the same first-order behavior as its homogeneous counterpart. As a result, for small dip angles the homogeneous AMO, as a tool for partial stacking, often enhances the image. Moderate to steep dips in complex $v(z)$ media requires the application of an algorithm that honors such velocity variations.

INTRODUCTION

For cost reasons, seismic surveys are designed so that multiple geophone arrays are deployed to record sound waves, typically emanating from a single source. These geophones, in a typical 3D survey, are rarely aligned along a single straight line that is parallel to the shooting direction. In most 3D marine, as well as land, survey designs, the source-receiver azimuth is not constant. In typical marine data, the source-receiver azimuth can range between $-45^\circ$ and $45^\circ$ for small offsets, where $0^\circ$ azimuth is the direction of the ship’s motion. Azimuth variation is often ignored and seismic traces are binned, after normal-moveout (NMO) correction, into a regularly sampled data set in offset and common midpoint (CMP). Although for isotropic homogeneous media, such binning has no bearing on reflections from horizontal events, ignoring the azimuth variation can harm reflections from dipping events (Biondi et al., 1998) resulting in the attenuation of such reflections when partial stacking is applied to reduce the volume of the data set (Hanson and Witney, 1995).

Biondi et al. (1998) introduced azimuth-moveout (AMO) correction as a single operator to correct for azimuth variations (and dip) in homogeneous isotropic media, and devised a Kirchhoff-type implementation on multiazimuth seismic data sets. Though the AMO operator had a 3D structure, it had an overall small aperture; thus, the Kirchhoff implementation of AMO is relatively cheap. Figure 1 shows an AMO operator in homogeneous media. It is clearly 3D in structure and has a general skewed-saddle shape. Like the dip-moveout (DMO) operator, the AMO operator is applied after NMO correction. Velocity increase with depth is very common in the subsurface, and an important question is how much of an error can be attributed to ignoring such vertical velocity variation. Using the 3D SEG/EAEG salt-dome model, Biondi (1998) showed that the homogeneous AMO operator produces reasonable results in areas with smooth vertical velocity variations. Is this a general conclusion or does it hold only for the cases he tested?

In this paper, we numerically construct the AMO operator for vertically inhomogeneous media and observe how the operator shape is influenced by vertical inhomogeneity. Next, we generate the residual AMO operator constructed by cascading a forward homogeneous-medium AMO operator and an inverse $v(z)$-medium AMO operator. The size and shape of
the residual operator provides us with valuable information regarding the impact of vertical inhomogeneity on AMO. The smaller the residual operator the less the impact of vertical velocity gradients on AMO. Since the AMO operator is generally small, the \( v(z) \) AMO operator might be useful even in relatively complex areas (with lateral velocity variation). Examples include three types of vertical velocity variations: linear increase as a function of depth, a low-velocity layer embedded in an overall increase in velocity with depth, and a high-velocity layer embedded in an increase in velocity with depth. The last example is similar to what can be observed in Texas as a result of the Austin Chalk layer.

**AZIMUTH MOVEOUT CORRECTION IN HOMOGENEOUS MEDIA**

The impulse response of the AMO operator in homogeneous media is a skewed saddle. The shape of the saddle depends on the offset vector of the input data \( \mathbf{h}_1 = h_1 \cos \theta_1 \hat{x} + h_1 \sin \theta_1 \hat{y} = h_1 (\cos \theta_1, \sin \theta_1) \) and on the offset vector of the desired output data \( \mathbf{h}_2 = h_2 (\cos \theta_2, \sin \theta_2) \), where the unit vectors \( \hat{x} \) and \( \hat{y} \) point respectively in the inline direction and the crossline direction, and \( \theta_1 \) and \( \theta_2 \) are respectively angles of the input and output azimuth from the inline axis. The time shift to be applied to the data is a function of the difference vector \( \Delta \mathbf{m} = \Delta m \cos \Delta \varphi, \sin \Delta \varphi \) between the midpoint of the input trace and the midpoint of the output trace. The analytical expression of the AMO saddle is

\[
t_2(\Delta \mathbf{m}, \mathbf{h}_1, \mathbf{h}_2, t_1) = t_1 \frac{h_2 \sin^2(\theta_1 - \theta_2) - \Delta m^2 \sin^2(\theta_2 - \Delta \varphi)}{h_1 \sin^2(\theta_1 - \theta_2) - \Delta m^2 \sin^2(\theta_1 - \Delta \varphi)}.
\]

The traveltimes \( t_1 \) and \( t_2 \) are the traveltime of the input data after normal moveout (NMO) has been applied and the traveltime of the results before inverse NMO has been applied, respectively.

Figure 2 shows three AMO operators that correspond to three different azimuth correction angles in a homogeneous medium. Though the general shape of the AMO operators is practically the same, the size is very much dependent on the amount of azimuth correction; the larger the azimuth correction the larger the AMO operator. Clearly, for zero azimuth correction the operator reduces to a point. The size dependence of the operator on azimuth holds regardless of the medium. The shape of the operator, however, is independent of azimuth correction.

In Figure 2 and throughout, the contour curves plotted represent lines of equal ray parameter. The curves provide information on the distribution of dip angles, as well as on the distribution of energy along the operator; denser contour lines imply higher amplitude.

**GENERATING THE AZIMUTH MOVEOUT OPERATOR IN \( v(z) \) MEDIA**

We build the AMO operator in \( v(z) \) media by cascading a forward and an inverse 3D \( v(z) \) DMO operator. An angular transformation, that depends on the azimuth correction, is applied to the inverse operator. Therefore, to build the AMO operator we must first build the 3D \( v(z) \) DMO operator. Artley et al. (1993) suggested an approach to build a kinematically exact 3D DMO operator. Following their approach, we construct the 3D DMO operator by solving a system of six nonlinear equations to obtain six unknowns that include, among other things, the zero-offset time and surface position of the specular reflection.
point. Artley's traveltimes are calculated and tabulated using \(v(z)\) ray tracing. The total traveltime is

\[
t_{\text{tg}} = t_s + t_g,
\]

where \(t_s\) is the traveltime from the source to the reflection point and \(t_g\) is the traveltime from the receiver to the reflection point, and therefore the gradient vector,

\[
\nabla t_{\text{tg}} = \nabla t_s + \nabla t_g = \mathbf{p}_s + \mathbf{p}_g.
\]

has a direction that is normal to a hypothetical reflector. Because the zero-offset slowness vector \(p_0\) is also in the direction that is normal to reflector dip, \(p_g\) is a scaled sum of the slownesses of the rays from the source \(p_s\) and receiver \(p_r\) to the specular point (see Figure 3). Therefore,

\[
p_0 = \lambda (p_s + p_r).
\]

Considering the \(z\)-component gives

\[
p_{0z} = \lambda (p_{sz} + p_{gz}).
\]

then,

\[
\lambda = \frac{p_{0z}}{p_{sz} + p_{gz}}.
\]

Since

\[
p_{0z} = \cos[\theta(p_0, t_0)]s(t_0),
\]

\[
p_{sz} = \cos[\theta(p_0, t_s)]s(t_s),
\] and

\[
p_{gz} = \cos[\theta(p_0, t_g)]s(t_g),
\]

where \(s(=|p|)\), for each of \(p_s, p_r\), and \(p_0\) is the slowness, \(\theta\) is the ray angle, and \(t_0\) is the two-way zero offset traveltime. Then,

\[
\lambda = \frac{\cos[\theta(p_0, t_0)]s(t_0)}{\cos[\theta(p_0, t_s)]s(t_s) + \cos[\theta(p_0, t_g)]s(t_g)}.
\]

Substituting equation (10) into the \(x\)- and \(y\)-components of equation (4) provides two of the six nonlinear equations needed to be solved. The other four equations are

\[
0 = \xi(p_g, t_g) \cos \phi_g - \xi(p_s, t_s) \cos \phi_s + 2h
\]

Equation (11) is the requirement that the surface distances \(\xi\) along the inline component from both the source and receiver to the specular reflection point (SRP) add to equal the source-receiver offset \(2h\). Equation (12) is the requirement that the distances along the crossline component to the SRP are equal for the source and receiver. Equations (13) and (14) imply that the vertical times \(t\) from the source, the receiver, and the zero-offset surface positions to the SRP are equal. Both \(\xi\) and \(t\) are calculated using ray tracing and then stored in a table as a function of ray parameter \(p\) and the traveltime \(t\).

The inverse operator is calculated in the same way as the forward operator, but now we must evaluate the NMO time \(t_x\) or the total traveltime \(t_{tg}\) instead of \(t_0\), which is known. Subsequently, \(t_0\) and \(y_0\) are calculated in the same way as for the forward approach.

To build the AMO operator, the outputs of the forward 3D DMO operator \(t_0(t_0, p_x, p_y), x_0(t_0, p_x, p_y), \) and \(y_0(t_0, p_x, p_y)\) are inserted into the inverse 3D DMO operator. Prior to applying the inverse operator, the axes are rotated with an angle given by the desired azimuth correction. The result is an AMO operator given by

\[
x_{\text{AMO}}(t_0, p_x, p_y) = x_0(t_0, p_x, p_y) - x_0(t_0, p_x, p_y).
\]

and

\[
y_{\text{AMO}}(t_0, p_x, p_y) = y_0(t_0, p_x, p_y) - y_0(t_0, p_x, p_y).
\]
AMO OPERATORS IN $v(z)$ MEDIA

Three examples of vertical velocity variations with depth are considered here. These examples provide us with a reasonable understanding on how the AMO operator is sensitive to vertical inhomogeneity. We show three different AMO operators for each example: the first corresponding to a correction in offset only, called typically the residual DMO operator; the second corresponding to a correction in offset and azimuth; the third has an azimuth correction only. The offset correction, used in most of the examples, is from 2.0 to 1.5 km and, as mentioned previously, the azimuth correction is $30^\circ$. For size comparison, we also display the full $v(z)$ DMO operator for an offset of 2 km. The NMO time for all operators in this paper is 2 s.

All the 3D graphs of AMO operators have an aperture that covers half the maximum possible ray parameter given by $1/v(z)$, where $v(z)$ is the velocity at the reflection point. Since the surface velocity for all three models is the same at 1.5 km/s, this range includes ray-emergence angles up to $30^\circ$. The cross-sections of the 3D operator, on the other hand, will include emerging angles up to the critical angle. Figure 6 shows two of the three velocity models considered in this paper. The left one will be referred to as the low-velocity-layer example; the right one will be referred to as the high-velocity-layer example. The third velocity model, not shown here, is a simple linear velocity increase with depth at a gradient of $0.6 \text{ s}^{-1}$. All velocity models have a surface velocity of 1.5 km/s. The gray curves in the figure correspond to the rms velocity for the same models.

Figure 7 shows the AMO operators for the first example, which is a linear velocity increase with depth. The AMO operator corresponding to a pure offset correction from offset of 2 to 1.5 km (shown upper left), has a similar shape to the full 3D DMO operator (shown lower right) which is generally a saddle, but much smaller in size. The corresponding residual DMO operator for homogeneous media is a purely 2D operator. The azimuth-correction-only operator (shown upper right) is very similar to the homogeneous-medium one shown in Figure 2, with an overall skewed-saddle shape. When the offset and azimuth corrections are combined in a single operator, it is given by the one shown in the lower left of Figure 7. The full DMO operator (shown lower right) is clearly the largest in size considering the scale difference of the plots. AMO operators that include offset correction alters the position of horizontal, as well as dipping reflections. This alteration is necessary to correct for the nonhyperbolic moveout associated with $v(z)$ media for horizontal and dipping events.

Figure 8 shows the inline and crossline components of the AMO operator shown in Figure 7 (upper left), which corrects for offset only from 2 to 1.5 km. However, the operator shown in this figure includes the full aperture, and thus includes the triplications at high angles. Surprisingly, the size of the operator in the crossline component is larger than that in the inline component. This emphasizes the importance of considering the true vertical velocity variation for the crossline component. This fact also stresses the importance of the crossline component of the residual DMO operator. Figure 9 shows the inline and crossline components of the AMO operator corresponding to...
azimuth correction of 30°. Again, we include the full possible aperture and, conveniently, no triplications exist. The absence of triplications simplify the application of such an operator in a Kirchhoff-type of implementation. Figure 10 shows the inline and crossline components of the AMO operator that includes both the offset and azimuth corrections. This operator includes triplications that are associated with the offset correction portion of the operator. This operator is simply the convolution of the two previous operators, combining elements from both operators.

An AMO (or residual DMO) correction from offset 1.5 to 2.0 km provides us with an operator that is adjoint to the operator shown in Figure 8, which corresponds to an offset correction from 2.0 to 1.5 km. Figure 11 shows the inline and crossline components of such an AMO operator with the full aperture included.

The velocity model used in the second example has a low-velocity zone as shown in Figure 6 (left). Figure 12 shows AMO operators for this velocity model: corresponding to a pure offset correction (upper left), a pure azimuth correction (upper right), the combination of offset and azimuth correction (lower left), and a full DMO operator (plotted at a larger scale, lower right). The operators that include offset corrections are more complicated than their linear velocity model counterparts, whereas the operators that includes only azimuth corrections show similar shapes as the linear velocity model ones as well as to the homogeneous model ones. This observation implies that vertical inhomogeneity has a greater impact on the offset correction part of the operator than on the azimuth correction part.

A more detailed view is given by the inline and crossline components shown in Figure 13, revealing the complications added to the operator by the offset correction. Specifically, the crossline component includes triplications at low reflector angles. These triplications will make any Kirchhoff-type application of this operator difficult. The AMO operator corresponding to only azimuth correction, on the other hand, does not include triplications at any angle, as shown in Figure 14. The

Figure 7. Four AMO operators in the linear \( v(z) \) model. The upper left operator corresponds to a correction in offset only, or residual DMO; the upper right operator corresponds to a correction in azimuth only; the lower left corresponds to a correction in both offset and azimuth; the lower right operator is the 3D \( v(z) \) DMO operator. The full 3D DMO operator is drawn at a large scale. The vertical axis corresponds to \( t_0(s) \).

Figure 8. The inline and crossline components of the AMO operator (or residual DMO) shown in Figure 7 (upper left), but with a wider aperture which includes the triplications.

Figure 9. The crossline and inline components of the AMO operator shown in Figure 7 (upper right), but with a wider aperture.
absence of triplications, despite the presence of a low-velocity zone, is encouraging.

Figure 15 shows the four AMO operators for the high-velocity layer model. Again, the AMO operators are smaller in general than the full DMO operator shown at the lower right. Interestingly, the full DMO operator and the residual DMO operator (upper left) have small crossline components and, in this aspect, they are similar to the constant-velocity operator. The azimuth correction gives the AMO operator a more 3D shape as shown in Figure 15 (upper right and lower left). Again, the AMO operator that includes only azimuth correction (of 30°) does not include triplications.

In summary, AMO operators correcting only the azimuth are much simpler than those that correct both offset and azimuth.

Figure 10. The crossline and inline components of the AMO operator shown in Figure 7 (lower left), but with a wider aperture which includes the triplications.

Figure 11. The crossline and inline components of the AMO operator (or residual DMO) that is the adjoint of that shown in Figure 8. This operator applies an offset correction from 1.5 to 2 km.

Figure 12. Same as in Figure 7, but using the low-velocity-layer model shown in Figure 6 (left). The vertical axis corresponds to $t_0(s)$.
These azimuth-only correction operators are overall triplication free, even for the case of the high-velocity layer. Therefore, using such operators in a Kirchhoff-type implementation should be straightforward. These AMO operators are also, especially for the simpler velocity examples, similar to the constant-velocity ones.

**THE RESIDUAL AMO OPERATOR**

The residual AMO operator includes a cascade of four 3D $v(z)$ DMO operations: two forward operations and two inverse ones. The difference between each of the pair of forward and inverse operations is the medium parameters. For example, a pair of forward and inverse DMOs, or AMO, is applied for a homogeneous medium followed by another pair corresponding to a $v(z)$ medium. The result is a residual AMO operator that corrects for the velocity perturbation from a background homogeneous model to a $v(z)$ one.

The size of the residual AMO operator depends directly on the size of velocity perturbation away from the homogeneous background model. Hence, the residual operator provides information on the impact of the perturbation in velocity on the AMO operator. Figure 16 shows a side and a top view of a residual AMO operator that corrects a homogeneous AMO operator to a linear-velocity AMO operator. In other words, this residual AMO operator, when convolved with the homogeneous-medium AMO operator, provides us with the linear-velocity AMO operator. This AMO operator corresponds to a pure azimuth correction of $30^\circ$. The resulting residual operator is about one-tenth the size of the corresponding full AMO operator shown in Figure 7 (upper left). In fact, the maximum time correction exerted by this residual AMO operator is less than 10 ms, even for dips around $50^\circ$. Such corrections are very small and, thus, the homogeneous medium AMO operator is sufficient to correct for azimuth in this linear velocity variations.

Figure 17 shows residual AMO operators for corrections in offset, as well as azimuth, for the linear velocity model. The residual operator corresponding to a correction in azimuth only (middle) is smaller in size than the operator that includes an offset and azimuth correction (right) or only offset correction (left). The crossline component of the residual AMO operator that includes offset correction is important since in homogeneous media the offset-correction (residual DMO) operator does not include a crossline component. In other words, the convolution of the residual DMO operator for homogeneous medium (a 2D operator) with the residual AMO operator in Figure 17 (left) should give us the AMO operator shown in Figure 7 (upper left). As expected, all residual AMO operators correcting for azimuth only for the linear velocity case are smooth (triplication free).

This is not so, for the low-velocity-layer case, where the perturbation of the model from a homogeneous background causes, among other things, huge triplications. However, the

Figure 16. A side (left) and a top (right) view of a residual AMO operator responsible for the correction from the linear velocity model to a homogeneous medium for a pure azimuth correction of $30^\circ$. 

Figure 17 shows residual AMO operators for corrections in offset, as well as azimuth, for the linear velocity model. The residual operator corresponding to a correction in azimuth only (middle) is smaller in size than the operator that includes an offset and azimuth correction (right) or only offset correction (left). The crossline component of the residual AMO operator that includes offset correction is important since in homogeneous media the offset-correction (residual DMO) operator does not include a crossline component. In other words, the convolution of the residual DMO operator for homogeneous medium (a 2D operator) with the residual AMO operator in Figure 17 (left) should give us the AMO operator shown in Figure 7 (upper left). As expected, all residual AMO operators correcting for azimuth only for the linear velocity case are smooth (triplication free).

This is not so, for the low-velocity-layer case, where the perturbation of the model from a homogeneous background causes, among other things, huge triplications. However, the
Figure 17. Residual AMO operators corresponding to the difference between AMO operators in a homogeneous medium and AMO operators in the linear velocity medium. Both media have the same rms velocity at the NMO-corrected time of 2 s. Left: corresponds to the AMO operator with a correction in offset only. Middle: corresponds to the AMO operator with a correction in azimuth only. Right: corresponds to the AMO operator with a correction in offset and azimuth. The vertical axis corresponds to $t_0(s)$.

Figure 18. The inline and crossline components of the residual AMO operator for the high-velocity model and offset correction only.

The residual operator, even for this case, is generally small. Therefore, the correction needed to adjust for the low-velocity layer model, when a homogeneous AMO is applied, is generally small. In fact, it is as small as the linear-velocity-case model. Again, the residual operator corresponding to a correction in azimuth is the smallest.

The inline and crossline component of the residual AMO operator corresponding to the high-velocity model and offset correction is shown in Figure 18. It displays a number of triplications associated with the operator.

The computation cost of applying the 3D AMO or DMO operators is an order of magnitude higher than that for 2D ones. However, the computational cost is also directly dependent on the size of the operator. Therefore, for a simple Kirchhoff-type implementation (Deregowski, 1995), the cost of the 3D AMO operator is proportional to the product of the inline and crossline grid points needed to represent the operator. Luckily, AMO operators are generally small in size and, as a result, the number of grid points which the operator covers are relatively few. However, with triplications the application of a Kirchhoff-type implementation is not trivial.

CONCLUSIONS

For smooth vertical inhomogeneities, the AMO operator has a shape and size similar to its homogeneous medium counterpart: a general skewed-saddle shape. This is especially the case when the AMO operator includes only azimuth corrections. In fact, such an operator is also free of triplications, which will ease its use in practice. AMO operators that include offset correction often show triplications. In general, AMO operators that correct only the azimuth are much simpler than those that also correct the offset. Therefore, using such operators in a Kirchhoff-type implementation should be straightforward.

The residual operators, derived by cascading a forward homogeneous-medium AMO and an inverse $v(z)$ AMO, confirm the small difference in AMO between the two media. In fact, for the linear-gradient and low-velocity-zone examples shown here, the vertical size of the operator is less than 10 ms. This is not the case for the high-velocity layer model, where the residual AMO operator has almost the size of the full AMO operator.

The computational efficiency of a Kirchhoff-type AMO implementation depends largely on the size of the AMO operator. We have shown that for smooth $v(z)$ media, as for homogeneous media, the AMO operator is generally small.

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