Converted waves azimuth moveout

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ABSTRACT

We introduce a new partial prestack migration operator to manipulate multicomponent data. This operator transforms prestack data with an arbitrary offset and azimuth to equivalent data with a new offset and azimuth position. We call this operator converted waves azimuth moveout (PS-AMO). The chain process of converted waves dip moveout and inverse converted waves dip moveout creates PS-AMO. We also present a reliable and efficient computational implementation of PS-AMO in the log-stretch frequency wavenumber domain. This implementation is computationally efficient. The main applications for the PS-AMO operator are: 1) geometry regularization; 2) data-reduction through partial stacking; and 3) interpolation of unevenly sampled data.

We evaluate our PS-AMO operator by solving 3-D acquisition geometry regularization problems for multicomponent ocean bottom seismic data. We present a methodology to formulate geometry regularization problems as a regularized least-squares problem. In order to preserve the resolution of dipping events the regularization term uses the PS-AMO operator. This regularization term fills the acquisition gaps by minimizing inconsistencies between cubes with similar offset and azimuth. We test our operator and geometry regularization methodology on a portion of the 3-D multicomponent
ocean bottom seismic Alba data set. In addition to introducing a new operator for converted waves, we also present results on a real data set using this operator in order to solve a real problem of geometry acquisition gaps.

INTRODUCTION

Multicomponent ocean bottom seismic (OBS) technology reestablishes the use and importance of converted waves data, and opens the door for a series of new and existing problems with the processing of PS data. One of the main problems with OBS data is the irregularity in the acquisition geometry. Irregular geometries are a serious impediment for accurate subsurface imaging (Beasley, 1994; Gardner and Canning, 1994; Chemingui, 1996). Irregularly sampled data affect the image with amplitude artifacts and phase distortions if the missing data are assumed to be zero traces. Irregular geometry problems are more evident in cases in which the amplitude information is one of the main goals of study. Processing OBS seismic data presents geometry problems similar to those of land data. Gardner and Canning (1994) demonstrate some of the effects of irregular sampling on 3-D prestack migration through synthetic examples using real 3-D land acquisition geometry. For converted waves, the problem of irregular sampling is especially crucial since most of the PS processing focuses on the estimation of rock properties from seismic amplitudes.

For PP data, dip moveout (DMO) is the most common partial prestack migration operator used to transform data from any offset to zero offset. Harrison (1990) first introduces the DMO operator for converted waves data (PS-DMO) by applying an integral-summation approach for the zero-offset mapping equation of the PS-DMO operator. Huub Den Rooijen (1991) also discusses the transformation to zero offset with a PS-DMO operator. The essential concept of his transformation is the property of his PS-DMO operator to transform CMP-sorted data into CRP-sorted data. In addition,
Xu et al. (2001) introduce a $f-k$ log-stretch PS-DMO operator that is computationally efficient and transforms data from the CMP domain into the CRP domain.

In this paper, we introduce a new partial prestack migration operator for converted waves that rotates the data’s azimuth and offset. This operator is the cascade of PS-DMO and its inverse. Because of its ability to modify the azimuth of the PS data we call this operator PS Azimuth Moveout (PS-AMO).

The application of inverse theory satisfactorily regularizes acquisition geometries of 3-D prestack seismic data (Duquet et al., 1998; Chemingui, 1999; Albertin et al., 1999; Bloor et al., 1999; Nemeth et al., 1999; Audebert, 2000; Duijndam et al., 2000; Rousseau et al., 2000). Partial prestack migration operators are useful tools in reducing the size of seismic data. The use of a partial prestack migration operator, Azimuth moveout (AMO) (Biondi et al., 1998), that rotates the data azimuth and changes the data’s absolute offset is a useful element for solving irregularities in the acquisition geometry for PP data (Biondi and Chemingui, 1994; Chemingui, 1999).

In order to solve the problem of reorganizing irregular geometries, there are two distinct approaches that can be applied: 1) data regularization before migration (Duijndam et al., 2000) and 2) irregular geometries correction during migration (Duquet et al., 1998; Nemeth et al., 1999; Albertin et al., 1999; Bloor et al., 1999; Audebert, 2000; Rousseau et al., 2000). Biondi and Vlad (2002) combine the advantages of the previous two approaches. Their methodology regularizes the data geometry before migration, filling in the acquisition gaps with an AMO operator that preserves the amplitudes in the frequency-wavenumber log-stretch domain.

A methodology that involves the PS-AMO operator can be used to solve for geometry irregularities of OBS data, particularly PS data. Due to the asymmetry of ray trajectories in PS data, there are more elements to consider in PS data regularization than in PP data regularization. Our method
for PS data regularization uses the PS-AMO operator in order to preserve the resolution of dipping events and correct for the lateral shift of the common reflection point.

We solve the irregular geometry problem using a preconditioned regularized least-squares scheme. Our methodology uses the PS-AMO operator in the regularization term in order to compare data cubes of different azimuths. In order to establish this comparison, the input data for this least-squares problem needs to be coherent among the traces to be stacked.

The 3-D OBS data set acquired above the Alba reservoir in the North Sea serves as test data for our PS geometry regularization methodology. We show how our methodology fills in the acquisition gaps using the information on neighboring traces and the physics of the converted waves propagation phenomena.

PS AZIMUTH MOVEOUT

Azimuth moveout is a partial prestack migration operator that transforms 3-D prestack data with a given offset and azimuth into equivalent data with a different offset and azimuth. Azimuth moveout for converted waves (PS-AMO) is especially designed to process PS data since it handles the asymmetry of the raypaths. PS-AMO moves events across common reflection point (CRP) gathers according to their geological dip.

The lateral movement between the CMP position and the CRP position is an inherent characteristic of converted waves seismic processing. Gathering converted waves traces into common reflection point gathers that can be subsequently stacked presents problems because of the asymmetry of the downgoing and upgoing travel paths in the subsurface. Figure 1 illustrates the asymmetry of this raypath geometry for converted waves data. The asymmetry is the result of the difference between the incidence angle of the downgoing wavefield and the reflection angle of the upgoing wavefield. A
lateral shift that transforms the trace from the CMP position to the CRP position is the proper way to
connect both common point. This lateral shift varies with depth, since the displacement depends on
the depth of the reflector and the P and S velocities ratio.

Theoretically, the cascade of any imaging operator with its corresponding forward-modeling op-
erator generates a partial prestack operator (Biondi et al., 1998). A cascade operation of PS-DMO
and its inverse (PS-DMO\(^{-1}\)) is the basic procedure that we follow to derive the PS-AMO operator.
First, we present a Kirchhoff integral derivation of the PS-AMO operator using a 3-D extension of
the 2-D PS-DMO operator presented by Harrison (1990).

Following the derivation of the PP-AMO operator (Biondi et al., 1998), we collapse the PS-DMO
operator with its inverse. Figure 2 schematically illustrates the PS-AMO transformation. The axes
are the \(x_x\) and \(x_y\) CMP coordinates. Figure 2 shows four important vectors. The vectors \(D_1\) and
\(D_2\) are transformation vectors which are extensions of the offset vectors \(h_1\) and \(h_2\) respectively. The
transformation vectors \((D_i)\) are responsible for the lateral shift needed for transforming a trace in the
CMP domain into the CRP domain and vice versa. These transformation vectors lie in the CRP do-
main. Figure 2 shows the surface representation of a trace with input offset vector \(h_1\), midpoint at the
origin, and azimuth \(\theta_1\). This trace is: 1) translated to its corresponding CRP position using the trans-
mformation vector \(D_1\); 2) transformed into zero offset \((x_0)\) by a time shift with the PS-DMO operator
(an intermediate step); 3) converted into equivalent data in the CRP domain with the transformation
vector \(D_2\); and finally, 4) turned into its corresponding CMP position with output offset vector \(h_2\),
midpoint \(x\), and azimuth \(\theta_2\).

Appendix A presents the extension of the 2-D PS-DMO equation to 3-D as well as the derivation
of the PS-AMO operator. The PS-AMO is a transformation that relates the input time \(t_1\) after PS-
NMO to the output time \(t_2\) before inverse PS-NMO in the CRP domain. Its analytical expression
Equation (1) represents an asymmetrical saddle on the CMP coordinates \((x, y)\). In these equations: 1) \(t_1\) is the input time after PS-NMO; 2) \(t_2\) is the time after PS-AMO and before inverse PS-NMO; 3) \(H_{10}\) is the scaled offset vector of the transformation from the original trace position to the intermediate zero offset position; 4) \(H_{02}\) is the scaled offset vector of the transformation from the intermediate zero offset position to the new trace position; 5) \(D_1\) and \(D_2\) are the transformation vectors; and 6) \(x\) is the final trace position. Similarly, \(D_{10}\) is the transformation vector from the original trace position to the intermediate zero offset position, and \(D_{02}\) is the transformation vector from the intermediate zero offset position to the final trace position. The scaled offset vectors \((H_i)\) are a modification of the offset vectors \(h_i\). Appendix A presents the derivation of equation (1) as well as the relation among the transformation vectors, the scaled offset vectors and the offset vectors.

Figure 3 explains the relationship between the offset vector \(h\) and the transformation vector \(D\). The transformation vectors belong to the CRP space. Since these vectors are a linear transformation of the offset vectors, the two vectors are parallel. If we align the coordinate system of the CRP domain with the coordinate system of the CMP domain, as in Figure 3, then the angle between the two vectors \((\lambda)\) is the same as the azimuth \((\theta)\). It is important to note that the angle \(\lambda\) changes after
applying PS-AMO. This characteristic shows how the PS-AMO operator handles the transformation to the CRP domain.

The PS-AMO operator is velocity dependent. Equation (1) shows this dependence reflected on the transformation vector $D$ and the offset vector $H$. As we show in Appendix A, $D$ depends on the traveltime after normal moveout ($t_n$), the P velocity ($v_p$), and the P and S velocities ratio ($\gamma$). Therefore, PS-AMO varies with the traveltime ($t_n$) and depends on the P velocity and the P and S velocities ratio. Because of the dependency of PS-AMO on the traveltime, PS-AMO is not constant even in constant velocities mediums since this operator varies with respect to the traveltime.

For implementation reasons it is important to have a PS-AMO operator that is computationally efficient. A log-stretch transformation of the traveltime axis enables this efficiency. Next, we discuss a computationally efficient implementation of the PS-AMO operator. This time, the operator is developed and implemented in the frequency-wavenumber log-stretch domain.

$f-k$ log-stretch PS-AMO

Since PS-AMO is conceived as a cascade of forward and reverse PS-DMO, the accuracy and speed of the PS-DMO operator used is highly important. The PS-DMO operator in the frequency wavenumber domain (Alfaraj, 1992) is accurate and conceptually simple, but computationally expensive because the operator is nonstationary in time.

The technique of logarithmic time-stretching, introduced by Bolondi et al. (1982), increases the computational efficiency because the PP-DMO operator is stationary in the log-stretch domain. Fast Fourier Transforms also can be used instead of slow Discrete Fourier Transforms. Zhou et al. (1996) create a PP-DMO operator that considers variations of the traveltime as well as variations in the midpoint position before and after PP-DMO; therefore, the operator has the main property of handling
the amplitudes properly. Xu et al. (2001) introduce a log-stretch frequency-wavenumber PS-DMO operator that is computationally efficient and kinematically correct. However, this implementation does not consider variation in the midpoint location before and after PS-DMO. By following a similar procedure as Zhou et al. (1996), we obtain a 3-D PS-DMO operator that considers both time shift and spatial shift after applying the operator to the data (Appendix B). This new 3-D PS-DMO operator is computationally efficient and kinematically correct. Since this operator is parallel to the one presented by Zhou et al. (1996), it might handle the amplitudes better than the existing PS-DMO operators.

By performing the cascade operation of PS-DMO in the frequency-wavenumber log-stretch domain with its inverse, we obtain the PS-AMO operator that is computationally efficient. This PS-AMO operator consists of two main operations. In the first operation, the input data \( P(t, x, h_1) \) is transformed to the wavenumber domain \( P(t, k, h_1) \) using FFT. Then, a lateral shift correction is applied using the transformation vectors \( \mathbf{D}_1 \) as

\[
\tilde{P}(t, k, h_1) = P(t, k, h_1)e^{ik(D_1-D_2)}. \tag{4}
\]

The final step of the first operation is to apply a log-stretch along the time axis with the relation:

\[
\tau = \ln \left( \frac{t}{t_c} \right). \tag{5}
\]

Therefore, the data set after the first operation is \( \tilde{P}(\tau, k, h_1) \). In the second operation, the log-stretched time domain \( \tau \) section is transformed into the log-stretched frequency domain \( \Omega \) using FFT. Then, the filters \( F(\Omega, k, h_1) \) and \( F(\Omega, k, h_2) \) are applied as follow:

\[
P(\Omega, k, h_2) = \tilde{P}(\Omega, k, h_1) \frac{F(\Omega, k, h_1)}{F(\Omega, k, h_2)}. \tag{6}
\]

8
The filter $F(\Omega, \mathbf{k}, \mathbf{h}_1)$ is given by either

$$F(\Omega, \mathbf{k}, \mathbf{h}_1) = \begin{cases} 0 & \text{for } \mathbf{k} \cdot \mathbf{h}_1 = 0 \\ 0 & \text{for } \Omega = 0 \\ e^{i\Omega \ln \frac{1}{2} \left( \sqrt{\left( \frac{2\mathbf{k} \cdot \mathbf{h}_1}{\pi} \right)^2 + 1} \right)} & \text{otherwise.} \end{cases}$$ (7)

or

$$F(\Omega, \mathbf{k}, \mathbf{h}_1) = \begin{cases} 0 & \text{for } \mathbf{k} \cdot \mathbf{h}_1 = 0 \\ \mathbf{k} \cdot \mathbf{H}_i & \text{for } \Omega = 0 \\ e^{i\Omega \left\{ \sqrt{1 + \left( \frac{2\mathbf{k} \cdot \mathbf{h}_1}{\pi} \right)^2} - i \ln \frac{1}{2} \left( \sqrt{\left( \frac{2\mathbf{k} \cdot \mathbf{h}_1}{\pi} \right)^2 + 1} \right) \right\}} & \text{otherwise.} \end{cases}$$ (8)

The main difference between equations (7) and (8) is observed in the next section and in Appendix B. Also, Appendix B shows the detailed derivation of the PS-AMO operator in the frequency-wavenumber log-stretch domain. In order to implement this PS-AMO operator, we: 1) calculate the FFT along the midpoint axis for each input offset cube; 2) compute the transformation vectors $\mathbf{D}_1$ and $\mathbf{D}_2$ with equation (A-4); 3) apply the lateral shift of the transformation vectors as a phase shift with equation (4); 4) perform the log-stretch transformation over the time axis using equation (5); 5) calculate the FFT along the transformed time axis; 6) compute the filters $F(\Omega, \mathbf{k}, \mathbf{h}_1)$ and $F(\Omega, \mathbf{k}, \mathbf{h}_2)$; 7) apply these filters to the data in the log-stretch frequency wavenumber domain with equation (6); and 8) perform inverse FFT and inverse log-stretch.

**Impulse response**

In order to have a better understanding of the PS-AMO operator, we evaluate and analyze its impulse response. Figure 4 compares the PP-AMO impulse responses obtained with the filter in both equation...
(7) (top) and equation (8) (bottom). Both are obtained with a value of $\gamma = 1$ and are kinematically equivalent.

Figure 4 also illustrates the differences in the dynamic behavior of the operator. Figure 4 (top) shows the operator without amplitude considerations [equation (7)], while Figure 4 (bottom) shows the behavior of the operator with a better distribution of the amplitudes [equation (8)]. Amplitude distribution is enhanced along a wider range of offset.

Figure 5 presents a similar comparison to Figure 4 for the case of converted waves. Here, we use $\gamma = 1.2$ and $v_p = 2.0$ km/s. As in the previous case, the kinematic behavior is the same for both operators, but the response with the filter in equation (8) presents an amplitude distribution that covers a wider range of offset.

Figure 6 shows the comparison between the PP-AMO impulse response and the PS-AMO impulse response. The PS-AMO not only has a saddle shape as the PP-AMO operator, but it also exhibits lateral movement. This lateral displacement corresponds to the asymmetry of the raypaths or the CMP to CRP transformation. The displacement is toward the lower-left part of the cube. Moreover, Figure 7 shows the variation of the PS-AMO operator with traveltime even for a constant velocity medium. This variation of the PS-AMO operator reflects the lateral displacement between the common midpoint and the common reflection point, a characteristic of converted waves data.

APPLICATIONS TO DATA REGULARIZATION

Irregular acquisition geometries are a serious impediment for accurate subsurface imaging. Irregularly sampled data degrades the final image with amplitude artifacts and phase distortions. Irregular geometry problems are more evident in cases in which the amplitude information is one of the main goals of study.
We tested our implementation of the log-stretch frequency wavenumber domain PS-AMO operator in the problem of irregular geometries, specifically for ocean bottom seismic and multicomponent data. The method used is based on a formulation of the geometry regularization problem as a regularized least-squares problem.

**PS-AMO regularization**

Partial stacking the data recorded with irregular geometries within offset and azimuth ranges yields uniformly sampled common offset/azimuth cubes. In order to enhance the signal and reduce the noise, the reflections should be coherent among the traces to be stacked. Normal Moveout (NMO) is commonly used to create this coherency among the traces.

Figure 8 shows a simple sketch of the problem to solve. The recorded data traces \( \mathbf{d} \) are in an irregular acquisition geometry. An interpolation operator \( \mathbf{A} \) links the data traces with the stacked volume defined on a regular grid \( \mathbf{m} \). Each data trace is the result of interpolating the stacked traces and is equal to the weighted sum of the neighboring stacked traces. In matrix notation, this transforms to

\[
\mathbf{d} = \mathbf{A}\mathbf{m},
\]

where \( \mathbf{d} \) is the data space, \( \mathbf{m} \) is the model space, and \( \mathbf{A} \) is an interpolation operator. Stacking can be represented as the application of the adjoint operator \( \mathbf{A}^\dagger \) to the data traces,

\[
\hat{\mathbf{m}} = \mathbf{A}^\dagger \mathbf{d}.
\]

In order to obtain better results, it is possible to express this problem in the least-squares sense.
This least-squares problem can be expressed as

\[ \min_{m} \ ||\mathbf{A}m - \mathbf{d}||^2. \quad (11) \]

In general, the operator \( \mathbf{A} \) is not square, and its inverse is not defined; therefore, we use its least-squares inverse (Strang, 1986). The least-squares solution to this overdetermined problem is given by

\[ \mathbf{m} = (\mathbf{A}'\mathbf{A})^{-1} \mathbf{A}'\mathbf{d}. \quad (12) \]

The least-squares solution is equivalent to applying the adjoint operator, \( \mathbf{A}' \), followed by a spatial filtering of the model space given. The inverse of \( \mathbf{A}'\mathbf{A} \) represents this spatial filtering. Note that a fold normalization, acting as a diagonal operator \( \mathbf{W}_m \), can be seen as a particular approximation of the inverse of \( \mathbf{A}'\mathbf{A} \).

Appendix C presents a detailed discussion of the solution of this least-squares problem. The least-squares problem is regularized using the PS-AMO operator as an important element on the regularization term. We apply an inversion scheme to solve this regularized least-squares problem, the solution of which is the center result of this PS-AMO regularization process.

In order to implement this regularization methodology to converted waves, several aspects must be considered. The main aspect is the dependency of the PS-AMO operator on the P and S velocities ratio. The following section discusses the implementation of the PS-AMO regularization methodology to satisfactorily handle converted waves data.
PS-AMO regularization implementation

As we know from the formulation of the PS-AMO operator, the operator depends on the ratio between the P and the S velocities ($\gamma$). Therefore, we need a priori velocity estimation. This suggests that for different $\gamma$ values we will have different regularization results.

Traditional PS processing sorts the data in the common reflection point (CRP) domain. This process has always been dependent on the $\gamma$ value; therefore, the PS processing community performs iterative processing (CRP binning, velocity analysis) until obtaining a satisfactory result.

The input data for the PS-AMO regularization process needs to be coherent among the traces to be stacked. This means that we must apply Normal Moveout (NMO) before the regularization process, a very important task for our methodology. Because of the non-hyperbolicity of PS data, we need to take extra care with this procedure. Huub Den Rooijen (1991) demonstrates that for small offset to depth ratio the moveout of converted waves data can be approximated by a hyperbola. Therefore, it is possible to apply conventional NMO to converted waves before the regularization process. However, this approximation has its limitations for shallow events and/or long offsets. In these cases, a higher order approximation of the converted waves moveout equation might bring better results.

In this work, we follow Huub Den Rooijen’s (1991) approach of applying conventional NMO to the data and leaving the lateral shift correction to the PS-AMO operator. When we apply traditional NMO to a converted waves data set, the stacking velocity is denominated effective velocity expressed as $v_{\text{eff}} = \sqrt{v_p v_s}$ (Huub Den Rooijen, 1991). Because the PS-AMO operator depends on the P and S velocity ratio ($\gamma$), we can obtain this $\gamma$ function with both the stacking P velocity function and the stacking velocity function of the converted waves section with
In order to proceed with the PS data regularization, we need to have the regularized PP section as well as the stacking velocity model.

The flow chart in Figure 9 represents our methodology. Since the PS-AMO operator depends on the $\gamma$ function, we need an iterative methodology that updates this function. We refer to the processing sequence of NMO, PS-AMO geometry regularization, and inverse NMO as the PS-AMO geometry regularization process.

We start our methodology with both PP and PS sections, estimating the stacking velocities of both. We compute the $\gamma$ function, proceeding with the PS-AMO geometry regularization process if this is the first iteration of our methodology. If this is not the first iteration, however, we then compare the present $\gamma$ function with the previous $\gamma$ function. If they are not the same, we apply the PS-AMO geometry regularization process. When the two $\gamma$ functions are equivalent we have achieved our final result.

The main disadvantages of this procedure are the time stretching between the PP and the PS sections and the hyperbolic moveout approximation of the PS traveltimes. These drawbacks might be overcome with a higher degree approximation of the PS traveltime equation that depends only on the PS stacking velocity and the $\gamma$ function.

FIELD DATA EXAMPLES

We apply PS-AMO regularization to a portion of a real OBS data set recorded above the Alba oil field. The Alba oil field is located in the UK North Sea and elongates along a NW-SE axis. The oil
reservoir is 9 km long, 1.5 km wide, and up to 90 m thick at a depth of 1,900 m subsea (Newton and Flanagan, 1993).

Figure 10 illustrates the main problem. Observe the gaps in both the PP-CMP (top) and the PS-CMP (bottom) gathers. Our goal is to fill these gaps with energy from the surrounding traces in a way that honors the physics of wave propagation.

We use a subsection of the entire 3-D cube. This portion consists of 17 crosslines with 719 CMPs each. The PP section uses only the absolute value of the offset, for a total of 121 offsets. The PS section uses the full offset; however, the maximum offset extension is reduced from 8000 m to 4000 m since these distant offset traces do not contribute to the data.

Figure 11 presents the PP data for one crossline of the data set in the study. The horizontal axes represent the common midpoint locations and the absolute offset of the data in meters. The vertical axis corresponds to the two-way traveltime in seconds. There are clear holes in the data due to irregularities in the acquisition geometry. Figure 12 shows the AMO geometry regularization result for the PP portion of the data.

Figure 13 exhibits the PS portion of the data set. This time, the horizontal axes represent the common midpoint location of the traces and the offset in meters. The vertical axis represents the two-way traveltime in seconds. Again, the holes in the data result from irregularities in the acquisition geometry. It is also possible to observe the presence of negative and positive offset.

We proceed with the methodology discussed in the previous section. We perform the PS-AMO regularization process because it is the only one that corrects for the lateral shift displacement of the common reflection point. There are only two iterations of the algorithm presented in Figure 9.

Figures 14 and 15 present a PS-AMO regularization results for two iterations of our methodology.
For each iteration note that the moveout of the events is not a perfect hyperbola. This is characteristic of the nature of propagation of PS waves. Observe the difference in the moveout of the events between the two iterations of our methodology (Figures 14 and 15). This is due to the different velocities on each iteration. However, both results satisfactorily fit the data.

Figure 16 presents a zoom of the original data (top), the results of one iteration (center), and the results of two iterations (bottom). Both results fit the data well. However, the second iteration is more realistic since it better follows the information of the surrounding traces. Moreover, the moveout of the events in the second iteration resembles the nonhyperbolic moveout of converted waves data better than the first iteration.

**DISCUSSION AND CONCLUSIONS**

We have presented a new partial prestack migration operator for converted waves, the PS-AMO operator, which is the cascade operation of PS-DMO and its inverse. A data trace with input offset vector $h_1$ and midpoint position $x$ is first transformed to its corresponding CRP position and zero offset. By defining the new offset and azimuth position and by applying inverse PS-DMO, we transform the data to a new CRP position and its corresponding CMP position. The operator is velocity dependent and also depends on the P to S velocities ratio ($\gamma$). Our operator reduces to the standard AMO operator for the case of $\gamma = 1$. We also presented an implementation of the PS-AMO operator in the $f$-$k$ log-stretch domain that is computationally efficient.

The PS-AMO operator that we use has the advantage of not demanding data in the CRP domain. This operator, as a cascade operation of PS-DMO and its inverse, internally performs the CMP to CRP lateral shift correction, since the PS-DMO operator does it as well. Therefore, a priori CRP binning is not necessary before applying azimuth moveout to converted waves data. The PS-AMO
operator has two main characteristics: 1) it preserves the resolution of the dipping events, and 2) it corrects for the spatial lateral shift of the common reflection point.

PS-AMO has several applications. In this work, we tested the operator for the problem of irregular geometries, more specifically the converted waves portion of OBS seismic data. In this case, the geometry regularization problem is handled in the least-squares sense.

The model space of this least-squares problem is composed of uniformly sampled common offset-azimuth cubes. The inclusion of a regularization term in the least-squares problem has the intention of filling the acquisition gaps by minimizing inconsistencies between cubes with similar offset and azimuth. To preserve the resolution of dipping events in the final image, the regularization term includes the PS-AMO operator. The method that we used is computationally efficient since we applied the PS-AMO operator in the Fourier domain and preconditioned the regularized least-squares problem. For this problem, an iterative procedure was needed due to the dependence of the PS-AMO operator on the $\gamma$ value.

A higher degree NMO approximation would produce better coherence among the traces to be stacked on the PS section and, consequently, better regularization results. Additionally, formulating the $\gamma$ estimation problem in a least-squares sense should allow a better constraint for its calculation, creating better PS regularized sections.

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REFERENCES


APPENDIX A

The 2-D PS-DMO smile (Harrison, 1990; Xu et al., 2001) extends to 3-D by replacing the offset and midpoint coordinates for the offset and midpoint vectors respectively. The 3-D expression for the PS-DMO is

\[
\frac{t_0}{t_0^2} + \frac{\|\mathbf{y}\|^2}{\|\mathbf{H}\|^2} = 1, \tag{A-1}
\]

where

\[
\|\mathbf{y}\|^2 = \|\mathbf{x} + \mathbf{D}\|^2, \tag{A-2}
\]

\[
\mathbf{H} = \frac{2\sqrt{\gamma}}{1 + \gamma} \mathbf{h}, \tag{A-3}
\]

and

\[
\mathbf{D} = \left[1 + \frac{4\gamma \|\mathbf{h}\|^2}{v_p^2 + \gamma(1 - \gamma)\|\mathbf{h}\|^2} \right] \frac{1 - \gamma}{1 + \gamma} \mathbf{h}. \tag{A-4}
\]

Here, \(\mathbf{x}\) is the midpoint position vector, \(\mathbf{h}\) is the offset vector, \(\mathbf{D}\) is the transformation vector responsible for the CMP to CRP coordinates change, and \(\gamma\) is the \(v_p/v_s\) ratio.

First, we refer to equations (A-1) and (A-2) in order to understand the relationship between CMP and CRP for the 3-D case. We rewrite equation (A-2) as

\[
\|\mathbf{y}\|^2 = \|\mathbf{x}\|^2 + \|\mathbf{D}\|^2 + 2\|\mathbf{x}\|\|\mathbf{D}\|\cos \lambda,
\]

where \(\lambda\) is the angle between the midpoint vector (\(\mathbf{x}\)) and the transformation vector (\(\mathbf{D}\)).
We can then rewrite equation (A-1) as

\[ t_0^2 \frac{\|x\|^2}{\|H\|^2} + \frac{\|D\|^2 + 2\|x\|\|D\|}{\|H\|^2} \cos \lambda = 1. \] (A-5)

The time after PS-AMO is defined by

\[ t_2^2 = t_1^2 \frac{H_{02}^2}{H_{10}^2} \left( \frac{H_{10}^2 - x_{10}^2 - D_{10}^2 - 2x_{10}D_{10}}{H_{02}^2 - x_{02}^2 - D_{02}^2 - 2x_{02}D_{02}} \right). \] (A-6)

Both \(x_{10}\) and \(x_{02}\) can be expressed as terms of the final midpoint position \(x\) by using the rule of sines in the triangle \((x, x_{10}, x_{02})\) in Figure 2 as

\[ x_{10} = x \frac{\sin(\theta_2 - \Delta \phi)}{\sin(\theta_1 - \theta_2)} \] (A-7)

\[ x_{02} = x \frac{\sin(\theta_1 - \Delta \phi)}{\sin(\theta_1 - \theta_2)}. \] (A-8)

The final expression takes the form of

\[ t_2^2 = t_1^2 \frac{H_{02}^2}{H_{10}^2} \left\{ \frac{H_{10}^2 \sin^2(\theta_1 - \theta_2) - x^2 \sin^2(\theta_2 - \Delta \phi) - D_1}{H_{02}^2 \sin^2(\theta_1 - \theta_2) - x^2 \sin^2(\theta_1 - \Delta \phi) - D_2} \right\}. \] (A-9)

This equation represents a time axis transformation where \(t_1\) is the input time after PS-NMO and \(t_2\) is the time before inverse PS-NMO. The remaining elements of this equation are: 1) \(H_{10}\) is the scaled offset vector of the transformation from the original trace position to the intermediate zero offset position; 2) \(H_{02}\) is the scaled offset vector of the transformation from the intermediate zero offset position to the new trace position; 3) \(D_1\) and \(D_2\) are the transformation vectors; and 4) \(x\) is the final trace position. In the same way, \(D_{10}\) is the transformation vector from the original trace position to the intermediate zero offset position, and \(D_{02}\) is the transformation vector from the intermediate zero
offset position to the final trace position. The scaled offset vectors $\mathbf{H}_i$ are a modification of the offset vectors $\mathbf{h}_i$. The relationship between these two offset vectors is given by equation (A-3). Moreover, the intermediate transformations ($\mathbf{H}_{10}, \mathbf{H}_{02}, \mathbf{D}_{10}, \mathbf{D}_{02}$) are computed respectively with equations (A-3) and (A-4) with an offset $\mathbf{h}$ equals to the distance from either the starting point 1 to the intermediate point 0 or the intermediate point 0 to the ending point 2.

APPENDIX B

Zhou et al. (1996) discuss that the PP-DMO operator in the $f$-$k$ domain is computationally expensive because the operator is temporarily nonstationary. They make use of an idea of Bolondi et al. (1982) to derive a more accurate PP-DMO operator by a logarithmic time stretching.

Xu et al. (2001) exploit the idea of computational efficiency of the logarithmic time stretching for the PS-DMO operator. We reformulate their work using the PS-DMO smile presented in Appendix A and following a procedure similar to Hale (1984) and Zhou et al. (1996). This operation accounts for constant velocity case.

Starting from the PS wave DMO smile,

$$\frac{t_0^2}{t_n^2} + \frac{y^2}{H^2} = 1. \quad (B-1)$$

By following Hale’s (1984) assumption that the DMO operator maps each sample of the NMO section $(p_n)$ from time $t_n$ to time $t_0$ without changing its midpoint location, $x \{ p_0(t_0, x, H) = p_n(t_n, x, H) \}$, we obtain the 2D PS-DMO operator in the $f$-$k$ domain:

$$P_0(\omega, k, H) = \int \int p_0(t_0, y, H)e^{i(\omega t_0 - ky)}dt_0dy. \quad (B-2)$$
Equation (B-1) implies a change of variable from $t_0$ to $t_n$. From equation (B-1) we have

$$t_0^2 = t_n^2 \left(1 - \frac{\nu^2}{H^2}\right)$$  \hspace{1cm} (B-3)

and

$$\frac{dt_0}{dt_n} = \sqrt{1 + \frac{H^2 k^2}{t_n^2 \omega^2}} \equiv A.$$  \hspace{1cm} (B-4)

Therefore, equation (B-2) becomes

$$P_0(\omega, k, H) = \int \int A^{-1} p_0(t_n, x, H) e^{i \omega A t} e^{-i k(x + D)} dt_n dx.$$  \hspace{1cm} (B-5)

Equation (B-5) is the foundation of PS-DMO in the $f$-$k$ domain. By using a time log-stretch transform pair,

$$\tau = \ln \left( \frac{t}{t_c} \right),$$  \hspace{1cm} (B-6)

$$t = t_c e^\tau.$$  \hspace{1cm} (B-7)

The PS-DMO operator in the $f$-$k$ log-stretch domain becomes

$$P_0(\Omega, k, h) = P_n(\Omega, k, h) e^{ikD} F(\Omega, k, h),$$  \hspace{1cm} (B-8)

where

$$F(\Omega, k, h) = e^{i \Omega \ln \frac{1}{2} \left(1 + \frac{\nu^2}{H^2} + 1\right)}.$$  \hspace{1cm} (B-9)

When either $kh$ or $\Omega$ has the value of 0, the filter becomes 0 as well.
The previous expression is equivalent to the one presented by Xu et al. (2001). Note that equation (B-8) is based on the assumption that \( p_0(t_0, x, H) = p_n(t_n, x, H) \). This does not include changes in midpoint position and/or common reflection point position. This leads to a correct kinematic operator but one with a poor amplitude distribution along the reflectors.

Zhou et al. (1996) solve for PP-DMO this problem in the log-stretch frequency wavenumber domain by reformulating Black’s (1993) \( f-k \) PP-DMO operator. This operator is based on the assumption that the midpoint location changes its location after applying the PP-DMO operator \( [p_0(t_0, x_0, H) = p_n(t_n, x_n, H)] \). The midpoint location also changes, leading to a more accurate distribution of amplitudes. Following the derivation used by Zhou et al. (1996), we derive a more accurate PS-DMO operator in the frequency-wavenumber log-stretch domain. This new operator differs from the previous one in the filter \( F(\Omega, k, h) \). The new filter is

\[
F(\Omega, k, h) = e^{\frac{i}{2} \Omega \left[ \sqrt{1 + \left( \frac{2 \Omega}{k H} \right)^2} - 1 - \ln \left[ \sqrt{1 + \left( \frac{2 \Omega}{k H} \right)^2} + 1 + 1 \right] \right]}. \tag{B-10}
\]

This filter reduces to 0 if \( kh = 0 \) or \( kH \) if \( \Omega = 0 \). Note that for a value of \( \gamma = 1 \), the filter reduces to the known expression for P waves data (Zhou et al., 1996).

The \( f-k \) log-stretch operator for PS-DMO in 3-D takes the form

\[
P(\Omega, k, h) = P(\Omega, k, h)e^{i \Omega D} F(\Omega, k, h). \tag{B-11}
\]

By applying the PS-DMO transformation to zero offset and a given azimuth, followed by the inverse PS-DMO from zero-offset to a different offset and azimuth, we obtain the PS-AMO operator in the frequency-wavenumber log-stretch domain. This operator consists of two main operators. The first one applies the lateral shift of the CMP to CRP transformation as
\[ P(t, k, h_1) = P(t, k, h_1) e^{ik(D_1 - D_2)}. \]  

The second one applies the filters \( F(\Omega, k, h_1) \) and \( F(\Omega, k, h_2) \) after a log-stretch transformation of the time axis. The variable \( \Omega \) is the Fourier par of the log-stretched time axis \( \tau \). The second operator is

\[ P(\Omega, k, h_2) = \frac{P(\Omega, k, h_1) F(\Omega, k, h_1)}{F(\Omega, k, h_2)}. \]  

The filters \( F(\Omega, k, h_1) \) and \( F(\Omega, k, h_2) \) take the form of

\[
F(\Omega, k, h_i) = \begin{cases} 
0 & \text{for } k \cdot h_i = 0 \\
0 & \text{for } \Omega = 0 \\
\frac{\Omega}{2} \ln \left( \sqrt{\left(\frac{2kH_i}{\Omega^2}\right)^2 + 1} \right) & \text{otherwise.} 
\end{cases}
\]  

To achieve a more accurate amplitude distribution the filter in equation B-14 takes the form of

\[
F(\Omega, k, h_i) = \begin{cases} 
0 & \text{for } k \cdot h_i = 0 \\
k \cdot H_i & \text{for } \Omega = 0 \\
e^{\frac{\Omega}{2} \ln \left( \sqrt{1 + \left(\frac{2kH_i}{\Omega^2}\right)^2} - 1 \ln \left[ \sqrt{\left(\frac{2kH_i}{\Omega^2}\right)^2 + 1} \right] \right)} & \text{otherwise.} 
\end{cases}
\]  

**APPENDIX C**

The least-squares solution to the irregular acquisition geometry is given by

\[ m = (A^t A)^{-1} A^t d. \]
This least-squares solution is equivalent to applying the adjoint operator $A'$ followed by a spatial filtering of the model space given by the inverse of $A'A$. Note that a fold normalization, acting as a diagonal operator $W_m$, can be seen as a particular approximation of the inverse of $A'A$.

The derivation of an analytical expression of the inverse of $A'A$ is not simple to obtain when we consider complex stacking operators such as imaging operators. In these cases we have two choices: 1) to compute an analytical approximation of the inverse or 2) to use an iterative method to compute a numerical approximation of the inverse. Even if we follow the second strategy, the availability of an analytical approximation to the inverse is useful.

There are several methods to compute an analytical approximation of the inverse. Rickett (2001) captures the most significant properties of $A'A$ by measuring its effects when applied to a reference model ($m_{\text{ref}}$). The approximation is then evaluated as

$$A'A \approx \frac{\text{diag}(A'A m_{\text{ref}})}{\text{diag}(m_{\text{ref}})} = W_m^{-1}. \quad (C-2)$$

Data gaps can bring problems in the least-squares solution of our problem, equation (C-1). To fill the gaps, we want to use the information from traces recorded with geometry similar to the missing ones. The challenge is to devise a method that maximizes the image resolution and minimizes artifacts.

Given no a priori knowledge of the reflectors geometry, using the information from traces from the surrounding midpoints and same offset-azimuth range can cause a resolution loss. On the other hand, because of physical constraints on the reflection mechanism, the reflection amplitudes can be assumed to be a smooth function of the reflection angle and azimuth. This observation leads to the idea that smoothing the data over offset and azimuth could be performed without losing resolution. Ideally, such smoothing should be done over aperture angles (dip and azimuth) at the reflection loca-
tion, not over offset and azimuth at the surface. However, smoothing at the reflectors would require a full migration of the data. The migration step would make the method dependent on the accurate knowledge of the interval velocity model. This reliance on the velocity model is inescapable when the imaging problems are caused by the complexities in the velocity model itself, (e.g. subsalt illumination (Muerdter et al., 1996; Prucha, 2002)), but it ought to be avoided when the imaging problems are caused by irregularities in the acquisition geometries.

In the context of least-squares inversion, smoothing along offset/azimuth in the model space (e.g. uniformly sampled offset/azimuth cubes) can be accomplished by introducing a model regularization term that penalizes variations of the seismic traces between the cubes. The simple least-squares problem of equation (C-1) then becomes

\[
\min_m \{ ||Am - d||^2 + ||\epsilon G_h G_m||^2 \}, \tag{C-3}
\]

where \(G_h\) is a roughener operator defined by

\[
G_h = \frac{1}{1 - \rho_G} \begin{bmatrix}
1 - \rho_G I & 0 & 0 & \ldots & 0 & 0 \\
-\rho_G T_{h,1,2} & I & 0 & \ldots & 0 & 0 \\
0 & -\rho_G T_{h,2,3} & I & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & -\rho_G T_{h,n-1,n} & I
\end{bmatrix} \tag{C-4}
\]

The coefficient \(\rho_G\) must be between 0 and 1. It determines the range over which we smooth the offset-azimuth cubes. The smaller the value we set for \(\rho_G\), the narrower the smoothing range is. The operator \(T_{h,i,i+1}\) is the PS-AMO operator that transforms the offset-azimuth cube \(i\) into the offset-azimuth cube \(i + 1\). Regularization with the \(G_h\) operator has the main property of preserving the resolution of the geological dips.
Regularization with a roughener operator, such as $G_h^r G_h$, has the computational drawback that it substantially worsens the conditioning of the problem, making the solution more expensive. However, the problem is easy to precondition because $G_h^r G_h$ is easy to invert. Because it is already factored in a lower block-diagonal operator $G_h$ and in an upper block-diagonal operator $G_h^r$, $G_h^r G_h$ can be inverted by recursion. Then, we can write the preconditioned regularized least-squares problem

$$\min_p \left\{ \| A (G_h^r G_h)^{-1} p - d \|^2 + \| \epsilon G p \|^2 \right\}, \quad (C-5)$$

where $p = G_h^r G_h m$ is the preconditioned model vector.

We can obtain an analytical approximation of the inverse for the preconditioned regularized least-squares problem (C-5) using equation (C-2) as

$$W_m^{-1} = \frac{\text{diag} \left\{ \left( (G_h^r G_h)^{-1} A A (G_h^r G_h)^{-1} + \epsilon G I \right) p_{\text{ref}} \right\}}{\text{diag}(p_{\text{ref}})}, \quad (C-6)$$

where $p_{\text{ref}} = G_h^r G_h m_{\text{ref}}$.

Therefore, the solution for the regularized preconditioned least-squares problem (C-5) is

$$\tilde{m} = (G_h^r G_h)^{-1} W_m (G_h^r G_h)^{-1} A d. \quad (C-7)$$

This is the final result of the regularization methodology. The methodology is based on the formulation of the geometry regularization problem in the least-squares sense. The least-squares needs to be regularized to account for gaps in the data. Because of the regularization operator it is feasible to precondition the regularized least-squares problem. The final solution for the regularized preconditioned least-squares problem includes a fold normalization with a diagonal operator $W_m$ [equation (C-7)].
LIST OF FIGURES

1 Raypath geometry asymmetry for converted waves data. Note that a lateral shift \((D)\) transforms a trace from its CMP position to its CRP position.

2 Schematic of the PS-AMO transformation. An input trace with offset vector \(h_1\) and midpoint at the origin is transformed into equivalent data with offset vector \(h_2\) and midpoint position \(x\) after a transformation in and out of the CRP domain with the transformation vectors \(D_1\) and \(D_2\).

3 Definition of offset vectors \(h_i\) and transformation vectors \(D_i\), before and after PS-AMO. The PS-AMO operator handles the transformation to the CRP domain and the event movement along the common reflection point.

4 PP-AMO impulse response comparison, filter in equation (7) (top), and filter in equation (8) (bottom), both with \(\gamma = 1\).

5 PS-AMO impulse response comparison, filter in equation (7) (top), and filter in equation (8) (bottom), with \(\gamma = 1.2\) and \(v_p = 2.0\text{km/s}\).

6 PS-AMO saddle with equations (4)-(6) and (8) for \(\gamma = 1\) (top) and \(\gamma \neq 1\) (bottom). The top figure resembles the PP-AMO skewed saddle impulse response.

7 PS-AMO impulses response variation with traveltime \(t\).

8 Sketch of the PS-AMO regularization process. The recorded data traces \((d)\) are linked to the stacked volume \((m)\) by an interpolation operator \((A)\).

9 Flow chart for OBS data regularization.

10 CMP gather for the PP (top) and the PS (bottom) components for one crossline of the 3-D cube for the Alba data set.

11 PP crossline section of the data in study. Note the gaps in the data as a result of irregularities in the acquisition geometry.
12 AMO geometry regularization result for the PP portion of the Alba data set. This cube section is the same section as in Figure 11.

13 PS section for the same crossline on Figure 11. Observe the holes in the data as well as the presence of positive and negative offset.

14 PS regularization results. First iteration result of our methodology on Figure 9.

15 PS regularization results. Second iteration result of our methodology on Figure 9.

16 Zoom of the results on Figures 14 and 15. From top to bottom, the original data, the first and the second iterations. Note how event A is more continuous in the second iteration, and event B presents a more realistic PS moveout (not a perfect hyperbola).
Figure 1.
Figure 2.
Figure 3.
Figure 4.
Figure 5.
Figure 6.
Figure 7.
Figure 8.
Sort data into CMP domain

Estimate \( V_p \) from PP section

Estimate \( V_{eff} \) from PS section

Estimate \( \gamma \) section

Equation (12)

Is it the first iteration?

Yes

Apply NMO

Apply PS–AMO geometry regularization

Apply inverse NMO

No

Compare the previous and the present \( \gamma \) sections

Are the sections equivalents?

Yes

END

No

Figure 9.
Figure 11.
Figure 15.