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LABORATORY PARTIAL SATURATION DATA AND POROELASTIC SHEAR DEPENDENCE ON FLUIDS

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OUTLINE

- Question: Do Fluids Affect Shear Behavior in Rocks?
  - Review of Gassmann's Predictions
  - What Lab Data Say About Shear/Fluid Interaction
  - What Effective Medium Modeling Says Is True
- A Physical Analogy/Metaphor from Optics
- Layered Media – Effects of Heterogeneity
- Soft Anisotropy – Effects of Pore/Crack Distribution
- An Effective Shear Modulus and What It Means
- Conclusions

Review of Gassmann's Predictions

Derivation of Gassmann's Equations

The presence of a saturating pore fluid in porous media introduces an additional control field p_f and an additional type of strain variable ζ. p_f is the fluid pressure, and ζ is the change in amount of fluid mass contained in the pores.

$$\begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ -\zeta \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{12} & -\beta \\ S_{12} & S_{11} & S_{12} & -\beta \\ S_{12} & S_{12} & S_{11} & -\beta \\ -\beta & -\beta & -\beta & \gamma \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ -p_f \end{pmatrix}$$

Gassmann Derivation (2)

The meanings of the compliances S_{11} and S_{12} do not change in any way from their meanings in elasticity:

$$S_{11} = \frac{1}{E_{dr}} = \frac{1}{9K_{dr}} + \frac{1}{3G_{dr}}$$

$$S_{12} = -\frac{\nu_{dr}}{E_{dr}} = \frac{1}{9K_{dr}} - \frac{1}{6G_{dr}}$$

K_{dr} and G_{dr} are the bulk and shear moduli of the drained (almost dry) porous medium. E_{dr} and ν_{dr} are the drained Young's modulus and Poissons's ratio.

Gassmann Derivation (3)

β and γ are poroelastic constants connecting p_f to the elastic strains e_{ii}, and connecting the elastic stresses σ_{ii} and p_f to the fluid increment ζ.

Now consider the saturated (and undrained, meaning that the liquid is trapped and cannot escape from the volume) case so that

$$\zeta \equiv 0.$$

Then, the previous equations show that

$$p_f = -\frac{\beta}{\gamma}(\sigma_{11} + \sigma_{22} + \sigma_{33}).$$

Gassmann Derivation (4)

The preceding equation is known as the pore pressure build-up equation, for the obvious reason that it shows how the pore pressure increases when an external compressive load is applied to a closed system.

We can now use this equation to eliminate both ζ and p_f from the poroelastic equations and we find

$$\begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \end{pmatrix} = \left[\begin{pmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{11} & S_{12} \\ S_{12} & S_{12} & S_{11} \end{pmatrix} - \frac{\beta^2}{\gamma} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right] \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{pmatrix}$$

Gassmann Derivation (5)

The resulting set of equations shows that, for isotropic poroelastic media, the compliances for the saturated system are given by

$$S_{11}^{sat} = S_{11} - \frac{\beta^2}{\gamma}$$

and

cl$S_{12}^{sat} = S_{12} - \frac{\beta^2}{\gamma}$.

Using our previous relations, we find

$$\frac{1}{9K^{sat}} + \frac{1}{3G^{sat}} = \frac{1}{9K_{dr}} + \frac{1}{3G_{dr}} - \frac{\beta^2}{\gamma}$$

and

$$\frac{1}{9K^{sat}} - \frac{1}{6G^{sat}} = \frac{1}{9K_{dr}} - \frac{1}{6G_{dr}} - \frac{\beta^2}{\gamma}.$$

Gassmann Derivation (6)

Taking the difference, we find $1/2G^{sat} = 1/2G_{dr}$, or

$$G^{sat} = G_{dr}.$$

This shows that the saturated shear modulus is not influenced at all mechanically by the liquid.

Then, substituting the shear modulus result back into either of the previous equations gives

$$\frac{1}{K^{sat}} = \frac{1}{K_{dr}} - \frac{9\beta^2}{\gamma},$$

which is one form of the result known as Gassmann's formula for the bulk modulus of a poroelastic system.

What Lab Data Say About
Shear/Fluid Interaction

What Effective Medium Modeling Says Is True

Polarized Light: Can a Vertically Polarized Beam Be Rotated to Horizontal?

Effect of a Polarizing Filter on a Light Beam (1)

Define a polarizer mathematically as a projection operator acting in the vertical plane on waves propagating normal to the plane. Then, θ is the polar angle and:

$$\mathcal{P}_\theta = |\theta\rangle \langle\theta| \,.$$

If a vertically polarized $\theta = 0$ wave hits the polarizer, the incoming beam of unit amplitude is represented by $|0\rangle$, and the outgoing wave is determined by

$$\mathcal{P}_\theta \; |0\rangle = cos\,\theta \; |\theta\rangle$$

since by definition $\langle\theta|0\rangle = cos\,\theta$.

Effect of a Polarizing Filter on a Light Beam (2)

Then, by sending the beam through a series of polarizers, we find that

$$\mathcal{P}_{\theta''}\mathcal{P}_{\theta'}\mathcal{P}_{\theta}\;|0\rangle = cos(\theta'' - \theta')\,cos(\theta' - \theta)\,cos\,\theta\;|\theta''\rangle,$$

thus, showing that we can rotate the polarization to any desired angle θ'' as long as we do not try to rotate it a full $90°$ in any one step. The wave amplitude is reduced by each rotation, but — by making each step of rotation small — the amplitude reduction can be minimized.

Does Heterogeneity Affect Shear/Fluid Interactions and, If So, How?

Soft Anisotropy:
Effects of Pore/Crack Distribution

If the Solid Is Isotropic, Are Shear/Fluid Interactions Still Possible?

An Effective Shear Modulus and What It Means

An Effective Shear Modulus (1)

In a layered medium made up of isotropic elastic layers, there are four easy eigenvectors of the system, and these are all shear modes. Two are determined by constant

$$c_{44} = \langle 1/\mu \rangle^{-1}$$

and two others by

$$c_{66} = \langle \mu \rangle$$

where $< \cdot >$ means a layer average (or equivalently a volume average in a simple layered medium), and $\mu(z)$ is shear modulus at depth z. The other eigenvectors are quasi-compressional and quasi-shear modes.

An Effective Shear Modulus (2)

Nevertheless, there is at least one other very well-defined quantity for this system, and that is the bulk modulus K_R, which turns out also to be exactly equal to the Reuss average (or harmonic mean). It would be helpful at least intellectually and probably also practically to have an effective shear modulus that is paired in some way with the bulk modulus of the system. It turns out there is such a modulus: the effective uniaxial shear modulus G_{eff} of this system.

An Effective Shear Modulus (3)

If we consider the Voigt average for shear, we find

$$G_V = \frac{1}{5}\left(G_{eff} + 2c_{44} + 2c_{66}\right),$$

and G_{eff} is given approximately by

$$G_{eff} \simeq c_{66} - O(\delta\mu^2)$$

when the layers have fluctuating shear modulus, as we normally assume to be the case. G_{eff} is defined in general (for TI media) by

$$G_{eff} = (c_{11} + c_{33} - 2c_{13} - c_{66})/3$$

and it satisfies a product formula:

$$6K_R G_{eff} = (\Lambda_+\Lambda_-)^{-1},$$

where $\Lambda_\pm$ are the eigenvalues of the compliance matrix.

CONCLUSIONS

• Data and theory agree that undrained shear moduli can depend on pore fluid mechanics if the conditions are right.

• Heterogeneity helps to produce the effect but local anisotropy (which might be the result of orientational effects of heterogeneity) is essential. Low permeability also helps.

• The theory shows that the magnitude of this effect has definite limits. The largest the effect can ever be is approximately a 20% change in overall shear modulus, and 10% changes or less would be more typical. These predictions are in agreement with both laboratory results, and effective medium theory estimates.