

SEP Seminar



Stanford University, March 1, 2004

Shear Wave Splitting Due to Pore Fluids: Gassmann Confounded by Earth Heterogeneity

James G. Berryman
University of California, LLNL
and Stanford University

OUTLINE



- Statement of the Problem: Shear Dependence on Pore Fluids
- Elasticity in VTI Media
- Backus Averaging for Finely Layered Media
- Thomsen Parameters
- Uniaxial Shear Strain: Its Special Role for Pore Fluids
- Analysis of Wave Dispersion
- Some Examples



With σ_{ij} being the ij component of stress, and e_{kl} being the kl component of strain:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} a & b & f \\ b & a & f \\ f & f & c \\ & & & 2l \\ & & & & 2m \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ e_{23} \\ e_{31} \\ e_{12} \end{pmatrix},$$

where $a = b + 2m$ (e.g., Musgrave, 1970; Auld, 1973).

Indices i, j, k, l range from 1 to 3 in Cartesian coordinates.



Following Backus (1962), we suppose that the region of interest is composed of fine layers having isotropic elastic constants, $\lambda(z)$ and $\mu(z)$, being functions of depth z . Then, the average over an arbitrary stack of such layers can be computed using a layer averaging method. This involves a Legendre transform that I will not present here. We use the layer averaging operator symbolized, for example, by brackets

$$\langle \mu \rangle \equiv \frac{1}{D} \int_0^D \mu(z) dz.$$

where D is the depth of the stack of layers.



The elastic anisotropy coefficients are then related to the layer parameters by the following expressions:

$$c = \left\langle \frac{1}{\lambda + 2\mu} \right\rangle^{-1}, \quad \text{and} \quad l = \left\langle \frac{1}{\mu} \right\rangle^{-1},$$

$$f = c \left\langle \frac{\lambda}{\lambda + 2\mu} \right\rangle,$$

$$a = \frac{f^2}{c} + 4m - 4 \left\langle \frac{\mu^2}{\lambda + 2\mu} \right\rangle,$$

$$m = \langle \mu \rangle, \quad \text{and} \quad b = a - 2m.$$

How Do Fluids Affect These Constants?



Within each isotropic layer, Gassmann says that the shear modulus μ is independent of all fluids present.

So all the dependence on fluids in this layered model comes in through the other Lamé constant

$$\lambda = K - 2\mu/3,$$

where K is the bulk modulus. Depending on the situation, K can be the drained bulk modulus K_d , or it can be the undrained bulk modulus K_u .



Gassmann's well-known result for fluid-substitution is:

$$K_u = K_d / (1 - \alpha B),$$

where K_d is the drained bulk modulus, $\alpha = 1 - K_d / K_s$ is the Biot-Willis or effective stress coefficient with K_s being a measure of the grain bulk moduli, while B is Skempton's coefficient, containing all the relevant information about the fluid moduli and porosity.

Note that $1 / (1 - \alpha B)$ is a magnification factor.



The Thomsen (1986) parameters ϵ , δ , and γ are related to these stiffness coefficients by

$$\epsilon \equiv \frac{a - c}{2c},$$

$$\delta \equiv \frac{(f + l)^2 - (c - l)^2}{2c(c - l)},$$

$$\gamma \equiv \frac{m - l}{2l}.$$



With σ_{ij} being the ij component of stress, and e_{kl} being the kl component of strain:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} a & b & f \\ b & a & f \\ f & f & c \\ & & & 2l \\ & & & & 2l \\ & & & & & & 2m \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ e_{23} \\ e_{31} \\ e_{12} \end{pmatrix},$$

where $a = b + 2m$ (e.g., Musgrave, 1970; Auld, 1973).

Indices i, j, k, l range from 1 to 3 in Cartesian coordinates.



We can immediately write down four singular vectors
(or eigenvectors):

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

and their corresponding singular values (eigenvalues), are
respectively: $2l$, $2m$, and $a - b = 2m$.

All four correspond to shear modes of the system.

Uniaxial Shear Strain



Uniaxial shear strain can be applied to this system
in the form:

$$\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & -2 & & \\ & & & 0 & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}.$$

Then, the associated energy per unit volume for such
an excitation can be easily determined.



Ignoring the parts of the matrix not relevant, we now have:

$$E_v = v^T \begin{pmatrix} a & b & f \\ b & a & f \\ f & f & c \end{pmatrix} v,$$

where the normalized vector of interest is

$$v^T = (1 \quad 1 \quad -2) / \sqrt{6},$$

and

$$E_v = 2[a - m + c - 2f]/3 \equiv 2G_{eff}.$$



Thus, based on energy alone, we can justify defining

$$G_{eff} \equiv [a - m + c - 2f]/3.$$

For this and several other reasons I will not have time to discuss, G_{eff} acts like an effective shear modulus and it is the only one of the five shear moduli that ever contains information about pore fluids.



It is not hard to show that $l \leq G_{eff} \leq m$, the precise value depending on αB . Therefore, define for utility

$$ratio_{\alpha B} \equiv \frac{m - G_{eff}}{m - l},$$

a quantity that ranges from 0 to 1. Then, the quantity

$$1 - \frac{ratio_{\alpha B}}{ratio_0}$$

also ranges at most from 0 to 1 and measures the effect of pore fluids on shear (for $B > 0$, $\alpha > 0$).

do is based again on an assumption that the bulk moduli in the layers are always proportional to the shear modulus so $K = s\mu$, for some fixed value of $s > 0$. For a given model, we can infer from this condition that

$$1 - \frac{ratio_{\alpha B}}{ratio_0} = \frac{\alpha B}{1 + 4(1 - \alpha B)/3s} \leq \alpha B, \quad (59)$$

in agreement with the empirical result from the synthetic data shown in Figure 1.

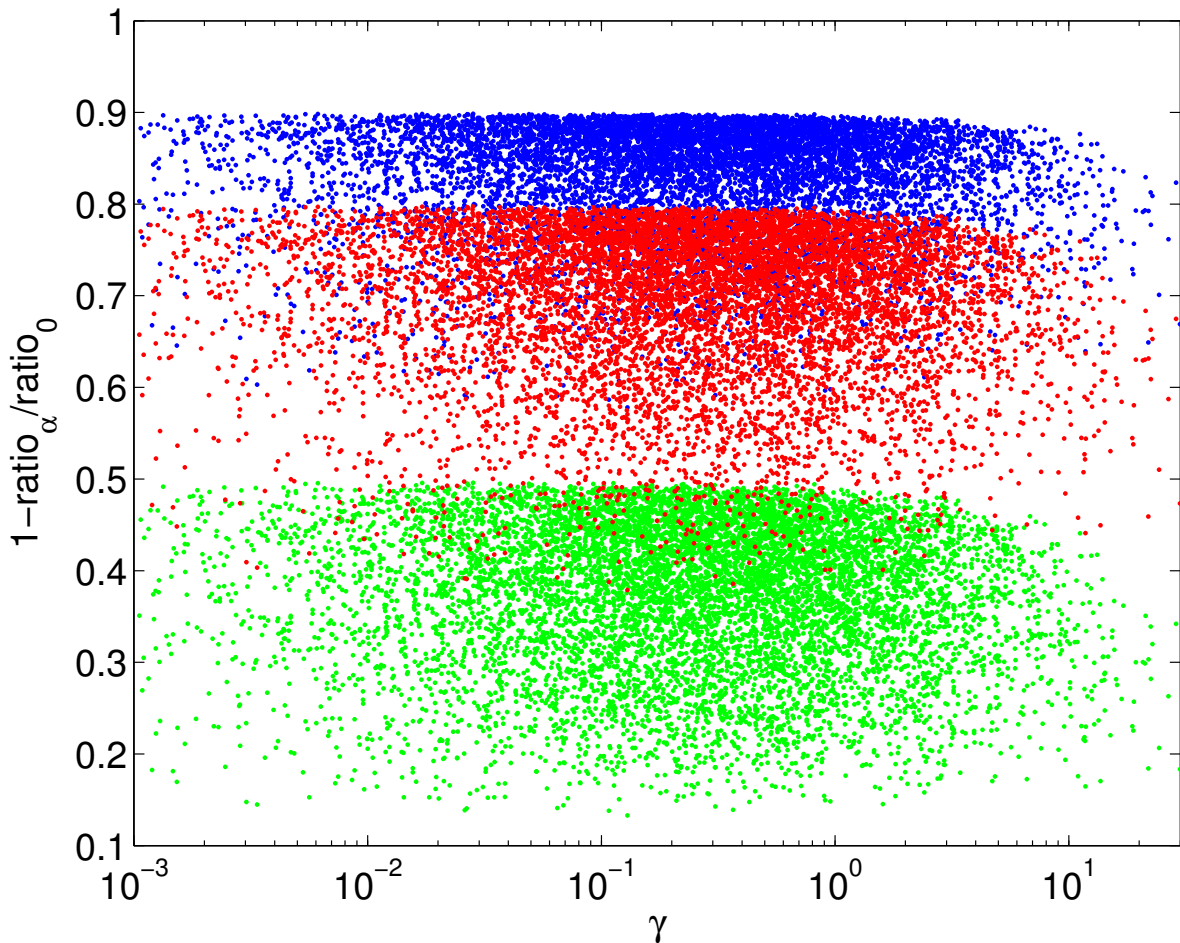


Figure 1: Scatter plot illustrating how G_{eff} varies over a physically sensible range of layered isotropic media (see text for details) with 2700 distinct models and $B = 1$ [see Eq. (58) in the text for the definition of $ratio_{\alpha}$]. Blue dots are for $\alpha = 0.9$, red for $\alpha = 0.8$, and green for $\alpha = 0.5$. Note, that in each case, all the points for a particular choice of α are bounded above precisely by the value of α . (A general proof of this empirical observation is currently lacking.)

To check the corresponding result for P-waves, we need to estimate δ . Making use of (50),



The general behavior of seismic waves in anisotropic media is well known, and the equations are derived in many places including Berryman (1979) and Thomsen (1986). The results are

$$\rho\omega_{\pm}^2 = \frac{1}{2} \left[(a+l)k_1^2 + (c+l)k_3^2 \pm \sqrt{[(a-l)k_1^2 - (c-l)k_3^2]^2 + 4(f+l)^2 k_1^2 k_3^2} \right],$$

for compressional (+) and vertically polarized shear (−) waves and

Dispersion Relations (continued)



$$\rho\omega_s^2 = mk_1^2 + lk_3^2,$$

for horizontally polarized shear waves, where ρ is the overall density, ω is the angular frequency, k_1 and k_3 are the horizontal and vertical wavenumbers (respectively), and the velocities are given simply by $v = \omega/k$ with

$$k = \sqrt{k_1^2 + k_3^2}.$$



The SH wave depends only on elastic parameters l and m , which are not dependent in any way on layer λ and, therefore, play no role in the poroelastic analysis. Thus, we can safely ignore SH except when we want to check for shear wave splitting (bi-refringence) – in which case the SH results will be useful for the comparisons.

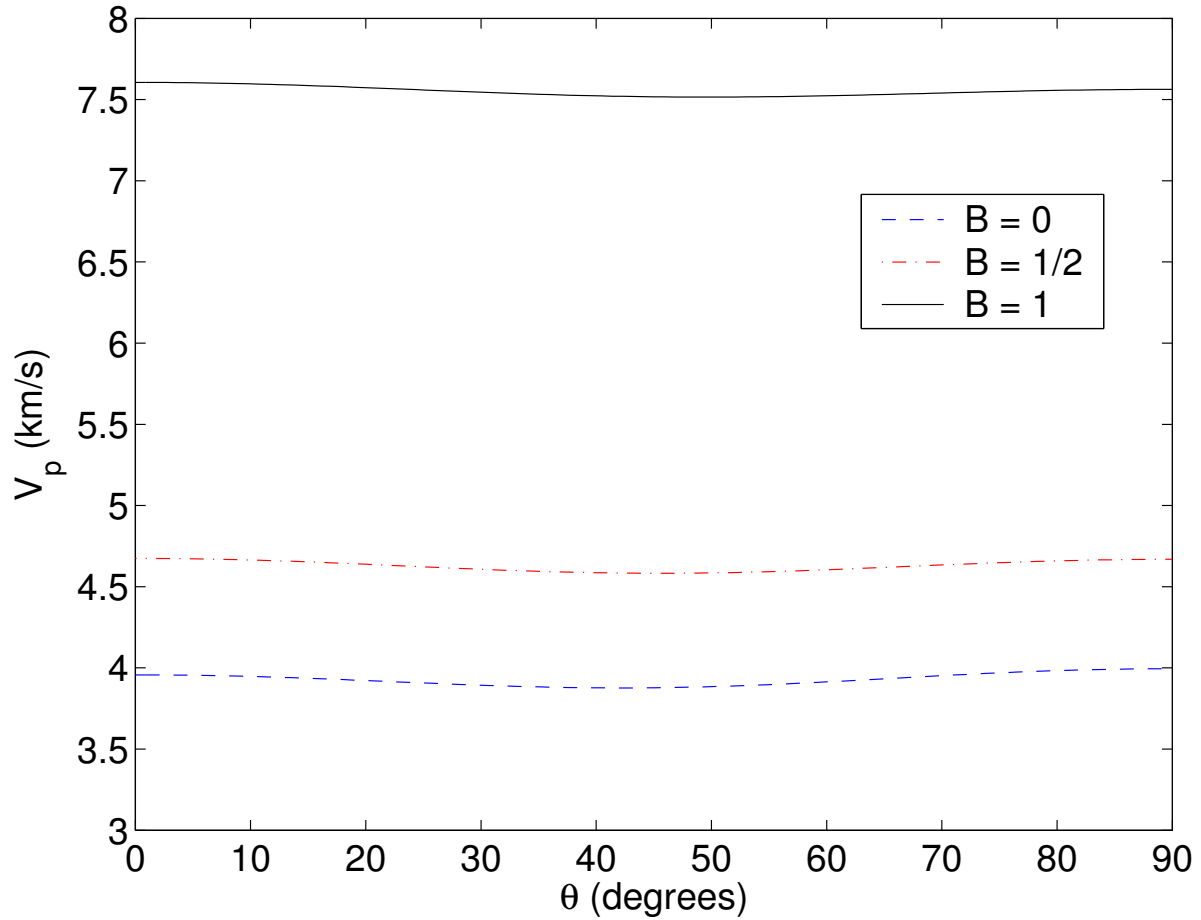


Figure 2: Compressional wave speed V_p as a function of angle θ from the vertical. Two curves shown correspond to choices of Skempton's coefficient $B = 0$ for the drained case (dashed line) and $B = 1$ for the undrained case (solid line). The case $B = \frac{1}{2}$ (dot-dash line) is used to model partial saturation conditions as described in the text. The Biot-Willis parameter was chosen to be $\alpha = 0.8$, constant in all layers.

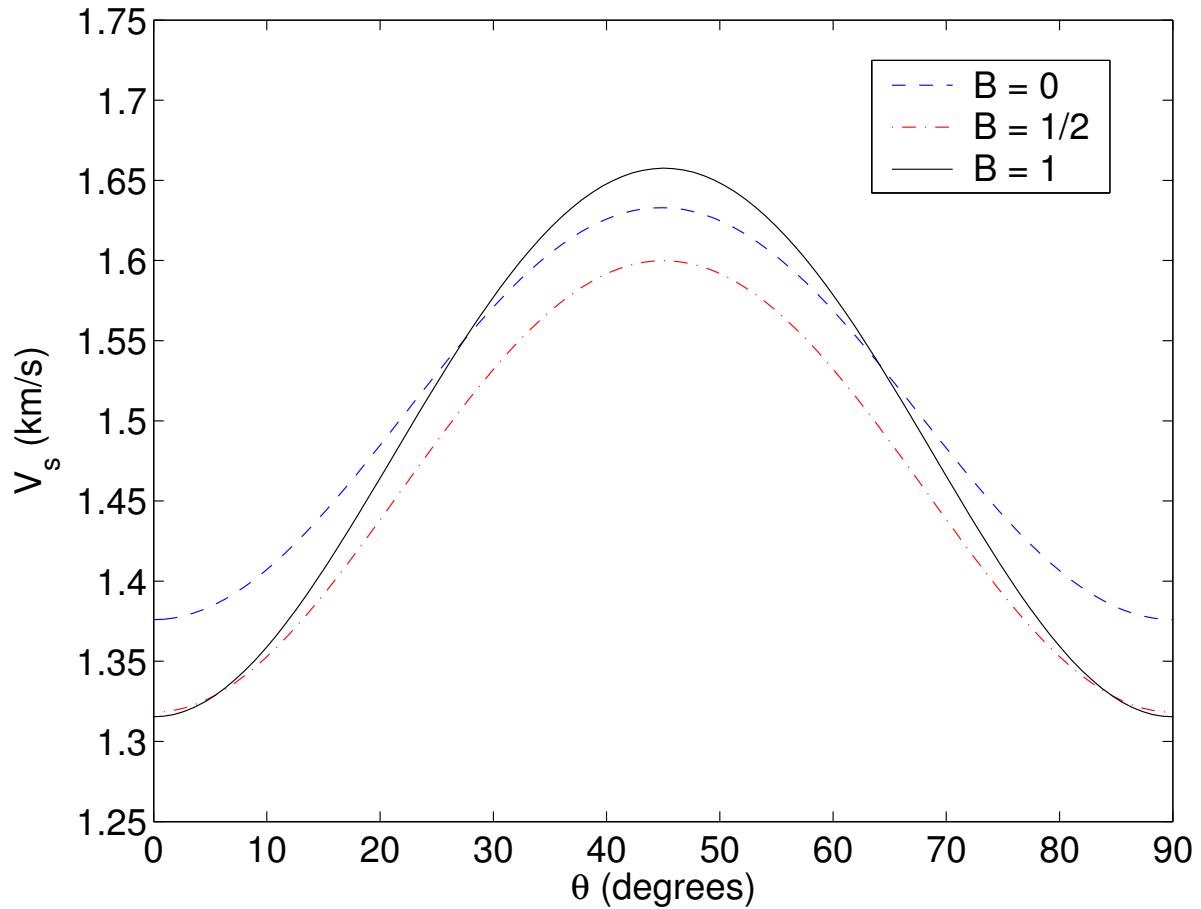


Figure 3: Vertically polarized shear wave speed V_{sv} as a function of angle θ from the vertical. Two curves shown correspond to choices of Skempton's coefficient $B = 0$ for the drained case (dashed line) and $B = 1$ for the undrained case (solid). The case $B = \frac{1}{2}$ (dot-dash line) is used to model partial saturation conditions as described in the text. The Biot-Willis parameter was chosen to be $\alpha = 0.8$, constant in all layers.



$$\rho\omega_+^2 \equiv ak_1^2 + ck_3^2 - \Delta,$$

and

$$\rho\omega_-^2 \equiv lk^2 + \Delta,$$

with Δ determined approximately by

$$\Delta \simeq \frac{[(a-l)(c-l) - (f+l)^2]}{(a-l)/k_3^2 + (c-l)/k_1^2}.$$



Recall that

$$(a - l)(c - l) - (f + l)^2 = 2c(c - l)(\epsilon - \delta).$$

We can also rewrite the first elasticity factor in the denominator as $a - l = (c - l)[1 + 2c\epsilon/(c - l)]$. Combining these results in the limit of $k_1^2 \rightarrow 0$ (for relatively small horizontal offset), we find that



$$\rho\omega_+^2 \simeq ck^2 + 2c\delta k_1^2,$$

and

$$\rho\omega_-^2 \simeq lk^2 + 2c(\epsilon - \delta)k_1^2,$$

since, in this limit, we have $\Delta \simeq 2c(\epsilon - \delta)k_1^2$.

Improved approximations to any desired order can be obtained with only a little more effort by keeping more terms in the expansion.

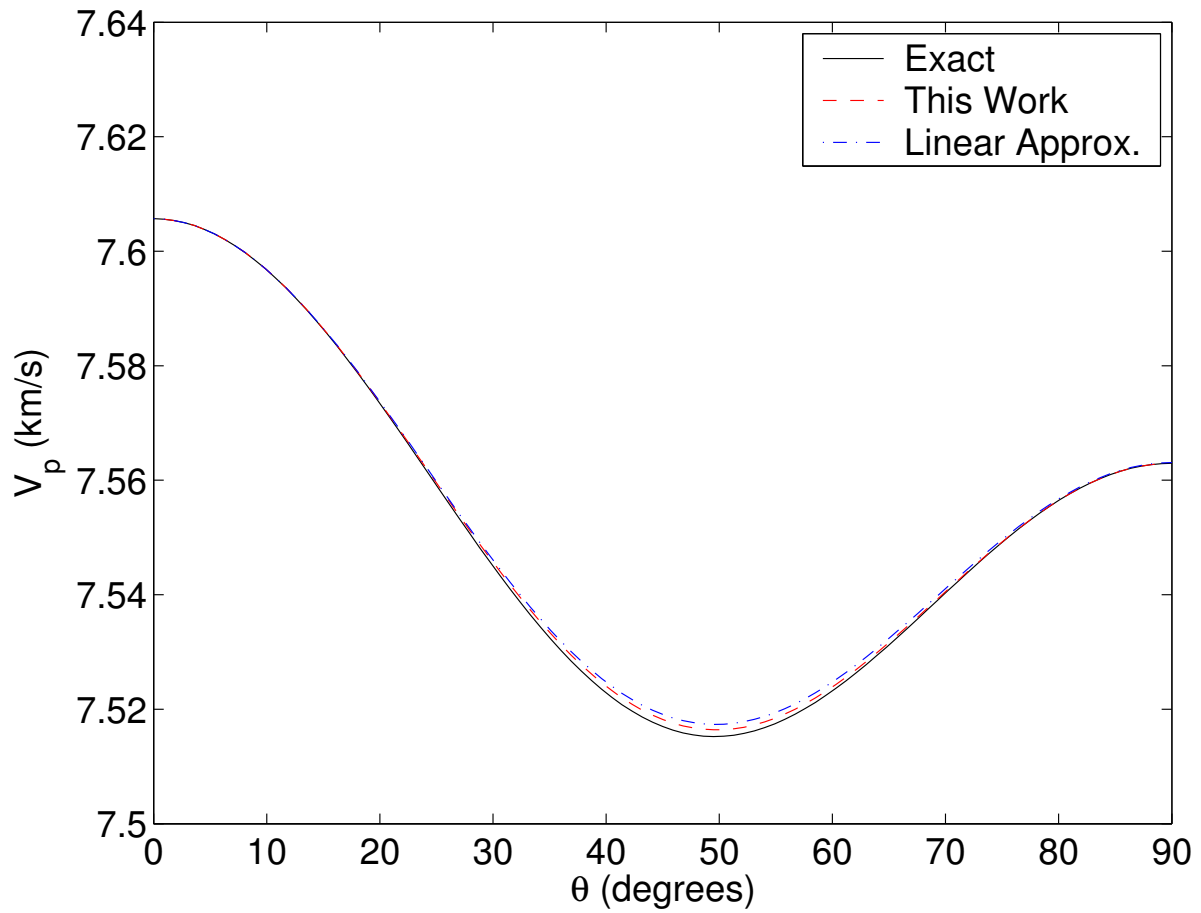


Figure 4: Compressional wave velocities as computed exactly from the dispersion relation (34), by (34) using approximation (37) for Δ , and by the linear approximation (64). This layered model is the same as in Figures 2 and 3 for the case $B = 1$.

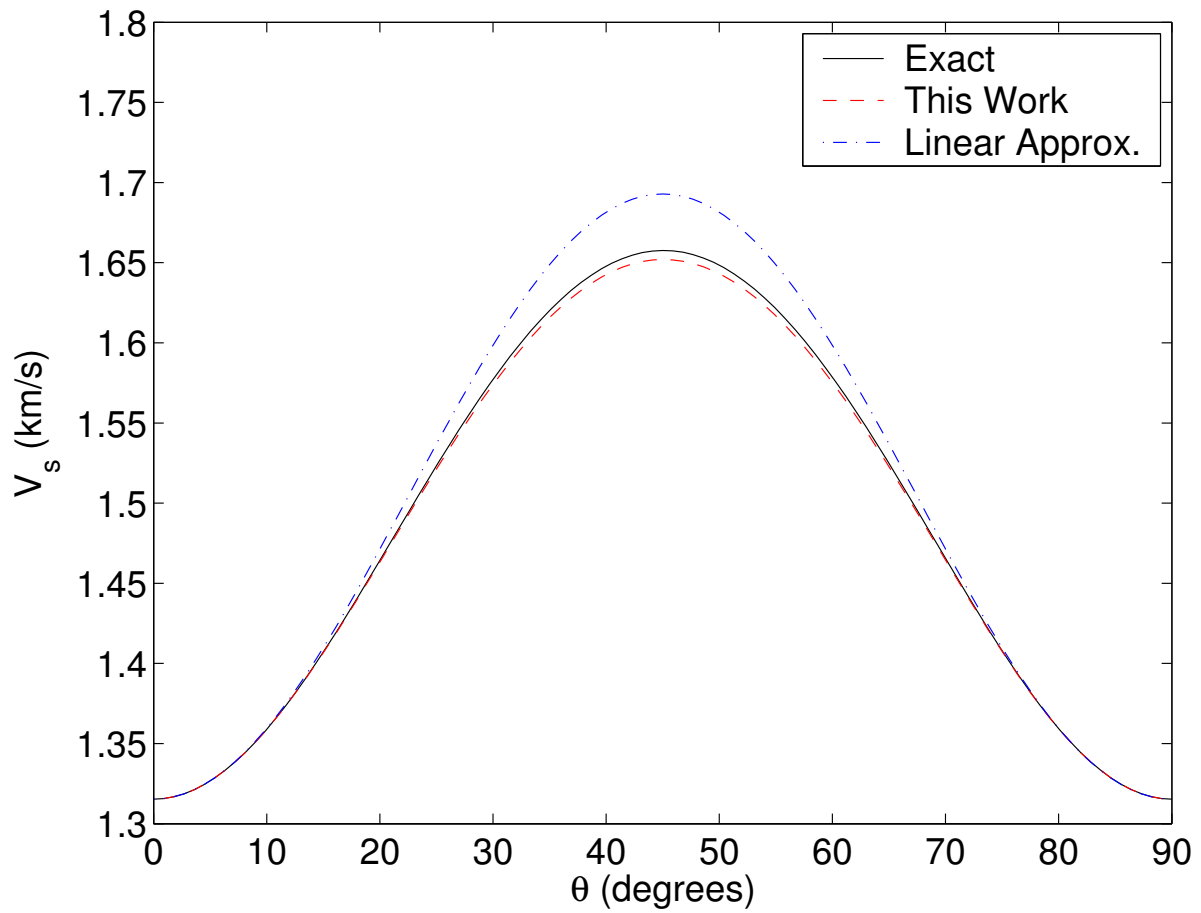


Figure 5: Shear wave velocities as computed exactly from the dispersion relation (35), by (35) using approximation (37) for Δ , and by the linear approximation (66). This layered model is the same as in Figures 2 and 3 for the case $B = 1$.

CONCLUSIONS



- Of the five shear moduli of a VTI system, only G_{eff} as defined here can ever contain information about pore fluids.
- Pore fluids have their biggest effects at $\theta = 45^\circ$, on both quasi-P and quasi-SV waves in layered VTI media.
- The stiffening effects of pore fluids can be substantial if the necessary condition of large shear fluctuation is met.
- Examples show that the magnitude of these effects on G_{eff} translate into 10-20% effects on shear wave speed.