WEAK CONSOLIDATION MODEL FOR WAVE PROPAGATION IN OCEAN SEDIMENTS

James G. Berryman
University of California
Lawrence Berkeley National Laboratory
Berkeley, CA

Collaborators:
Steve Pride, LBNL; Todd Hefner and Kevin Williams,
APL, Univ. of Washington – Seattle
OUTLINE

- Biot’s equations of poroelasticity
  - Dispersion relations
  - Low frequency asymptotics
- Up-scaling
- Double-porosity dual-permeability
- Why not multi-porosity, multi-permeability?
- Examples (not available online)
- Summary and conclusions
- References
Biot’s (1962) Strain Energy Functional

\[ 2E = He^2 - 2Ce\zeta + M\zeta^2 - 4\mu I_2 \]

where \( H, C, M, \) and \( \mu \) are poroelastic constants,
\( e = \nabla \cdot \vec{u} = \) frame dilatation,
\( \zeta = -\nabla \cdot \vec{w} = \) increment of fluid content,
\( \phi = \) porosity,
\( \vec{u} = \) solid frame displacement,
\( \vec{u}_f = \) pore fluid displacement,
\( \vec{w} = \phi(\vec{u}_f - \vec{u}) = \) relative displacement, and
\( I_2 = e_x e_y + e_y e_z + e_z e_x - \frac{1}{4}(\gamma_x^2 + \ldots) = \) a strain invariant.
Biot’s Equations of Dynamic Poroelasticity

\[ \omega^2 \rho \ddot{u} + (H - \mu) \nabla e + \mu \nabla^2 \ddot{u} = -\omega^2 \rho_f \ddot{w} + C \nabla \zeta, \]

\[ \omega^2 q(\omega) \ddot{w} - M \nabla \zeta = -\omega^2 \rho_f \ddot{u} - C \nabla e, \]

where

\[ \omega = 2\pi f = \text{angular frequency}, \]
\[ \rho = \phi \rho_f + (1 - \phi) \rho_m = \text{the average density}, \]
\[ q(\omega) = \rho_f [\tau/\phi + iF(\xi)\eta/\kappa \omega], \text{ and} \]
\[ p_f = -M \nabla \cdot \ddot{w} - C \nabla \cdot \ddot{u} = \text{fluid pressure}. \]
Some Relations Among Poroelastic Constants

\[ H = K_u + \frac{4}{3} \mu, \]

\[ C = BK_u, \]

\[ M = BK_u / \alpha = C / \alpha, \]

where \( \alpha = 1 - K/K_m = \) the effective stress coefficient, and \( B \) is Skempton’s coefficient, and \( K_u = 1/(1 - \alpha B) \) is the undrained (or Gassmann) bulk modulus of the system.
Dispersion Relations

- For shear wave:
  \[ k_s^2 = \omega^2 (\rho - \rho_f^2 / q) / \mu \]

- For fast and slow compressional waves:
  \[ k_{\pm}^2 = \frac{1}{2} \left[ b + f \mp [(b - f)^2 + 4cd]^{1/2} \right] \]
  
  \[ b = \omega^2 (\rho M - \rho_f C) / \Delta, \quad c = \omega^2 (\rho_f M - qC) / \Delta \]
  
  \[ d = \omega^2 (\rho_f H - \rho C) / \Delta, \quad f = \omega^2 (qH - \rho_f C) / \Delta \]

  where

  \[ \Delta = HM - C^2. \]
Compressional and shear waves have very similar asymptotic behavior in Biot’s theory at low frequencies as $\omega \to 0$.

At such low frequencies with $\omega \ll \omega_{crit}$, the wavenumber for shear wave propagation $k_s$ is controlled by the factor $q(\omega)$ which goes like

$$q(\omega) \to \frac{i\rho_f\eta}{\kappa\omega}.$$ 

Then, we have

$$k_s^2 = \frac{\omega^2 \rho}{\mu_d} \left[ 1 + i\frac{\rho_f\kappa\omega}{\rho\eta} \right] = \frac{\omega^2}{v_s^2} \left[ 1 + i\frac{1}{Q_s} \right].$$
And, similarly, the shear wave quality (attenuation) factor is given by

\[
\frac{1}{Q_s} \approx \frac{\rho_f \kappa \omega}{\rho \eta} \sim O(\kappa \omega).
\]
UP-SCALING
via effective medium theory or homogenization methods

- Electrical Conductivity (scale invariant)
  \[ J = \sigma E \rightarrow \langle J \rangle = \sigma^* \langle E \rangle \]
- Navier-Stokes equation → Darcy’s equation
definitely not scale invariant!
- Linear elasticity + Navier-Stokes equations →
  Biot’s equations of poroelasticity
- Heterogeneous Biot → ????
  One possibility is a double-porosity model, which is
  known to work in a variety of circumstances (i.e., high
  and low frequencies, fractured reservoirs, etc.).
Double-Porosity Concept

Main features:

- Two types of porosity: equant (aspect ratio \(\approx 1\)) and fracture.

- Equant porosity (\(\phi \approx 0.2\)) has large volume but small permeability (\(\kappa \approx 1mD\)).

- Fracture porosity (\(\phi \approx 0.001\)) has small volume but large permeability (\(\kappa \approx 10D\)).
DOUBLE-POROSITY/DUAL-PERMEABILITY

Two classic references:


NEW
DOUBLE-POROSITY CONCEPT

Main features:

- Two distinct types of porous media: different frame constants, different porosity values, different permeabilities

- No need for fractures

- No need for any special correlation between flow properties and mechanical properties
Two more recent references:


WHY NOT MULTI-POROSITY?

- Of course, real heterogeneous media will generally contain more than two kinds of porosity. So why should we be satisfied with a double-porosity model? Why not multi-porosity?

- In principle, it is possible to make such a generalization of the approach. However, there are no known exact results for systems more complex than double-porosity, at this time.
Also, since there is only one permeability that dominates in each of the two extremes, *i.e.*, high and low frequency → low and high permeability \((\kappa)\), that means that we will usually see effects in data only of the extreme values present: just the highest permeability at low frequencies and the lowest permeability at high frequencies. So, either as a reservoir modeling tool or as an ocean sediment modeling tool, two-component double-porosity modeling is expected to be both necessary & sufficient.
Single- and Double-Porosity Geomechanics

Single-porosity:

\[
\begin{pmatrix}
\delta e \\
-\delta \zeta
\end{pmatrix} =
\begin{pmatrix}
a_{11} & a_{12} \\
a_{12} & a_{22}
\end{pmatrix}
\begin{pmatrix}
-\delta p_c \\
-\delta p_f
\end{pmatrix}.
\]

Double-porosity:

\[
\begin{pmatrix}
\delta e \\
-\delta \zeta^{(1)} \\
-\delta \zeta^{(2)}
\end{pmatrix} =
\begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{12} & a_{22} & a_{23} \\
a_{13} & a_{23} & a_{33}
\end{pmatrix}
\begin{pmatrix}
-\delta p_c \\
-\delta p_f^{(1)} \\
-\delta p_f^{(2)}
\end{pmatrix}.
\]

where \( \delta e \) is total volume dilatation,
\( \delta \zeta \) is the increment of fluid mass content
\( \delta p_c \) and \( \delta p_f \) are the changes in
confining pressure and pore-fluid pressure.
Mechanics of Gassmann Constituents

\[
\begin{pmatrix}
\delta e^{(1)} \\
-\delta \zeta^{(1)}
\end{pmatrix} = \frac{1}{K^{(1)}} \begin{pmatrix}
1 & -\alpha^{(1)} \\
-\alpha^{(1)} & \alpha^{(1)}/B^{(1)}
\end{pmatrix} \begin{pmatrix}
-\delta p_c^{(1)} \\
-\delta p_f^{(1)}
\end{pmatrix}
\]

where \( K^{(1)} \) is the jacketed or drained bulk modulus, 
\[
\alpha^{(1)} = 1 - K^{(1)}/K_s^{(1)}
\]
is the Biot-Willis parameter, 
\( B^{(1)} \) is Skempton’s coefficient for material 1. 
Porosity is fairly uniform and, if cracks are present, 
all the porosity is of that type.
A similar equation applies to other Gassmann constituents.
DOUBLE-POROSITY APPLICATIONS (1)

\[
\begin{pmatrix}
\delta e \\
-\delta \zeta^{(1)} \\
-\delta \zeta^{(2)}
\end{pmatrix} =
\begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{12} & a_{22} & a_{23} \\
a_{13} & a_{23} & a_{33}
\end{pmatrix}
\begin{pmatrix}
-\delta p_c \\
-\delta p_f^{(1)} \\
-\delta p_f^{(2)}
\end{pmatrix},
\]

where

\[
a_{11} = \frac{1}{K^*},
\]

\[
a_{22} = \frac{v^{(1)}}{K^{(1)}} \left( \frac{1}{B^{(1)}} - \frac{\alpha^{(1)}(1-Q_1)}{1-K^{(1)}/K^{(2)}} \right),
\]

\[
a_{12} = -\frac{v^{(1)}Q_1}{K^{(1)}} \alpha^{(1)},
\]

\[
a_{23} = \frac{\alpha^{(1)} \alpha^{(2)} K^{-1}}{[K^{(2)}-K^{(1)}]^2} \left[ \frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*} \right],
\]
and where

\[ v^{(1)} Q_1 = \frac{1 - \frac{K^{(2)}}{K^*}}{1 - \frac{K^{(2)}}{K^{(1)}}}. \]

The remaining coefficients can be found using phase-interchange symmetry.
DOUBLE-POROSITY APPLICATIONS (3)

\[ a_{11} = \frac{1}{K^*} \quad \text{and} \quad D = \frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*} \]

\[ a_{12} = -\frac{\alpha^{(1)}}{K^{(1)}} \frac{1-K^{(2)}/K^*}{1-K^{(2)}/K^{(1)}} \]

\[ a_{13} = -\frac{\alpha^{(2)}}{K^{(2)}} \frac{1-K^{(1)}/K^*}{1-K^{(1)}/K^{(2)}} \]

\[ a_{22} = \frac{v^{(1)} \alpha^{(1)}}{B^{(1)K^{(1)}}} - \left( \frac{\alpha^{(1)}}{1-K^{(1)}/K^{(2)}} \right)^2 D \]

\[ a_{23} = \frac{K^{(1)} K^{(2)} \alpha^{(1)} \alpha^{(2)}}{(K^{(2)}-K^{(1)})^2} D \]
The main point to observe is that the double-porosity coefficients are now completely determined except that we do not necessarily know the value of the overall drained bulk modulus $K^*$ that appears in all the formulas. So, it would clearly be advantageous (if we have the choice, as we sometimes do) to choose a microstructure for which we can obtain good estimates or formulas for $K^*$. This choice does not affect the preceding formulas as their form is independent of the microstructure! But of course their value depends on the crucially on microstructure entirely through the value of $K^*$. 
It is not hard to show that the general equations for the $a_{ij}$'s in the double-porosity model have finite limits if one of the drained constants (either $K^{(1)}$ or $K^{(2)}$) is taken continuously to zero. We imagine this limit as being the case of finite modulus for grains in the force chains, and zero modulus for the grains in the fluid suspension (and therefore not participating in the force chains) contained within the cages created by the force chains throughout the granular sedimentary materials.
Generalized Biot Coefficients (1)

\[ D = \text{Drained}, \ B = \text{Skempton}, \ U = \text{Undrained} \]

\[ \frac{1}{K_D} \equiv a_{11} - \frac{a_{13}^2}{a_{33} - \gamma(\omega)/i\omega} \]

\[ B = \frac{a_{13}(a_{23} + \gamma(\omega)/i\omega) - a_{12}(a_{33} - \gamma(\omega)/i\omega)}{(a_{22} - \gamma(\omega)/i\omega)(a_{33} - \gamma(\omega)/i\omega) - (a_{23} + \gamma(\omega)/i\omega)^2} \]

\[ \frac{1}{K_U} \equiv \frac{1}{K_D} + B\left(a_{12} - \frac{a_{13}(a_{23} + \gamma(\omega)/i\omega)}{a_{33} - \gamma(\omega)/i\omega}\right) \]

\[ \gamma(\omega) = \gamma_m \sqrt{1 - i\frac{\omega}{\omega_m}} \text{ where } \gamma_m \approx \frac{v(1)\kappa(1)}{\eta L_1^2} \]
Generalized Biot Coefficients (2)

\[ H = K_U + 4G/3 \]

\[ C = BK_U \]

\[ M = \frac{B^2}{1-K_D/K_U} K_U \]

These are exactly the coefficients from Biot’s (1962) equations, but they are now complex and frequency dependent.
SUMMARY AND CONCLUSIONS

- Double porosity model provides a different mechanism for additional wave attenuation, due to mesoscopic flow between stiffer and weaker components of an ocean sediment.
- The magnitude of these predicted effects is of the right magnitude and in the right frequency band to explain some of the observed extra attenuation present in these systems.
- Further work is clearly needed to establish this mechanism as viable by considering the patch sizes of the various heterogeneities. But semiquantitative work so far shows good promise.
REFERENCES

- Experiment Results
- Double-Porosity Geomechanics (1)
- Double-Porosity Geomechanics (2)
- Other Mesoscale Analyses
EXPERIMENTAL RESULTS


OTHER MESOSCALE ANALYSES
