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ROLE OF DOUBLE-POROSITY DUAL-PERMEABILITY MODELS FOR MULTI-RESONANCE GEOMECHANICAL SYSTEMS

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OUTLINE

- Biot's equations of poroelasticity
  - Dispersion relations
  - Low frequency asymptotics
- Line integrals in tomography
- High frequency asymptotics
- Up-scaling for seismology
- Double-porosity dual-permeability
- Why not multi-porosity, multi-permeability?
- Summary and conclusions

Biot's (1962) Strain Energy Functional

$$2E = He^2 - 2Ce\zeta + M\zeta^2 - 4\mu I_2$$

where H, C, M, and μ are poroelastic constants,

$e = \nabla \cdot \vec{u} =$ frame dilatation,

$\zeta = -\nabla \cdot \vec{w} =$ increment of fluid content,

$\phi =$ porosity,

$\vec{u} =$ solid frame displacement,

$\vec{u}_f =$ pore fluid displacement,

$\vec{w} = \phi(\vec{u}_f - \vec{u}) =$ relative displacement, and

$I_2 = e_x e_y + e_y e_z + e_z e_x - \frac{1}{4}(\gamma_x^2 + \ldots) =$ a strain invariant.

Biot's Equations of Dynamic Poroelasticity

$$\omega^2 \rho \vec{u} + (H - \mu)\nabla e + \mu \nabla^2 \vec{u} = -\omega^2 \rho_f \vec{w} + C\nabla\zeta,$$

$$\omega^2 q(\omega)\vec{w} - M\nabla\zeta = -\omega^2 \rho_f \vec{u} - C\nabla e,$$

where

$\omega = 2\pi f =$ angular frequency,

$\rho = \phi\rho_f + (1-\phi)\rho_m =$ the average density,

$q(\omega) = \rho_f[\tau/\phi + iF(\xi)\eta/\kappa\omega]$, and

$p_f = -M\nabla\cdot\vec{w} - C\nabla\cdot\vec{u} =$ fluid pressure.

Some Relations Among Poroelastic Constants

$$H = K_u + \tfrac{4}{3}\mu,$$

$$C = BK_u,$$

$$M = BK_u/\alpha = C/\alpha,$$

where

$\alpha = 1 - K/K_m =$ the effective stress coefficient, and

K_u is the undrained or Gassmann bulk modulus of the system.

Dispersion Relations

- For shear wave:

$$k_s^2 = \omega^2(\rho - \rho_f^2/q)/\mu$$

- For fast and slow compressional waves:

$$k_\pm^2 = \frac{1}{2}\left[b + f \mp \left[(b-f)^2 + 4cd\right]^{1/2}\right]$$

$$b = \omega^2(\rho M - \rho_f C)/\Delta, \quad c = \omega^2(\rho_f M - qC)/\Delta$$
$$d = \omega^2(\rho_f H - \rho C)/\Delta, \quad f = \omega^2(qH - \rho_f C)/\Delta$$

where

$$\Delta = HM - C^2.$$

LOW FREQUENCY ASYMPTOTICS

Compressional and shear waves have very similar asymptotic behavior in Biot's theory at low frequencies as $\omega \to 0$.

At such low frequencies with $\omega << \omega_{crit}$, the wavenumber for shear wave propagation k_s is controlled by the factor $q(\omega)$ which goes like

$$q(\omega) \to \frac{i\rho_f \eta}{\kappa\omega}.$$

Then, we have

$$k_s^2 = \frac{\omega^2 \rho}{\mu_d}\left[1 + i\frac{\rho_f \kappa\omega}{\rho\eta}\right] = \frac{\omega^2}{v_s^2}\left[1 + \frac{i}{Q_s}\right],$$

and the shear wave quality (attenuation) factor is given by

$$\frac{1}{Q_s} \simeq \frac{\rho_f \kappa\omega}{\rho\eta} \sim O(\kappa\omega).$$

LINE INTEGRALS IN TOMOGRAPHY

We know that a plane shear wave in some homogeneous Biot material will have amplitude and phase that vary according to

$$u_s(z,t) = A_s\, exp\,[i(k_s \cdot z - \omega t)],$$

but in heterogeneous media the result is modified slightly by noting that the wavenumber can depend on location z so that $k_s(z)$ is the relevant wavenumber locally, and furthermore the wave amplitude and phase along a ray path (see diagram) are determined instead by

$$u_s(z,t) = exp\,[i \int_{path} dz k_s(z) - i\omega t].$$

LINE INTEGRALS IN TOMOGRAPHY (2)

The phase therefore looks like

$$\text{phase} = \int_{path} dz \frac{\omega}{v_s(z)} - \omega t,$$

while the amplitude decays like

$$\text{amplitude} = A_s\, exp\, [- \int_{path} dz \frac{\omega}{2 v_s(z) Q_s(z)}].$$

Then, since $1/Q(z)$ is proportional to $\omega\kappa(z)$, any attempt to make use of Biot's theory to make direct measurements of reservoir permeability will be frustrated unless tomographic methods are used to deconstruct (or invert for) the local values of permeability $\kappa(z)$ from the line integrals

$$\int_{path} dz \kappa(z),$$

which are all that can ever be measured directly from field data.

HIGH FREQUENCY ASYMPTOTICS

Furthermore, the problem is still more difficult if we try to use data gathered in multiple frequency bands. At high frequencies $\omega > \omega_{crit}$ (which would be the case for well-logging or laboratory data), the predictions of Biot's theory take a different asymptotic form:

$$\frac{1}{Q_s} \sim O(\frac{1}{\sqrt{\kappa\omega}}).$$

And therefore it follows, for exactly the same reasons as before, That the amplitude decay of a shear wave will depend now on the line integral

$$\int_{path} dz \frac{1}{\sqrt{\kappa(z)}}.$$

HIGH FREQUENCY ASYMPTOTICS (2)

These asymptotic results for the attenuation both imply that the measured attenuation will not be well-predicted by using measured permeabilities of earth materials. (This is demonstrated by a simple thought experiment.) And this is one reason why double-porosity modeling is needed for analysis of heterogeneous reservoirs.

UP-SCALING

via effective medium theory or homogenization methods

- Electrical Conductivity (scale invariant)

$$J = \sigma E \to \langle J \rangle = \sigma^* \langle E \rangle$$

- Navier-Stokes equation $\to$ Darcy's equation

  definitely not scale invariant!

- Linear elasticity + Navier-Stokes equations $\to$

  Biot's equations of poroelasticity

- Heterogeneous Biot $\to$????

  One possibility is the double-porosity model, which is known to work in a variety of circumstances.

DOUBLE-POROSITY CONCEPT

Main features:

- Two types of porosity: equant (aspect ratio $\simeq 1$) and fracture
- Equant porosity has large volume but small permeability
- Fracture porosity has small volume but large permeability

DOUBLE-POROSITY/DUAL-PERMEABILITY

Two classic references:

G. I. Barenblatt and Y. P. Zheltov, "Fundamental equations of filtration of homogeneous liquids in fissured rocks," *Sov. Phys. Dokl.* **5**, pp. 522-525 (1960).

J. E. Warren and P. J. Root, "The behavior of naturally fractured reservoirs," *Soc. Pet. Eng. J.* **3**, pp. 245-255 (1963).

DOUBLE-POROSITY GEOMECHANICS

Two more recent references:

J. G. Berryman and H. F. Wang, "The elastic coefficients of double-porosity models for fluid transport in jointed rocks," *J. Geophys. Res.* **100**, pp. 24611-24627 (1995).

J. G. Berryman and S. R. Pride, "Models for computing geomechanical constants of double-porosity materials from the constituents' properties," *J. Geophys. Res.* **107**, paper 2000JB000108 (2002).

WHY NOT MULTI-POROSITY?

• Of course, real reservoirs will generally contain more than two kinds of porosity. So why should we be satisfied with a double-porosity model? Why not multi-porosity?

• In principle, it is possible to make such a generalization of the approach. However there are no known exact results for systems more complex than double-porosity.

WHY NOT MULTI-POROSITY? (2)

- Also, since there is only one permeability that dominates in each of the two extremes, *i.e.*, high and low frequency $\rightarrow$ low and high permeability, that means that we will usually see effects in data only of the extreme values present: just the highest permeability at low frequencies and the lowest permeability at high frequencies. So, as a reservoir modeling tool, only double-porosity modeling is expected to be necessary (and often sufficient).

SUMMARY AND CONCLUSIONS

• When there is significant variation in permeability $\kappa(z)$ throughout a reservoir, Biot's single-porosity equations do not provide enough flexibility to incorporate all the effects of interest.

• In particular, the sound wave attenuation cannot be modeled correctly over the entire frequency range from seismic (1-100 Hz) to well-logging (1-100 kHz) to ultrasonic (100kHz - 5 MHz) frequencies of practical importance.

SUMMARY AND CONCLUSIONS (2)

- One good way to handle this heterogeneity is to model the reservoir as a double-porosity dual-permeability medium. Then, the high permeability regions contribute most to the attenuation at low frequencies, and the low permeability regions contribute most at high frequencies.

- This conceptual model has the advantage that it does not change qualitatively as the size of the macroscopic averaging volumes changes due to further attempts at up-scaling. So the **form of these equations** is invariant to further up-scaling.

Other recent references:

S. R. Pride and J. G. Berryman, "Linear dynamics of double-porosity dual-permeability materials. I. Governing equations and acoustic attenuation," *Physical Review E* **68**, 036603 (September 9, 2003).

S. R. Pride and J. G. Berryman, "Linear dynamics of double-porosity dual-permeability materials. II. Fluid transport equations," *Physical Review E* **68**, 036604 (September 9, 2003).

DOUBLE-POROSITY GEOMECHANICS

Other recent references:

J. G. Berryman and H. F. Wang, ‘Elastic wave propagation and attenuation in a double-porosity dual-permeability medium,” *Int. J. Rock Mech.* **37**, 63–78 (2000).

S. R. Pride, J. G. Berryman, and J. M. Harris, “Seismic attenuation due to wave-induced flow,” *J. Geophys. Res.* **109**, B01201 (January 14, 2004).

DOUBLE-POROSITY GEOMECHANICS

Another recent reference:

J. G. Berryman, "Comparison of up-scaling methods in poroelasticity and its generalizations," *ASCE J. Engineering Mechanics* **131**(9), 928–936 (2005).