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# POROMECHANICS OF RESERVOIRS ESTIMATED FOR THE RANDOM POLYCRYSTALS OF POROUS LAMINATES MODEL

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# REFERENCES (1)

## Double-porosity papers:

- J. G. Berryman and H. F. Wang, “The elastic coefficients of double-porosity models for fluid transport in jointed rock,” *J. Geophys. Res.* **100**, 24611–24627 (1995).
- JGB and S. R. Pride, “Models for computing geomechanical constants of double-porosity materials from the constituents’ properties,” *JGR* **107**(B3), 10.1029/2000JB000108 (2002).
- JGB, “Extension of poroelastic analysis to double-porosity materials,” *ASCE JEM* **128**, 840–847 (2002).

# REFERENCES (2)

## Random polycrystals of laminates papers:

- ○ JGB, "Bounds on elastic constants for random polycrystals of laminates," *J. Appl. Phys.* **96**, 4281–4287 (2004).

- ○ JGB, "Bounds and estimates for elastic constants of random polycrystals of laminates," *IJSS* **42** (13), 3730–3743 (2005).

# OUTLINE

- Double-Porosity (DP) geomechanics
  - Phenomenology: coefficients from experiments
  - Rigorous two-component analysis: modeling approach
- Random polycrystals of (porous) laminates geomechanics
  - Bounds
  - Self-consistent estimates
- Combining these two ideas ($K^* \rightarrow$ DP)
- Examples
- Conclusions

# Mechanics of Gassmann Constituents

$$\begin{pmatrix} \delta e^{(1)} \\ -\delta\zeta^{(1)} \end{pmatrix} = \frac{1}{K^{(1)}} \begin{pmatrix} 1 & -\alpha^{(1)} \\ -\alpha^{(1)} & \alpha^{(1)}/B^{(1)} \end{pmatrix} \begin{pmatrix} -\delta p_c^{(1)} \\ -\delta p_f^{(1)} \end{pmatrix}$$

where $K^{(1)}$ is the jacketed or drained bulk modulus,

$\alpha^{(1)} = 1 - K^{(1)}/K_s^{(1)}$ is the Biot-Willis parameter,

$B^{(1)}$ is Skempton's coefficient for material 1.

Porosity is fairly uniform and, if cracks are present,
all the porosity is of that type.

A similar equation applies to other Gassmann constituents.

# Single- and Double-Porosity Geomechanics

Single-porosity:

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}.$$

Double-porosity:

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}.$$

where $\delta e$ is total volume dilatation,

$\delta\zeta$ is the increment of fluid mass content

$\delta p_c$ and $\delta p_f$ are the changes in

confining pressure and pore-fluid pressure.

# Phenomenology Example (1): Equations for Very Long Times

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} + a_{23} \\ a_{12} + a_{23} & a_{22} + a_{33} + 2a_{23} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}$$

where $\delta p_c$ is confining (total external) pressure change,

$\delta p_f = \delta p_f^{(1)} = \delta p_f^{(2)}$ is uniform

pressure change of saturating fluid at long times,

$\delta e$ is overall volume dilatation of the sample, and

$\delta\zeta$ is the increment of fluid mass content

in the whole system.

# Poromechanics of Overall System (Long Times)

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \frac{1}{K^*} \begin{pmatrix} 1 & -\alpha^* \\ -\alpha^* & \alpha^*/B^* \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}$$

where $K^*$ is the jacketed or drained overall bulk modulus,

$\alpha^* = 1 - K^*/K_s^*$ is the overall Biot-Willis parameter,

$B^*$ is overall Skempton's coefficient for the system.

All porosity is lumped together. Heterogeneity of permeability does not matter any more at long times.

A similar equation applies to all Gassmann constituents.

# LONG TIME RESULTS

$$a_{11} = 1/K^*$$

$$a_{12} + a_{23} = -\alpha^*/K^*$$

$$a_{22} + a_{33} + 2a_{23} = \alpha^*/B^*K^*$$

These equations provide three constraints on the six unknown double-porosity coefficients. Coefficients $K^*$, $\alpha^*$, $B^*$ must obviously be determined either by experiment or by using theoretical estimates.

# Phenomenology Example (2): Equations for Poromechanics of Core Sample

$$v^{(1)}\begin{pmatrix}\delta e^{(1)}\\ -\delta\zeta^{(1)}/v^{(1)}\end{pmatrix}=\begin{pmatrix}a_{11}+2a_{13}+\bar{a}_{33} & a_{12}+a_{23}\\ a_{12}+a_{23} & a_{22}\end{pmatrix}\begin{pmatrix}-\delta p_c^{(1)}\\ -\delta p_f^{(1)}\end{pmatrix}$$

where $\delta p_c^{(1)}$ is confining (external) pressure change on core sample,

$\delta p_f^{(1)}$ is pressure change of saturating fluid in matrix,

$v^{(1)} \simeq 1$ is volume fraction of matrix in total,

$\delta e^{(1)}$ is volume dilatation of core sample, and

$\delta\zeta^{(1)}$ is the increment of fluid mass content in the matrix.

# Identifying Coefficients for Core Sample

$$\begin{pmatrix} \delta e^{(1)} \\ -\delta\zeta^{(1)} \end{pmatrix} = \frac{1}{K^{(1)}} \begin{pmatrix} 1 & -\alpha^{(1)} \\ -\alpha^{(1)} & \alpha^{(1)}/B^{(1)} \end{pmatrix} \begin{pmatrix} -\delta p_c^{(1)} \\ -\delta p_f^{(1)} \end{pmatrix}$$

where $K^{(1)}$ is the jacketed or drained bulk modulus,

$\alpha^{(1)} = 1 - K^{(1)}/K_s^{(1)}$ is the Biot-Willis parameter,

$B^{(1)}$ is Skempton's coefficient for matrix.

Each constant is measured for rock matrix with no large-scale fractures (cores are too small).

# FORMULAS FOR DP COEFFICIENTS FROM PHENOMENOLOGY

$$a_{11} = 1/K^*$$

$$a_{12} = -\alpha^{(1)}K_s^{(1)}/K^{(1)}K_s^*$$

$$a_{13} = -a_{12} - \alpha^*/K^*$$

$$a_{22} = v^{(1)}\alpha^{(1)}/B^{(1)}K^{(1)}$$

$$a_{23} = -a_{12} - v^{(1)}\alpha^{(1)}/K^{(1)}$$

$$a_{33} = v^{(2)}/K_f + v^{(1)}/K^{(1)} - 2a_{13} - 1/K^*$$

where $v^{(1)}$ is volume fraction of the matrix material, and

$v^{(2)}$ is volume fraction of the fracture: $v^{(1)} + v^{(2)} = 1$.

$K_f$ is pore fluid bulk modulus.

References: Berryman and Wang (1995); Berryman and Pride (2002)

# Phenomenology Versus Two-Component Model

- The analysis of the phenomenological approach is harder than that of the second approach we outline next, but it is also more closely tied to experiments on real systems.

- The basic concepts used in both approaches are the same, and the key issue in both is also the same – namely that knowledge of the overall system bulk modulus is essential, but usually unknown. This fact is why we are motivated to use the theory to estimate this parameter.

# Modeling Example:
# Uniform Expansion Thought Experiment

Suppose that both $\delta e = \delta e^{(1)} = \delta e^{(2)}$ (uniform strain),
and $\delta p_c = \delta p_c^{(1)} = \delta p_c^{(2)}$ (uniform confining pressure) hold.

If these conditions can all be met simultaneously, then **the system behaves self-similarly**, and the effective constants of the system can be determined algebraically.

The condition of uniform strain implies that

$$-\frac{1}{K^{(1)}}(\delta p_c - \alpha^{(1)}\delta p_f^{(1)}) = -\frac{1}{K^{(2)}}(\delta p_c - \alpha^{(2)}\delta p_f^{(2)}).$$

Thus, when $\delta p_c$ and either of the two $\delta p_f$'s is specified, the other $\delta p_f$ is then determined.

# DOUBLE-POROSITY APPLICATIONS (1)

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix},$$

where

$$a_{11} = \frac{1}{K^*},$$

$$a_{22} = \frac{v^{(1)}\alpha^{(1)}}{K^{(1)}}\left(\frac{1}{B^{(1)}} - \frac{\alpha^{(1)}(1-Q_1)}{1-K^{(1)}/K^{(2)}}\right),$$

$$a_{12} = -\frac{v^{(1)}Q_1}{K^{(1)}}\alpha^{(1)},$$

$$a_{23} = \frac{\alpha^{(1)}\alpha^{(2)}K^{1)}K^{(2)}}{[K^{(2)}-K^{(1)}]^2}\left[\frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*}\right],$$

# DOUBLE-POROSITY APPLICATIONS (2)

and where

$$v^{(1)}Q_1 = \frac{1-K^{(2)}/K^*}{1-K^{(2)}/K^{(1)}}.$$

The remaining coefficients can be found using phase-interchange symmetry.

# DOUBLE-POROSITY APPLICATIONS (3)

$$a_{11} = \frac{1}{K^*} \quad \text{and} \quad D = \frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*}$$

$$a_{12} = -\frac{\alpha^{(1)}}{K^{(1)}} \frac{1 - K^{(2)}/K^*}{1 - K^{(2)}/K^{(1)}}$$

$$a_{13} = -\frac{\alpha^{(2)}}{K^{(2)}} \frac{1 - K^{(1)}/K^*}{1 - K^{(1)}/K^{(2)}}$$

$$a_{22} = \frac{v^{(1)}\alpha^{(1)}}{B^{(1)}K^{(1)}} - \left(\frac{\alpha^{(1)}}{1 - K^{(1)}/K^{(2)}}\right)^2 D$$

$$a_{23} = \frac{K^{(1)}K^{(2)}\alpha^{(1)}\alpha^{(2)}}{(K^{(2)} - K^{(1)})^2} D$$

# DOUBLE-POROSITY APPLICATIONS (4)

The main point to observe is that the double-porosity coefficients are now completely determined except that we do not necessarily know the value of the overall drained bulk modulus $K^*$ that appears in all the formulas. So, it would clearly be advantageous (if we have the choice, as we do here) to choose a microstructure for which we can obtain good estimates or formulas for $K^*$. This choice does not affect the preceding formulas as their form is independent of the microstructure! But of course their value depends on the microstructure entirely through the value of $K^*$.

# Random Polycrystals of Laminates (1)

- Assume building blocks (crystalline grains) composed of layers

  ○ Use Backus averaging scheme to compute effective properties of these grains

  ○ Use Hashin-Shtrikman bounds based on layer properties to estimate behavior using only volume fraction and layer property information

# Random Polycrystals of Laminates (2)

- Assume also that the grains are equi-axed: when all grains are considered, the axis of anisotropic grain symmetry due to the layering has no preferred direction

  ◦ Use bounds based on these “anisotropic crystals” to estimate overall behavior of the resulting random polycrystal

  ◦ Use self-consistent method to provide one type of direct estimate of the overall behavior

# Random Polycrystals of Laminates (3)

- For poroelasticity, we also have two kinds of exact results:
  - ◦ If layers are poroelastic (Gassmann – i.e., microhomogeneous) materials, then with just two types of layers exact results are available for Biot-Willis parameter and Skempton's coefficient.

  - ◦ If, in addition, the permeability of these two types of layers are very different, then double-porosity modeling can also be pursued and this also gives exact results for two components.

  - ◦ The exact results do not predict the drained constants, but the random polycrystals of laminates model gives very close bounds.

# Uniaxial Shear Energy per Unit Volume and the Product Formula

For an applied uniaxial shear *strain* applied along thesymmetry axis

i.e., $(e_{11}, e_{22}, e_{33}) = (1, 1, -2)/\sqrt{6}$

$$G^v_{eff} \equiv (c_{11} + c_{33} - 2c_{13} - c_{66})/3$$

For an applied uniaxial shear *stress* applied along thesymmetry axis

i.e., $(\sigma_{11}, \sigma_{22}, \sigma_{33}) = (1, 1, -2)/\sqrt{6}$

$$G^r_{eff} \equiv K_{Reuss} G^v_{eff} / K_{Voigt}.$$

The latter expression is the product formula, relating the shear energies per unit volume to Voigt and Reuss bounds on $K$.

# BOUNDS ON K FOR POLYCRYSTALS

Hashin-Shtrikman-type bounds for elastic constants of isotropic random polycrystals are known, given first by Peselnick and Meister (1965), later improved by Watt and Peselnick (1980).

The bounds for the bulk modulus can be expressed in terms of these uniaxial shear energies per unit volume as

$$K_{PM}^{\pm} = K_V \frac{G_{eff}^r + \zeta_\pm}{G_{eff}^v + \zeta_\pm}$$

where

$$\zeta_\pm = \frac{G_\pm}{6} \left( \frac{9K_\pm + 8G_\pm}{K_\pm + 2G_\pm} \right).$$

Parameters $G_\pm$, $K_\pm$ were defined by Watt and Peselnick.

# BOUNDS ON G FOR POLYCRYSTALS

The bounds on shear modulus can be expressed similarly as

$$\frac{5}{G^{\pm}_{PM}+\zeta_{\pm}} = \frac{1-X_{\pm}}{G^{v}_{eff}+\zeta_{\pm}+Y_{\pm}} + \frac{2}{c_{44}+\zeta_{\pm}} + \frac{2}{c_{66}+\zeta_{\pm}}$$

where $X_{\pm}$ and $Y_{\pm}$ are additional parameters depending on $G_{\pm}$ and $K_{\pm}$.

Note that in both cases when $\zeta_{-} \to 0$ the bounds go to the Reuss average (lower bound), and when $\zeta_{+} \to \infty$ the bounds go to the Voigt average (upper bound). For example,

$$K^{-}_{PM} \to K_{V} G^{r}_{eff}/G^{v}_{eff} \equiv K_{R}$$

from the product formulas.

# SELF-CONSISTENT MODULI $K^*_{SC}$ AND $G^*_{SC}$

Self-consistent estimates are obtained (approximately) by taking $K_\pm \rightarrow K^*_{SC}$ and $G_\pm \rightarrow G^*_{SC}$. The resulting formulas are:

$$K^*_{SC} = K_V \frac{G^r_{eff}+\zeta^*_{SC}}{G^v_{eff}+\zeta^*_{SC}}$$

where

$$\zeta^*_{SC} = \frac{G^*_{SC}}{6}\left(\frac{9K^*_{SC}+8G^*_{SC}}{K^*_{SC}+2G^*_{SC}}\right),$$

and

$$\frac{5}{G^*_{SC}+\zeta^*_{SC}} = \frac{1-X^*_{SC}}{G^v_{eff}+\zeta^*_{SC}} + \frac{2}{c_{44}+\zeta^*_{SC}} + \frac{2}{c_{66}+\zeta^*_{SC}}.$$

These formulas can then be used together with the bounds to compute bounds and estimates on the double-porosity coefficients.

# CONCLUSIONS

- Double-porosity coefficients can be determined exactly when the medium is composed of two Gassmann porous media (i.e., microhomogeneous constituents), but the overall bulk modulus $K^*$ must be determined to make the formulas useful.
- The random polycrystals of laminates model imposes a microstructure for which very good estimates (both bounds and self-consistent-type estimates) are available.
- Numerical results show that the formulas obtained should be very useful for benchmarking double-porosity geomechanics codes.

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