McMat2005 JOINT CONFERENCE ON MECHANICS AND MATERIALS

Baton Rouge, LA, June 1–3, 2005

BOUNDS ON TRANSPORT COEFFICIENTS OF POROUS MEDIA

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REFERENCES

- J. G. Berryman, "Thermal conductivity of porous media," *Appl. Phys. Lett* **86**, 032905-1–3 (2005).
- JGB, "Bounds and estimates for transport coefficients of random and porous media with high contrasts," *J. Appl. Phys.* **97**, 063504-1–11 (2005).
- JGB, "Measures of microstructure to improve estimates and bounds on elastic constants and transport coefficients in heterogeneous media," *Mech. Materials*, under review.

OUTLINE

- Electrical and Thermal Conductivity
  - Electrical formation factor F_e
  - Thermal formation factor F_t
- Estimates and Bounds on Transport Coefficients
  - Estimates of permeability using F_e
  - Bounds on σ and κ using F_e and F_t
  - Comparisons to Hashin-Shtrikman and Bergman bounds
- Conclusions

Formation Factor and Tortuosity

For electrical conductivity of a porous medium, the formation factor F_e is defined as

$$F_e = \frac{\sigma_0}{\sigma^*} > 1,$$

where σ_0 is the conductivity of the pore fluid and σ^* is the overall conductivity of the sample.

The conductivity of the solid grains does not appear here because the grains are essentially electrical insulators.

If the porosity is ϕ, then formation factor is related to electrical tortuosity τ_e by

$$\tau_e = \phi F_e \geq 1.$$

Formation Factor and Thermal Conductivity

Note that it is also possible to define a second formation factor for the same porous medium when air is in the pores. This formation factor is significant for thermal conductivity.

Since the grains are strong heat conductors but air is not,

$$F_t = \frac{\kappa_0}{\kappa^*} > 1,$$

where κ_0 is the thermal conductivity of the grains and κ^* is the overall thermal conductivity of the sample.

It is also possible to define a thermal tortuosity τ_t for thermal conductivity, related to this formation factor by

$$\tau_t = (1 - \phi) F_t \geq 1.$$

Fluid Permeability and F_e

Steve Blair and I showed many years ago (1987) that a useful (Kozeny-Carman) formula relating fluid permeability to formation factor and other information contained in two-point correlation functions $S(r)$ is:

$$k \simeq \frac{\phi^2}{2F_e\bar{s}^2},$$

where k is the fluid permeability, F_e is the electrical formation factor, $\phi = S(0)$ is the porosity, and $\bar{s}$ is a smoothed approximation of the specific surface area — not difficult to obtain from the slope of the correlation function for small lag, since $S'(0) = -s/4$.

Fluid Permeability and F_e (continued)

The approach is simple to apply when all the data it requires are available, and it gives good estimates of the permeability. Since permeability ranges over many orders of magnitude, "good" in this context means within about a factor of two — often better than field measurements using other means.

See reference: S. C. Blair, P. A. Berge, and J. G. Berryman, "Using two-point correlation functions to characterize microgeometry and estimate permeabilities of sandstones and porous glass," *J. Geophys. Res.* **101**, 20359–20375 (1996).

Recent Confirmation of these Results

This work has been reassessed recently (about 15 years after the original work!) by a group working at Imperial College London and they concluded also that this method gives very good results.

Recent reference:

P. A. Lock, X. D. Jing, R. W. Zimmerman, and E. M. Schlueter, "Predicting the permeability of sandstone from image analysis of pore structure," *J. Appl. Phys.* **92**, 6311-6319 (2002).

Formation Factor Bounds

There is a well-known method in composites theory called the Bergman-Milton analytic continuation method, based on rigorous general formulas for the analytic structure that the conductivity function must have. One relevant form is

$$\kappa^* = \frac{\kappa_1}{F_1} + \frac{\kappa_2}{F_2} + \int_0^1 \frac{dx\mathcal{A}(x)}{x/\kappa_1 + (1-x)/\kappa_2},$$

where F_1 and F_2 might be, respectively, the electrical and thermal formation factors of a porous medium, and $\mathcal{A}(x)$ is a real valued density functional.

Formation Factor Bounds (2)

Considering the formula,

$$\kappa^* = \frac{\kappa_1}{F_1} + \frac{\kappa_2}{F_2} + \int_0^1 \frac{dx\mathcal{A}(x)}{x/\kappa_1 + (1-x)/\kappa_2}$$

we note that the first two terms on the right hand side are just conduction in parallel, while the integral is basically capturing all of the conduction in series throughout the random medium.

Formation Factor Bounds (3)

Such formulas are often employed for complex constants such as the complex dielectric constant. But the formula is also valid and useful when κ_1 and κ_2, and thus κ^*, are all real valued.

It is "not difficult to show" from the analytic form of κ^* that it must satisfy the following nontrivial bounds:

$$min(L_1, L_2) \leq \kappa^*(\kappa_1, \kappa_2) \leq max(L_1, L_2),$$

where L_1 and L_2 are defined by

$$L_1(\kappa_1, \kappa_2) = \kappa_2 + (\kappa_1 - \kappa_2)/F_1$$

and

$$L_2(\kappa_1, \kappa_2) = \kappa_1 + (\kappa_2 - \kappa_1)/F_2.$$

Formation Factor Bounds (4)

If one of the κ_i's stays constant while the other varies (for example, when the pore fluid changes while the grain matrix remains the same), these expressions are straight lines, crossing when

$$\kappa_1 = \kappa_2.$$

One expression is the upper bound, and the other the lower bound, but it depends on which side of the crossing point we are considering.

EXAMPLES

- Formation factor bounds, Hashin-Strikman bounds, and thermal conductivity data from Asaad (1955): 3 examples.

- Fluid permeability bounds and numerical simulation data from Warren and Price (1961).

- Random polycrystals of laminates model and comparisons to various theoretical estimates and bounds.

CONCLUSIONS

- Image processing estimates using both correlation functions and formation factor information provide very good, quantitative estimates of permeability
- Formation factor (FF) bounds provide very useful lower bounds on permeability, thermal and electrical conductivity
- FF bounds give very simple formulas
- FF bounds use data different from data used in Hashin-Shtrikman bounds and because of this give better lower bounds
- Another method by Bergman uses both volume fraction and FF data, and serves as an interpolation scheme between HS and FF bounds.

ACKNOWLEDGMENT

This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under contract No. W-7405-ENG-48 and supported specifically by the Geosciences Research Program of the DOE Office of Energy Research within the Office of Basic Energy Sciences, Division of Chemical Sciences, Geosciences and Biosciences. All support of the work is gratefully acknowledged.