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**GEOMECHANICS OF
FRACTURED RESERVOIRS**

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REFERENCES – RECENT PAPERS

- J. G. Berryman and V. Grechka, “Random polycrystals of grains containing cracks: Model of quasistatic elastic behavior for fractured systems,”
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OUTLINE

- Anisotropy Due to Fractures
 - Sayers-Kachanov method
 - Thomsen parameters and Rayleigh waves
- Deconstruction
 - Seismic reflection data
 - Surface wave data
- Gassmann's Equations and Mavko-Jizba
- Higher Crack Densities
 - Higher order Sayers-Kachanov η 's
 - Thomsen weak anisotropy parameters
- Discussion and Conclusions

STIFFNESS VERSUS COMPLIANCE

The isotropic stiffness matrix (inverse of the compliance matrix) for an elastic material is often written as $C =$

$$\begin{pmatrix} \lambda + 2G & \lambda & \lambda & & & & \\ \lambda & \lambda + 2G & \lambda & & & & \\ \lambda & \lambda & \lambda + 2G & & & & \\ & & & G & & & \\ & & & & G & & \\ & & & & & G & \end{pmatrix}$$

where λ and G are the two Lamé parameters,

G is shear modulus, and $K = \lambda + 2G/3$ is bulk modulus.

ANISOTROPY DUE TO FRACTURES (1)

The isotropic compliance matrix (inverse of the stiffness matrix) for an elastic material is often written as $S =$

$$\begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & & & & \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & & & & \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & & & & \\ & & & \frac{1}{G} & & & \\ & & & & \frac{1}{G} & & \\ & & & & & \frac{1}{G} & \end{pmatrix}$$

where E is Young's modulus, ν is Poisson's ratio, and $G = E/2(1 + \nu)$ is the shear modulus.

ANISOTROPY DUE TO FRACTURES (2)

We assume all the fractures/cracks are in the form of penny-shaped cracks. These are defined as oblate spheroids, being round/flat holes, circular in cross-section with area $A = \pi b^2$, and volume $V_c = 4\pi ab^2/3$, or $V_c = 4\pi\alpha b^3/3$, where $\alpha \equiv a/b$ is the aspect ratio. We assume $\alpha = a/b \ll 1$, so the crack porosity $\phi = V_c/V \ll 1$.

The crack density parameter $\rho = \rho_c = Nb^3/V$, where N/V is the number of cracks per unit volume, and b is the radius of the (assumed) penny-shaped crack.

ANISOTROPY DUE TO FRACTURES (3)

Sayers and Kachanov (1991) show that corrections to the isotropic matrix S_{ij} , caused by low crack densities ($\rho \ll 1$), can be written as

$$\Delta \left(\frac{1}{G} \right) = 4\eta_2 \rho / 3,$$

$$\Delta \left(-\frac{\nu}{E} \right) = 2\eta_1 \rho / 3,$$

$$\Delta \left(\frac{1}{E} \right) = 2(\eta_1 + \eta_2) \rho / 3,$$

where η_1 and η_2 are parameters to be found from EMT.

ANISOTROPY DUE TO FRACTURES (4)

Thus, in the isotropic case, we have

$$\Delta S_{ij} = (2\rho/3) \times \begin{pmatrix} (\eta_1 + \eta_2) & \eta_1 & \eta_1 & & & & \\ \eta_1 & (\eta_1 + \eta_2) & \eta_1 & & & & \\ \eta_1 & \eta_1 & (\eta_1 + \eta_2) & & & & \\ & & & 2\eta_2 & & & \\ & & & & 2\eta_2 & & \\ & & & & & 2\eta_2 & \\ & & & & & & \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (6)

For vertical cracks whose axis of symmetry is randomly oriented in the xy -plane, we have another anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} (\eta_1 + \eta_2) & \eta_1 & \eta_1/2 & & & & \\ \eta_1 & (\eta_1 + \eta_2) & \eta_1/2 & & & & \\ \eta_1/2 & \eta_1/2 & 0 & & & & \\ & & & \eta_2 & & & \\ & & & & \eta_2 & & \\ & & & & & 2\eta_2 & \\ & & & & & & \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (7)

Examples of the values of the η 's found from various effective medium theories are:

<u><i>EMT</i></u>	<u>η_1</u>	<u>η_2</u>
<i>NI</i>	-0.000216	0.0287
<i>DS</i>	-0.000216	0.0290
<i>CPA</i>	-0.000258	0.0290
<i>SC</i>	-.0000207	0.0290

THOMSEN PARAMETERS (1)

For the case of randomly oriented vertical fractures, two of the Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{C_{66} - C_{44}}{2C_{44}} = -\eta_2 \rho \frac{E}{4(1+\nu)} = -\eta_2 \rho \frac{G}{2}$$

$$\epsilon \equiv \frac{C_{11} - C_{33}}{2C_{33}} \simeq -\eta_2 \rho \frac{G}{1-\nu}.$$

The remaining Thomsen parameter δ , which is the one that determines the degree of anellipticity in angular dependence of the wave speeds is given exactly by $\delta = \epsilon$, which means there is no deviation from ellipticity.

THOMSEN PARAMETERS (2)

For the case of horizontal fractures, the same two Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{C_{66} - C_{44}}{2C_{44}} = \eta_2 \rho G$$

$$\epsilon \equiv \frac{C_{11} - C_{33}}{2C_{33}} \simeq \eta_2 \rho \frac{2G}{1-\nu}.$$

Note that these results both differ exactly by a factor of -2 from the previous results for randomly oriented vertical fractures. This fact can be easily understood in terms of the Sayers and Kachanov style of analysis.

RAYLEIGH WAVE SPEED

For a transversely isotropic medium with vertical axis of symmetry (which is true of both the cases described so far), the Rayleigh surface wave has a speed determined by the following equation:

$$\frac{1}{16}q^3 - \frac{1}{2}q^2 + \left(\frac{3}{2} - \frac{C_{66}}{C_{11}}\right)q + \left(\frac{C_{66}}{C_{11}} - 1\right) = 0,$$

where $q \equiv v_R^2/v_s^2$ and $v_s^2 = C_{66}/\rho_0$,

with ρ_0 being the inertial mass density of the medium.

Recall that $C_{66} = C_{44}(1 + 2\gamma)$ and that $C_{11} = C_{33}(1 + 2\epsilon)$ in terms of Thomsen parameters.

GASSMANN'S EQUATIONS (1)

$$K_u = K_d + \frac{\alpha^2}{(\alpha - \phi)/K_m + \phi/K_f},$$

where K_u is the undrained bulk modulus, K_d is the drained bulk modulus, K_m is the mineral (or solid) modulus, K_f is the pore fluid bulk modulus, ϕ is the porosity, and $\alpha = 1 - K_d/K_m$.

Rearranging into compliance form, we have

$$\frac{1}{K_u} - \frac{1}{K_d} = -\frac{\alpha}{K_d} \times \left[1 + \frac{K_d \phi}{K_f \alpha} \left(1 - \frac{K_f}{K_m} \right) \right]^{-1}.$$

Also, porosity $\phi = \frac{4\pi a}{3b} \frac{Nb^3}{V} = \frac{4\pi a}{3b} \rho$.

GASSMANN'S EQUATIONS (2)

Compliance correction matrix for fluid inclusions:

$$\Delta S_{ij} = -\gamma^{-1} \begin{pmatrix} \beta_1^2 & \beta_1\beta_2 & \beta_1\beta_3 & & & \\ \beta_1\beta_2 & \beta_2^2 & \beta_2\beta_3 & & & \\ \beta_1\beta_3 & \beta_2\beta_3 & \beta_3^2 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$$

The fluid effects (K_f) appear only in the overall factor

γ . The coefficients β_i , $i = 1, 2, 3$, satisfy a sumrule

of the form $\beta_1 + \beta_2 + \beta_3 = 1/K_d - 1/K_m \equiv \alpha/K_d$.

K_d is the drained bulk modulus. K_m is the mineral (solid)

modulus. α is the effective stress coefficient.

GASSMANN CORRECTIONS TO SAYERS-KACHANOV FORMULAS

With $\rho = Nr^3/V$ being the crack density parameter,
and η_2 being the Sayers-Kachanov parameter that
perturbs the isotropic inverse Young's modulus, we have

$$\Delta S_{ij} \simeq \frac{2\eta_2\rho}{3} \times \begin{pmatrix} (-K_f/K_m) & 0 & 0 & 0 & 0 & 0 \\ 0 & (-K_f/K_m) & 0 & 0 & 0 & 0 \\ 0 & 0 & (-K_f/K_m) & 0 & 0 & 0 \\ 0 & 0 & 0 & (-K_f/K_m) & 0 & 0 \\ 0 & 0 & 0 & 0 & (-K_f/K_m) & 0 \\ 0 & 0 & 0 & 0 & 0 & (-K_f/K_m) \end{pmatrix}$$

MAVKO-JIZBA REDERIVED

It is now easy to show that

$$\frac{1}{K_u} - \frac{1}{K_d} \equiv \Delta \left(\frac{1}{K_u} \right) = 2\eta_2\rho(-K_f/K_m)$$

and

$$\frac{1}{G_u} - \frac{1}{G_d} \equiv \Delta \left(\frac{1}{G_u} \right) = \frac{2}{5} \left(\frac{1}{G_{eff}^r} \right) + \frac{3}{5} \times 0$$

where

$$\Delta \left(\frac{1}{G_{eff}^r} \right) = 4\eta_2\rho(-K_f/K_m)/3$$

So

$$\Delta \left(\frac{1}{G_u} \right) = \frac{4}{15} \Delta \left(\frac{1}{K_u} \right).$$

CRACK-INFLUENCE PARAMETERS

Examples of the values of the η 's found from simulation results supplied by V. Grechka are:

<u>Parameter</u>	<u>$\nu_0 = 0.00$</u>	<u>$\nu_0 = 0.4375$</u>
η_1	0.0000	-0.0192
η_2	0.1941	0.3994
η_3	-0.3666	-1.3750
η_4	0.0000	0.0000
η_5	-0.0917	0.5500

THOMSEN WEAK ANISOTROPY FORMULAS

$$v_p(\theta) = v_p(0) (1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta),$$

$$v_{sv}(\theta) = v_s(0) (1 + (c_{33}/c_{44})(\epsilon - \delta) \sin^2 \theta \cos^2 \theta),$$

$$v_{sh}(\theta) = v_s(0) (1 + \gamma \sin^2 \theta).$$

$$\delta = \left(\frac{c_{13} + c_{33}}{2c_{33}} \right) \left(\frac{c_{13} + 2c_{44} - c_{33}}{c_{33} - c_{44}} \right)$$

EXAMPLES OF SEISMIC VELOCITIES

Series of examples have been computed using the “empirical” crack influence parameters.

CONCLUSIONS

- The Sayers and Kachanov (1991) approach has some powerful advantages for both forward and inverse modeling in fractured systems, especially when used in conjunction with measured Thomsen parameters.
- For a given material (*e.g.*, quartz) and a given crack shape (*e.g.*, penny-shaped), η_1 and η_2 can be computed once and for all.
- Measured Thomsen parameters can then be inverted for crack density.
- Incorporating fluid dependence rigorously into this problem is easy using the Sayers-Kachanov parameters.

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