

**SEG 2009 – Annual Meeting**  
**Special Session: Michael Schoenberg**  
**Houston, TX, October 27, 2009**

**SCHOENBERG'S ANGLE ON FRACTURES AND  
ANISOTROPY: A STUDY IN ORTHOTROPY**

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# OUTLINE

- Fracture Analysis & Seismic Wave Speeds
  - Stiffness versus compliance
  - Anisotropy due to fractures
- Two Fracture Modeling Methods
  - Schoenberg (1980) – linear slip interface
  - Kachanov (1980) – damage (frac. influ.) factors
  - Sayers and Kachanov (1995) – applications
- Vertical Fractures in VTI Earth Media
  - Schoenberg & Sayers (1991)
  - Schoenberg & Helbig (1997)
- Vanishing of Anellipticity Parameters
- Discussion and Conclusions

# FRACTURE ANALYSIS & SEISMIC VELOCITY

Seismologists usually consider elastic stiffnesses ( $c_{ij}$ 's) when computing seismic wave speeds. For example the wave speed in the  $x_1$ - $x_3$  plane for an anisotropic system is determined by the dispersion relation:

$$2\rho\omega_{\pm}^2 \equiv \left[ (c_{11} + c_{55})k_1^2 + (c_{33} + c_{55})k_3^2 \right. \\ \left. \pm \sqrt{[(c_{11} - c_{55})k_1^2 - (c_{33} - c_{55})k_3^2]^2 + 4(c_{13} + c_{55})^2 k_1^2 k_3^2} \right],$$

where the two pertinent wave speeds are then given by

$$V_{\pm}^2 \equiv \frac{\omega_{\pm}^2}{k^2}.$$

## SEISMIC WAVE SPEED (continued)

The factor  $k^2 = k_1^2 + k_3^2$  is the wavenumber squared, where directional components of the wavenumber are given respectively by

$$k_1 = k \sin \theta \text{ and } k_3 = k \cos \theta.$$

The angle  $\theta$  of wave propagation is measured relative to the vertical ( $\theta = 0^\circ$ ), and the inertial density (assumed uniform) of the earth itself is  $\rho$ .

## STIFFNESS VERSUS COMPLIANCE

The isotropic stiffness matrix (inverse of the compliance matrix) for an elastic material is often written as  $\mathbf{C} =$

$$\begin{pmatrix} \lambda + 2G & \lambda & \lambda & & & \\ \lambda & \lambda + 2G & \lambda & & & \\ \lambda & \lambda & \lambda + 2G & & & \\ & & & G & & \\ & & & & G & \\ & & & & & G \end{pmatrix}$$

where  $\lambda$  and  $G$  are the two Lamé parameters,  $G$  is shear modulus, and  $K = \lambda + 2G/3$  is bulk modulus.

Wave speed squared is proportional to stiffness/density.

## ANISOTROPY DUE TO FRACTURES (1)

The isotropic compliance matrix (inverse of the stiffness matrix) for an elastic material is often written as  $\mathbf{S} =$

$$\begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & & & \\ & & & \frac{1}{G} & & \\ & & & & \frac{1}{G} & \\ & & & & & \frac{1}{G} \end{pmatrix}$$

where  $E$  is Young's modulus,  $\nu$  is Poisson's ratio, and  $G = E/2(1 + \nu)$  is the shear modulus.

## ANISOTROPY DUE TO FRACTURES (2)

We assume all the fractures are treated adequately as penny-shaped holes in the rock. These are commonly modelled as oblate spheroids, being round/flat holes, circular in cross-section with area  $A = \pi b^2$ , and volume  $V_f = 4\pi ab^2/3$ , or  $V_f = 4\pi\alpha b^3/3$ , where  $\alpha \equiv a/b$  is the aspect ratio. We assume  $\alpha = a/b \ll 1$ , so the fracture porosity  $\phi = V_f/V \ll 1$ .

The fracture density  $\rho_f \equiv Nb^3/V$ , where  $N/V$  is the number of cracks per unit volume, and  $b$  is radius of (here assumed) penny-shaped fractures.

## ANISOTROPY DUE TO FRACTURES (3)

Sayers and Kachanov (1991) show that corrections to the isotropic compliance  $S_{ij}$ , caused by lower fracture densities ( $\rho_f \ll 1$ ), are given by the changes:

$$\Delta_f \left( \frac{1}{G} \right) = 4\eta_2 \rho_f / 3,$$

$$\Delta_f \left( -\frac{\nu}{E} \right) = 2\eta_1 \rho_f / 3,$$

$$\Delta_f \left( \frac{1}{E} \right) = 2(\eta_1 + \eta_2) \rho_f / 3,$$

where  $\eta_1$  and  $\eta_2$  are known from experiment or EMT ( *i.e.*, Effective Medium Theories  $\simeq$  CPA, DEM, NI).

## ANISOTROPY DUE TO FRACTURES (4a)

Thus, in case of isotropic fracture distribution, we have

$$\Delta_f S_{ij} = (2\rho_f/3) \times \begin{pmatrix} (\eta_1 + \eta_2) & \eta_1 & \eta_1 & & & & \\ \eta_1 & (\eta_1 + \eta_2) & \eta_1 & & & & \\ \eta_1 & \eta_1 & (\eta_1 + \eta_2) & & & & \\ & & & 2\eta_2 & & & \\ & & & & 2\eta_2 & & \\ & & & & & 2\eta_2 & \\ & & & & & & 2\eta_2 \end{pmatrix}.$$

## ANISOTROPY DUE TO FRACTURES (4b)

By considering the isotropic problem for randomly oriented cracks in the **NIA = Non-Interaction Approximation**, we can understand the significance of the two main crack-influence parameters:

$$\eta_2 = \frac{8(1-\nu_0)(5-\nu_0)}{15G_0(2-\nu_0)} \quad \text{and} \quad \eta_1 = -\frac{4\nu_0(1-\nu_0)}{15G_0(2-\nu_0)}.$$

The ratio of these two is therefore

$$\eta_1/\eta_2 = -\frac{\nu_0}{2(5-\nu_0)}$$

Typically  $0 \leq \nu_0 \leq 0.5$ , and therefore we find that

$$|\eta_1/\eta_2| \leq 0.05,$$

so the magnitude of  $\eta_1$  is less than about 5% of  $\eta_2$ .

## ANISOTROPY DUE TO FRACTURES (5)

For horizontal fractures only, we get an anisotropic medium whose correction matrix is of the polar or uniaxial form:

$$\Delta_f S_{ij} = \rho_f \times$$

$$\begin{pmatrix} 0 & 0 & \eta_1 & & & & \\ 0 & 0 & \eta_1 & & & & \\ \eta_1 & \eta_1 & 2(\eta_1 + \eta_2) & & & & \\ & & & 2\eta_2 & & & \\ & & & & 2\eta_2 & & \\ & & & & & 0 & \end{pmatrix}.$$

## ANISOTROPY DUE TO FRACTURES (6)

For vertical fractures whose axis of symmetry is randomly oriented in the  $xy$ -plane, we have another anisotropic medium whose correction matrix is

$$\Delta_f S_{ij} = \rho_f \times$$

$$\begin{pmatrix} (\eta_1 + \eta_2) & \eta_1 & \eta_1/2 & & & & \\ \eta_1 & (\eta_1 + \eta_2) & \eta_1/2 & & & & \\ \eta_1/2 & \eta_1/2 & 0 & & & & \\ & & & \eta_2 & & & \\ & & & & \eta_2 & & \\ & & & & & 2\eta_2 & \\ & & & & & & \end{pmatrix}.$$

## ANISOTROPY DUE TO FRACTURES (7)

Examples of the values of the  $\eta$ 's (in units of  $\text{GPa}^{-1}$ ) found (for one particular choice of background medium) using various effective medium theories are:

<u><i>EMT</i></u>	<u><math>\eta_1</math></u>	<u><math>\eta_2</math></u>
<i>NI</i>	-0.000216	0.0287
<i>DS</i>	-0.000216	0.0290
<i>CPA</i>	-0.000258	0.0290
<i>SC</i>	-.0000207	0.0290

Note the important fact that  $|\eta_1| \ll \eta_2$ , in all cases, showing it is often sufficient to use the single  $\eta_2 \neq 0$ .

## THOMSEN PARAMETERS (1)

For the case of randomly oriented vertical fractures, two of the Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{c_{66} - c_{44}}{2c_{44}} = -\eta_2 \rho_f \frac{E}{4(1+\nu)} = -\eta_2 \rho_f \frac{G}{2}$$

$$\epsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}} \simeq -\eta_2 \rho_f \frac{G}{1-\nu}.$$

The remaining Thomsen parameter  $\delta$ , which is the one that determines the degree of anellipticity in angular dependence of the wave speeds is given exactly by  $\delta = \epsilon$ , which means there is no deviation from ellipticity.

## THOMSEN PARAMETERS (2)

For the case of horizontal fractures, the same two Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{c_{66} - c_{44}}{2c_{44}} = \eta_2 \rho_f G$$

$$\epsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}} \simeq \eta_2 \rho_f \frac{2G}{1-\nu}.$$

Note that these results both differ exactly by a factor of -2 from previous results for randomly oriented vertical fractures. This fact can be easily understood in terms of the Sayers and Kachanov style of analysis.

## FRACTURE-INFLUENCE PARAMETERS

Examples of the values of the  $\eta$ 's found from simulation results supplied by V. Grechka (including higher order  $\eta$ 's than those discussed here) are:

<u><i>Parameter</i></u>	<u><math>\nu_0 = 0.00</math></u>	<u><math>\nu_0 = 0.4375</math></u>
$\eta_1$	0.0000	-0.0192
$\eta_2$	0.1941	0.3994
$\eta_3$	-0.3666	-1.3750
$\eta_4$	0.0000	0.0000
$\eta_5$	-0.0917	0.5500

## THOMSEN WEAK ANISOTROPY FORMULAS

$$v_p(\theta) = v_p(0) (1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta),$$

$$v_{sv}(\theta) = v_s(0) (1 + (c_{33}/c_{44})(\epsilon - \delta) \sin^2 \theta \cos^2 \theta),$$

$$v_{sh}(\theta) = v_s(0) (1 + \gamma \sin^2 \theta).$$

$$\epsilon \equiv \frac{c_{11} - c_{33}}{2c_{33}}$$

$$\delta \equiv \left( \frac{c_{13} + c_{33}}{2c_{33}} \right) \left( \frac{c_{13} + 2c_{44} - c_{33}}{c_{33} - c_{44}} \right)$$

$$\gamma \equiv \frac{c_{66} - c_{44}}{2c_{44}}$$

## COMPLIANCE CORRECTION FACTORS

Compliance **S** change factors for vertical fractures:

$$A = \rho_f + [\rho_f^2 - 4\rho_a\rho_b \sin^2 \phi]^{1/2},$$

$$B = \rho_f - [\rho_f^2 - 4\rho_a\rho_b \sin^2 \phi]^{1/2},$$

when the angle between two fracture sets is  $\phi$ .

Special case when  $\rho_a = \rho_b = \rho_f/2$  gives the case

$$A = \rho_f(1 + \cos \phi)$$

$$B = \rho_f(1 - \cos \phi)$$

considered in the examples that follow:

**Table 1**

<u>Compliance to Change</u>	<u><math>\Delta\mathbf{S}(\text{GPa}^{-1})</math></u>
$S_{11}$	$(\eta_1 + \eta_2)A$
$S_{22}$	$(\eta_1 + \eta_2)B$
$S_{33}$	0
$S_{12}$	$\eta_1\rho_f$
$S_{13}$	$\eta_1 A/2$
$S_{23}$	$\eta_1 B/2$
$S_{44}$	$\eta_2 B$
$S_{55}$	$\eta_2 A$
<u><math>S_{66}</math></u>	<u><math>2\eta_2\rho_f</math></u>

# MODEL RESERVOIR FRACTURE PARAMETERS

Fracture-influence parameters  $\eta$  used in a model reservoir having isotropic background with Poisson's ratio  $\nu = 0.4375$ ,  $V_p = 3.0$  km/s, and  $V_s = 1.0$  km/s.

**Table 2**

<u>Fracture Parameter</u>	<u><math>\eta(\text{GPa}^{-1})</math></u>
$\eta_1$	- 0.0192
<u><math>\eta_2</math></u>	<u>0.3944</u>

# CONSTANT $V_{sv}$ FOR FIXED FRACTURE ANGLE $\phi$

Examples of the values of the  $V_{sv}$ 's found using the formulas quoted previously:

**Table 3**

<u><math>\phi</math></u>	<u><math>V_{sv}</math> (km/s)</u>
$0^\circ$	0.8602
$30^\circ$	0.8678
$45^\circ$	0.8771
$60^\circ$	0.8896
<u><math>90^\circ</math></u>	<u>0.9222</u>

## EXAMPLES FOR SEISMIC VELOCITIES (1)

This result for quasi-SV-waves was obtained rather easily above and was also already known to Gassmann (1964) and to Schoenberg and Sayers (1995). The exact condition in terms of stiffnesses for vanishing of the anellipticity parameter is:

$$c_{11}c_{33} - c_{13}^2 = c_{55}(c_{11} + c_{33} + 2c_{13}).$$

This condition can be rewritten to first order in fracture density using compliances as:

$$\Delta S_{55} = \Delta S_{11} + \Delta S_{33} - 2\Delta S_{13}.$$

This equation is satisfied identically when the expressions in **Table 1** are substituted.

## EXAMPLES FOR SEISMIC VELOCITIES (2)

The observed equality obtains for the general forms of these definitions, and not just for a few particular examples. The curiously symmetric nature of these equations can be highlighted further by noting that two more expressions analogous to the previous one are:

$$\Delta S_{44} = \Delta S_{22} + \Delta S_{33} - 2\Delta S_{23}$$

and

$$\Delta S_{66} = \Delta S_{11} + \Delta S_{22} - 2\Delta S_{12}.$$

These two equations are also satisfied identically by the same set of equations in **Table 1**.

## EXAMPLES FOR SEISMIC VELOCITIES (3)

These results show that the model being studied has vanishing anellipticity factors in all directions for sufficiently low fracture densities.

Another result due to Schoenberg and Helbig (1997), among others, is a model of VTI earth systems — having vertical axis of symmetry in the background medium (instead of isotropic) — but also with vertical fractures superimposed. This type of model results in a different condition:

$$c_{13}(c_{22} + c_{12}) = c_{23}(c_{11} + c_{12}).$$

## EXAMPLES FOR SEISMIC VELOCITIES (4)

It is not hard to show using the same ideas presented already that this condition for the fractured medium amounts to:

$$S_{13} = S_{23},$$

which is a special case of orthotropy. The stated condition is exact, but the relation to the ideas discussed here so far is most easily seen by returning to the low fracture density results. Then, we find that the result is equivalent to:

$$\Delta S_{13} = \Delta S_{23},$$

since  $S_{13} = S_{23}$  in the unfractured background VTI medium.

## Table 4

Taking  $\eta_1 \rightarrow 0$  in Table 1.

<u>Compliance to Change</u>	<u><math>\Delta\mathbf{S}(\text{GPa}^{-1})</math></u>
$S_{11}$	$\eta_2 A$
$S_{22}$	$\eta_2 B$
$S_{33}$	0
$S_{12}$	0
$S_{13}$	0
$S_{23}$	0
$S_{44}$	$\eta_2 B$
$S_{55}$	$\eta_2 A$
<u><math>S_{66}</math></u>	<u><math>2\eta_2 \rho_f</math></u>

## EXAMPLES FOR SEISMIC VELOCITIES (5)

This result can be traced through the facts in **Table 1** but requires the additional approximation  $\eta_1 \rightarrow 0$ .

With  $\phi = 0^\circ$  and all the fractures vertical (as well as aligned), this set of constraints is exactly what Schoenberg and Helbig (1997) used.

The main point of all this analysis is to emphasize that it can be very useful to try out different viewpoints in order to understand more fully why certain elastic symmetries result in such special seismic wave propagation behavior.

## SUMMARY AND CONCLUSIONS

- Methods currently in use, including Schoenberg's method and also Sayers and Kachanov's method, for quantifying effects of fracture sets on reservoir geomechanics provide a consistent and presumably quite accurate measure of fracture influence on seismic waves.
- We concentrated here on elastic orthotropy, but this symmetry and higher symmetries such as VTI and HTI are not universal in the earth.
- Clearly, more work will be needed to reach a complete understanding of the influence of fractures and anisotropy on seismic wave speeds.

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## **ACKNOWLEDGMENT**

This work was performed under the auspices of the U.S. Department of Energy (DOE) by the University of California, Lawrence Berkeley National Laboratory under contract No. DE-AC02-05CH11231 and supported specifically by the Geosciences Research Program of the DOE Office of Basic Energy Sciences, Division of Chemical Sciences, Geosciences and Biosciences. All support of the work is gratefully acknowledged.