

DEPARTMENT SEMINAR
Earth and Planetary Sciences
Berkeley, CA, September 21, 2006

GEOMECHANICS OF FRACTURED RESERVOIRS

James G. Berryman

Earth Sciences Division

Lawrence Berkeley National Laboratory

Collaborator: Vladimir Grechka

Shell International Exploration and Prod., Houston, TX

OUTLINE

- Part I: Anisotropy Due to Fractures
 - Sayers-Kachanov (1991) method
- Part II:
 - Double-Porosity (DP) geomechanics
 - Geomechanics of random polycrystals of porous laminates
 - Combining these two ideas ($K^* \rightarrow$ DP)
- Part III: Analysis for dense systems of fractures
- Conclusions

PART I:
DEFINITION OF CRACK DENSITY ρ

A key parameter is crack density $\rho = na^3$, where a is the radius of a penny-shaped crack and $n = N/V$ is the number (N) of cracks per unit volume (V).

A penny-shaped crack is an oblate spheroidal void *i.e.*, an ellipsoid having principal axis dimensions $a \geq b \geq c$ such that $a = b$ and $c = \alpha a$, where $\alpha < 1$ is the aspect ratio. For flat cracks, α can be very small. Crack density does not depend on α .

However, porosity $\phi = 4\pi\rho\alpha/3$.

Anisotropy Due to Fractures (1)

The isotropic compliance matrix (inverse of the stiffness matrix) for an elastic material is often written as $S =$

$$\begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & & & \\ & & & \frac{1}{G} & & \\ & & & & \frac{1}{G} & \\ & & & & & \frac{1}{G} \end{pmatrix}$$

where E is Young's modulus, ν is Poisson's ratio, and $G = E/2(1 + \nu)$ is the shear modulus.

Anisotropy Due to Fractures (2)

Sayers and Kachanov (1991) show that corrections to the isotropic matrix S_{ij} , caused by low crack densities ($\rho \ll 1$), can be written as

$$\Delta \left(\frac{1}{G} \right) = 4\eta_2\rho/3,$$

$$\Delta \left(-\frac{\nu}{E} \right) = 2\eta_1\rho/3,$$

$$\Delta \left(\frac{1}{E} \right) = 2(\eta_1 + \eta_2)\rho/3,$$

where η_1 and η_2 are parameters to be found from EMT.

Anisotropy Due to Fractures (3)

Thus, in the isotropic case, we have

$$\Delta S_{ij} = (2\rho/3) \times \begin{pmatrix} (\eta_1 + \eta_2) & \eta_1 & \eta_1 & & & & \\ \eta_1 & (\eta_1 + \eta_2) & \eta_1 & & & & \\ \eta_1 & \eta_1 & (\eta_1 + \eta_2) & & & & \\ & & & 2\eta_2 & & & \\ & & & & 2\eta_2 & & \\ & & & & & 2\eta_2 & \\ & & & & & & 2\eta_2 \end{pmatrix}.$$

Anisotropy Due to Fractures (4)

For horizontal cracks, we get an anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times \begin{pmatrix} 0 & 0 & \eta_1 & & & \\ 0 & 0 & \eta_1 & & & \\ \eta_1 & \eta_1 & 2(\eta_1 + \eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 0 \end{pmatrix}.$$

Anisotropy Due to Fractures (5)

For vertical cracks whose axis of symmetry is randomly oriented in the xy -plane, we have another anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} (\eta_1 + \eta_2) & \eta_1 & \eta_1/2 & & & & \\ \eta_1 & (\eta_1 + \eta_2) & \eta_1/2 & & & & \\ \eta_1/2 & \eta_1/2 & 0 & & & & \\ & & & \eta_2 & & & \\ & & & & \eta_2 & & \\ & & & & & 2\eta_2 & \\ & & & & & & \end{pmatrix}.$$

Anisotropy Due to Fractures (6)

Examples of the values of the η 's for isotropic quartz found using various effective medium theories are:

<u><i>EMT</i></u>	<u>η_1</u>	<u>η_2</u>
<i>NI</i>	-0.000216	0.0287
<i>DS</i>	-0.000216	0.0290
<i>CPA</i>	-0.000258	0.0290
<i>SC</i>	-.0000207	0.0290

Note that $|\eta_1|/\eta_2 < 0.01$ in all cases here. So only one parameter is important in this particular case.

NIA EXAMPLES (1)

Since it makes little difference at low crack densities which theoretical method we use, we might as well consider the simplest one which is surely the non-interaction approximation (NIA).

It is well-known (e.g., see Zimmerman's book) that:

$$\frac{1}{K_{NI}} - \frac{1}{K} = \frac{\rho}{K} \frac{16(1-\nu^2)}{9(1-2\nu)}$$
$$\frac{1}{G_{NI}} - \frac{1}{G} = \frac{\rho}{G} \frac{32(1-\nu)(5-\nu)}{45(2-\nu)},$$

both obviously linear expressions in the crack density ρ .

NIA EXAMPLES (2)

But we also have

$$\eta_2 = \frac{3}{4\rho} \left(\frac{1}{G^*} - \frac{1}{G} \right)$$

and

$$\eta_1 = \frac{1}{6\rho} \left(\frac{1}{K^*} - \frac{1}{K} \right) - \frac{1}{4\rho} \left(\frac{1}{G^*} - \frac{1}{G} \right).$$

NIA EXAMPLES (3)

Thus, we find directly that

$$\eta_2 = \frac{1}{G} \frac{8(1-\nu)(5-\nu)}{15(2-\nu)}$$

and

$$\eta_1 = -\frac{1}{G} \frac{4\nu(1-\nu)}{15(2-\nu)},$$

where we also used the fact that

$$1/2G = (1 + \nu)/3K(1 - 2\nu).$$

Especially note that, in the NIA, η_1 is negative and directly proportional to Poisson's ratio ν .

PART II: Mechanics of Gassmann Constituents

$$\begin{pmatrix} \delta e^{(1)} \\ -\delta \zeta^{(1)} \end{pmatrix} = \frac{1}{K^{(1)}} \begin{pmatrix} 1 & -\alpha^{(1)} \\ -\alpha^{(1)} & \alpha^{(1)}/B^{(1)} \end{pmatrix} \begin{pmatrix} -\delta p_c^{(1)} \\ -\delta p_f^{(1)} \end{pmatrix}$$

where $K^{(1)}$ is the jacketed or drained bulk modulus,

$\alpha^{(1)} = 1 - K^{(1)}/K_s^{(1)}$ is the Biot-Willis parameter,

$B^{(1)}$ is Skempton's coefficient for material 1.

Porosity is fairly uniform and, if cracks are present,

all the porosity is of that type.

A similar equation applies to other Gassmann constituents.

Single- and Double-Porosity Geomechanics

Single-porosity:

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}.$$

Double-porosity:

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}.$$

where δe is total volume dilatation,

$\delta\zeta$ is the increment of fluid mass content

δp_c and δp_f are the changes in

confining pressure and pore-fluid pressure.

Phenomenology Example (1): Equations for Very Long Times

$$\begin{pmatrix} \delta e \\ -\delta\zeta \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} + a_{23} \\ a_{12} + a_{23} & a_{22} + a_{33} + 2a_{23} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}$$

where δp_c is confining (total external) pressure change,

$\delta p_f = \delta p_f^{(1)} = \delta p_f^{(2)}$ is uniform pressure change

of saturating fluid at long times,

δe is overall volume dilatation of the sample, and

$\delta\zeta$ is the increment of fluid mass content

in the whole system.

Poromechanics of Overall System (Long Times)

$$\begin{pmatrix} \delta e \\ -\delta \zeta \end{pmatrix} = \frac{1}{K^*} \begin{pmatrix} 1 & -\alpha^* \\ -\alpha^* & \alpha^*/B^* \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f \end{pmatrix}$$

where K^* is the drained (jacketed) overall bulk modulus,

$\alpha^* = 1 - K^*/K_s^*$ is the overall Biot-Willis parameter,

B^* is overall Skempton's coefficient for the system.

All porosity is lumped together. Heterogeneity of

permeability does not matter any more at long times.

A similar equation applies to all Gassmann constituents.

LONG TIME RESULTS

$$a_{11} = 1/K^*$$

$$a_{12} + a_{23} = -\alpha^*/K^*$$

$$a_{22} + a_{33} + 2a_{23} = \alpha^*/B^*K^*$$

These equations provide three constraints on the six unknown double-porosity coefficients. Coefficients K^* , α^* , B^* must obviously be determined either by experiment or by using theoretical estimates.

Phenomenology Example (2): Equations for Poromechanics of Core Sample

$$v^{(1)} \begin{pmatrix} \delta e^{(1)} \\ -\delta \zeta^{(1)}/v^{(1)} \end{pmatrix} = \begin{pmatrix} a_{11} + 2a_{13} + \bar{a}_{33} & a_{12} + a_{23} \\ a_{12} + a_{23} & a_{22} \end{pmatrix} \begin{pmatrix} -\delta p_c^{(1)} \\ -\delta p_f^{(1)} \end{pmatrix}$$

where

$\delta p_c^{(1)}$ is confining pressure change on core sample,

$\delta p_f^{(1)}$ is pressure change of saturating fluid in matrix,

$v^{(1)} \simeq 1$ is volume fraction of matrix in total,

$\delta e^{(1)}$ is volume dilatation of core sample, and

$\delta \zeta^{(1)}$ is increment of fluid mass content in matrix.

Identifying Coefficients for Core Sample

$$\begin{pmatrix} \delta e^{(1)} \\ -\delta \zeta^{(1)} \end{pmatrix} = \frac{1}{K^{(1)}} \begin{pmatrix} 1 & -\alpha^{(1)} \\ -\alpha^{(1)} & \alpha^{(1)}/B^{(1)} \end{pmatrix} \begin{pmatrix} -\delta p_c^{(1)} \\ -\delta p_f^{(1)} \end{pmatrix}$$

where $K^{(1)}$ is the jacketed or drained bulk modulus,

$\alpha^{(1)} = 1 - K^{(1)}/K_s^{(1)}$ is the Biot-Willis parameter,

$B^{(1)}$ is Skempton's coefficient for matrix.

Each constant is measured for rock matrix with no large-scale fractures (cores are too small).

FORMULAS FOR DP COEFFICIENTS FROM PHENOMENOLOGY

$$a_{11} = 1/K^*$$

$$a_{12} = -\alpha^{(1)} K_s^{(1)} / K^{(1)} K_s^*$$

$$a_{13} = -a_{12} - \alpha^* / K^*$$

$$a_{22} = v^{(1)} \alpha^{(1)} / B^{(1)} K^{(1)}$$

$$a_{23} = -a_{12} - v^{(1)} \alpha^{(1)} / K^{(1)}$$

$$a_{33} = v^{(2)} / K_f + v^{(1)} / K^{(1)} - 2a_{13} - 1/K^*$$

where $v^{(1)}$ is volume fraction of the matrix material, and
 $v^{(2)}$ is volume fraction of the fracture: $v^{(1)} + v^{(2)} = 1$.
 K_f is pore fluid bulk modulus.

Phenomenology Versus Two-Component Model

- The analysis of the phenomenological approach is harder than that of the second approach we outline next, but it is also more closely tied to experiments on real systems.
- Basic concepts used in both approaches are the same, and the key issue in both is also the same – namely that knowledge of overall system bulk modulus is essential, but usually unknown. Thus we are motivated to use theory to estimate this parameter.

Modeling Example:

Uniform Expansion Thought Experiment

Suppose that both $\delta e = \delta e^{(1)} = \delta e^{(2)}$ (uniform strain), and $\delta p_c = \delta p_c^{(1)} = \delta p_c^{(2)}$ (uniform confining pressure) hold.

If these conditions can all be met simultaneously, then **the system behaves self-similarly**, and the effective constants of the system can be determined algebraically.

The condition of uniform strain implies that

$$-\frac{1}{K^{(1)}}(\delta p_c - \alpha^{(1)}\delta p_f^{(1)}) = -\frac{1}{K^{(2)}}(\delta p_c - \alpha^{(2)}\delta p_f^{(2)}).$$

Thus, when δp_c and either of the two δp_f 's is specified, the other δp_f is then determined.

DOUBLE-POROSITY APPLICATIONS (1)

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix},$$

where

$$\begin{aligned} a_{11} &= \frac{1}{K^*}, \\ a_{22} &= \frac{v^{(1)}\alpha^{(1)}}{K^{(1)}} \left(\frac{1}{B^{(1)}} - \frac{\alpha^{(1)}(1-Q_1)}{1-K^{(1)}/K^{(2)}} \right), \\ a_{12} &= -\frac{v^{(1)}Q_1}{K^{(1)}}\alpha^{(1)}, \\ a_{23} &= \frac{\alpha^{(1)}\alpha^{(2)}K^{(1)}K^{(2)}}{[K^{(2)}-K^{(1)}]^2} \left[\frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*} \right], \end{aligned}$$

DOUBLE-POROSITY APPLICATIONS (2)

and where

$$v^{(1)}Q_1 = \frac{1 - K^{(2)}/K^*}{1 - K^{(2)}/K^{(1)}}.$$

The remaining coefficients can be found using phase-interchange symmetry.

DOUBLE-POROSITY APPLICATIONS (3)

$$a_{11} = \frac{1}{K^*} \quad \text{and} \quad D = \frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*}$$

$$a_{12} = -\frac{\alpha^{(1)}}{K^{(1)}} \frac{1-K^{(2)}/K^*}{1-K^{(2)}/K^{(1)}}$$

$$a_{13} = -\frac{\alpha^{(2)}}{K^{(2)}} \frac{1-K^{(1)}/K^*}{1-K^{(1)}/K^{(2)}}$$

$$a_{22} = \frac{v^{(1)}\alpha^{(1)}}{B^{(1)}K^{(1)}} - \left(\frac{\alpha^{(1)}}{1-K^{(1)}/K^{(2)}} \right)^2 D$$

$$a_{23} = \frac{K^{(1)}K^{(2)}\alpha^{(1)}\alpha^{(2)}}{(K^{(2)}-K^{(1)})^2} D$$

DOUBLE-POROSITY APPLICATIONS (4)

The main point to observe is that the double-porosity coefficients are now completely determined except that we do not necessarily know the value of the overall drained bulk modulus K^* that appears in all the formulas. So, it would clearly be advantageous (if we have the choice, as we do here) to choose a microstructure for which we can obtain good estimates or formulas for K^* . This choice does not affect the preceding formulas as their form is independent of the microstructure! But of course their value depends on the microstructure entirely through the value of K^* .

Random Polycrystals of Laminates (1)

- Assume building blocks (crystalline grains) composed of layers
 - Use Backus averaging scheme to compute effective properties of these grains
 - Use Hashin-Shtrikman bounds based on layer properties to estimate behavior using only volume fraction and layer property information

Random Polycrystals of Laminates (2)

- Assume also that the grains are equi-axed: when all grains are considered, the axis of anisotropic grain symmetry due to the layering has no preferred direction
 - Use bounds based on these “anisotropic crystals” to estimate overall behavior of the resulting random polycrystal
 - Use self-consistent method to provide one type of direct estimate of the overall behavior

Random Polycrystals of Laminates (3)

- For poroelasticity, we have two kinds of exact results:
 - If layers are poroelastic Gassmann materials (i.e., microhomogeneous), then with just two types of layers exact results are available for Biot-Willis parameter and Skempton's coefficient.
 - If, in addition, the permeability of these two types of layers are very different, the double-porosity modeling can also be pursued and this also gives exact results for two components.

Random Polycrystals of Laminates (4)

- The exact results do not predict the drained constants, but the random polycrystals of laminates model gives very close bounds.

Uniaxial Shear Energy per Unit Volume and the Product Formula

For an applied uniaxial shear *strain* applied along the symmetry axis, i.e., $(e_{11}, e_{22}, e_{33}) = (1, 1, -2)/\sqrt{6}$

$$G_{eff}^v \equiv (c_{11} + c_{33} - 2c_{13} - c_{66})/3$$

For an applied uniaxial shear *stress* applied along the symmetry axis, i.e., $(\sigma_{11}, \sigma_{22}, \sigma_{33}) = (1, 1, -2)/\sqrt{6}$

$$G_{eff}^r \equiv K_{Reuss} G_{eff}^v / K_{Voigt}.$$

The latter expression is the product formula, relating the shear energies per unit volume to Voigt and Reuss bounds on K .

BOUNDS ON K FOR POLYCRYSTALS (1)

Hashin-Shtrikman-type bounds for elastic constants of isotropic random polycrystals are known, given first by Peselnick and Meister (1965), later improved by Watt and Peselnick (1980).

The bounds for the bulk modulus can be expressed in terms of these uniaxial shear energies per unit volume as

$$K_{PM}^{\pm} = K_V \frac{G_{eff}^r + \zeta_{\pm}}{G_{eff}^v + \zeta_{\pm}}$$

BOUNDS ON K FOR POLYCRYSTALS (2)

where

$$\zeta_{\pm} = \frac{G_{\pm}}{6} \left(\frac{9K_{\pm} + 8G_{\pm}}{K_{\pm} + 2G_{\pm}} \right).$$

Parameters G_{\pm} , K_{\pm} were defined by Watt and Peselnick.

BOUNDS ON G FOR POLYCRYSTALS (1)

The bounds on shear modulus can be expressed similarly as

$$\frac{5}{G_{PM}^{\pm} + \zeta_{\pm}} = \frac{1 - X_{\pm}}{G_{eff}^v + \zeta_{\pm} + Y_{\pm}} + \frac{2}{c_{44} + \zeta_{\pm}} + \frac{2}{c_{66} + \zeta_{\pm}}$$

where X_{\pm} and Y_{\pm} are additional parameters depending on G_{\pm} and K_{\pm} .

Note that in both cases when $\zeta_{-} \rightarrow 0$ the bounds go to the Reuss average (lower bound), and when $\zeta_{+} \rightarrow \infty$ the bounds go to the Voigt average (upper bound).

BOUNDS ON G FOR POLYCRYSTALS (2)

For example,

$$K_{PM}^- \rightarrow K_V G_{eff}^r / G_{eff}^v \equiv K_R$$

from the product formulas.

SELF-CONSISTENT MODULI K_{SC}^* & G_{SC}^* (1)

Self-consistent estimates are obtained (approximately) by taking $K_{\pm} \rightarrow K_{SC}^*$ and $G_{\pm} \rightarrow G_{SC}^*$.

The resulting formulas are:

$$K_{SC}^* = K_V \frac{G_{eff}^r + \zeta_{SC}^*}{G_{eff}^v + \zeta_{SC}^*}$$

where

$$\zeta_{SC}^* = \frac{G_{SC}^*}{6} \left(\frac{9K_{SC}^* + 8G_{SC}^*}{K_{SC}^* + 2G_{SC}^*} \right),$$

and

$$\frac{5}{G_{SC}^* + \zeta_{SC}^*} = \frac{1 - X_{SC}^*}{G_{eff}^v + \zeta_{SC}^*} + \frac{2}{c_{44} + \zeta_{SC}^*} + \frac{2}{c_{66} + \zeta_{SC}^*}.$$

SELF-CONSISTENT MODULI K_{SC}^* & G_{SC}^* (2)

These formulas can then be used together with the bounds to compute bounds and estimates for the double-porosity coefficients.

PART III:
Analysis for dense systems of fractures

NUMERICAL EXAMPLES

CONCLUSIONS

- Double-porosity coefficients can be determined exactly when the medium is composed of two Gassmann porous media (i.e., microhomogeneous constituents), but the overall bulk modulus K^* must be determined to make the formulas useful.
- The random polycrystals of laminates model imposes a microstructure for which very good estimates (both bounds and self-consistent-type estimates) are available.
- Numerical results show that the formulas obtained should be very useful for benchmarking double-porosity geomechanics codes.

REFERENCES – EMT (1)

- JGB, 1995, Mixture theories for rock properties: in *American Geophysical Union Handbook of Physical Constants*, edited by T. J. Ahrens, AGU, New York, pp. 205–228.
- JGB, 2005, Bounds and self-consistent estimates for elastic constants of random polycrystals with hexagonal, trigonal, and tetragonal symmetries: *J. Mech. Phys. Solids* **53**(10), 2141–2173.
- V. Grechka, 2005, Penny-shaped fractures revisited: *Stud. Geophys. Geod.*, **49**, 365–381.

REFERENCES – EMT (2)

- V. Grechka and M. Kachanov, 2006, Effective elasticity of fractured rocks: *The Leading Edge*, **25**, 152–155.
- C. M. Sayers and M. Kachanov, 1991, A simple technique for finding effective elastic constants of cracked solids for arbitrary crack orientation statistics: *Int. J. Solids Struct.*, *27*, 671–680.
- R. W. Zimmerman, 1991, *Compressibility of Sandstones*, Elsevier, Amsterdam.

REFERENCES (3)

Double-porosity papers:

- JGB and H. F. Wang, “The elastic coefficients of double-porosity models for fluid transport in jointed rock,” *J. Geophys. Res.* **100**, 24611–24627 (1995).
- JGB and S. R. Pride, “Models for computing geomechanical constants of double-porosity materials from the constituents’ properties,”
JGR **107**(B3), 10.1029/2000JB000108 (2002).
- JGB, “Extension of poroelastic analysis to double-porosity materials,” *ASCE JEM* **128**, 840–847 (2002).

REFERENCES (4)

Random polycrystals of laminates papers:

- JGB, “Bounds on elastic constants for random polycrystals of laminates,” *J. Appl. Phys.* **96**, 4281–4287 (2004).
- JGB, “Bounds and estimates for elastic constants of random polycrystals of laminates,” *IJSS* **42** (13), 3730–3743 (2005).

REFERENCES (5)

Fractured reservoirs:

- V. Grechka and M. Kachanov, “Effective elasticity of rocks with closely spaced and intersecting cracks,” *Geophysics* **71**, D85 (2006).
- V. Grechka and M. Kachanov, “Seismic characterization of multiple fracture sets: Does orthotropy suffice?” *Geophysics* **71**, D93 (2006).
- JGB and V. Grechka, “Random polycrystals of grains containing cracks: Model of quasistatic elastic behavior for fractured systems,” *J. Appl. Phys.*, under review, 2006.

ACKNOWLEDGMENT

This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Berkeley National Laboratory under contract No. DE-AC03-76SF00098 and supported specifically by the Geosciences Research Program of the DOE Office of Energy Research within the Office of Basic Energy Sciences, Division of Chemical Sciences, Geosciences and Biosciences. All support of the work is gratefully acknowledged.