Schlumberger EMI Seminar
Richmond, CA, February 1, 2007

TIME-REVERSAL PROCESSING OF EM ARRAY DATA AND TARGET CHARACTERIZATION

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OUTLINE OF TALK

- Motivation
  - Electromagnetic imaging/detection through foliage/clutter
  - Characterization: If we do see something, what is it?
- History: Acoustic and Elastic T/R
  - Chambers and Gautesen (JASA, 2001)
- Summary of the Electromagnetic Analysis for a Small Dielectric or Conducting Sphere
- Numerical Examples
- Conclusions

TIME-REVERSAL ACOUSTICS PROBLEMS

- Liliana Borcea, Chrysoula Tsogka, and George Papanicolaou (Stanford - Math): acoustic imaging through random media using statistical stability concepts
- John Sylvester (UW - Math): more precise acoustic imaging using angular dependence of the far-field scattering operator
- Chris Jones, Darrell Jackson, and Dan Rouseff (APL - UWash): super-resolution or super-focusing (for communications) in waveguides (like the ocean) and also random media (like the turbulent ocean)

Source Array Setup

Consider an array of N short, crossed-dipole elements lying in the plane $z = -z_a$, where z_a is the distance between the plane and the scattering sphere (which is located at the origin). The position of the nth element of the array is given by $\vec{r}_n = (x_n, y_n, -z_a)$.

The standard result for the electric field at point $\vec{r}$ radiated from the nth element is given by

$$\vec{E}_n^{(i)} = \frac{ike^{ikR_n}}{4\pi\epsilon_0 c R_n} \hat{R}_n \times [\hat{R}_n \times (d_H I_n^H \hat{e}_x + d_V I_n^V \hat{e}_y)],$$

where c is the speed of light, k is the wavenumber, ϵ_0 is the electrical permittivity, and $\vec{R}_n = \vec{r} - \vec{r}_n$.

The scalar $R_n = |\vec{R}_n|$ is the vector's magnitude.

Source Array Setup (continued)

The horizontal and vertical dipoles in the element (having lengths d_H and d_V) are driven by the currents I_n^H and I_n^V, respectively). The horizontal dipole is oriented parallel to the x-axis and the vertical dipole parallel to the y-axis.

There is also a magnetic field radiated from the nth element, which is given similarly by

$$\vec{H}_n^{(i)} = \frac{ike^{ikR_n}}{4\pi\epsilon_0 c R_n}[\hat{R}_n \times (d_H I_n^H \hat{e}_x + d_V I_n^V \hat{e}_y)].$$

Scattered Field

With a sphere of radius $a << z_a$ at the origin and a also much smaller than the wavelength, the scattered field to leading order is given by

$$\vec{E}^{(s)} = -\frac{k^2 e^{ikr}}{r}[\hat{r} \times (\vec{m} + \hat{r} \times \vec{p})].$$

The induced electric dipole moment is $\vec{p}$ and the induced magnetic dipole moment is $\vec{m}$. These moments are generated at the sphere as if a plane wave were incident at this distance.

Scattered Field (continued)

The moments are related to the incident field evaluated at the position of the sphere $\vec{r} = 0$:

$$\vec{m} = -m_0 \hat{r}_n \times \vec{E}_n^{(i)}(-\vec{r}_n),$$

$$\vec{p} = p_0 \vec{E}_n^{(i)}(-\vec{r}_n).$$

The scalar factors are $p_0 = a^3(\tilde{n}^2 - 1)/(\tilde{n}^2 + 2)$, where $\tilde{n}^2 = \epsilon + i4\pi\sigma/\omega$, and $m_0 = -iB_1^m/k_3$. The sphere relative permittivity is ϵ, its conductivity is σ, the angular frequency is ω, and B_1^m determines the strength of the magnetic moment. In general, m_0 and p_0 are complex numbers.

Induced Fields at the Array

The scattered field induces voltages on each dipole element of the array. The result at the mth element can be expressed as

$$V_m^H = -d_H[\hat{r}_m \times (\hat{r}_m \times \hat{e}_x)] \cdot \vec{E}^{(s)}(\vec{r}_m),$$
$$V_m^V = -d_V[\hat{r}_m \times (\hat{r}_m \times \hat{e}_y)] \cdot \vec{E}^{(s)}(\vec{r}_m).$$

Combining all these expressions (incident field, scattered field, and induced voltages) will produce the full transfer matrix for this problem.

The Scattering Matrix

Since all three of these steps involve double cross-product formulas, the resulting final expressions will be rather tedious unless we can find some way to simplify them. We found that, by introducing a special type of projection operator (a 3×3 matrix) defined by

$$\Delta_{mn} = \hat{r}_m \cdot \hat{r}_n \Im - \hat{r}_n \hat{r}_m^T,$$

we could collapse the equations very efficiently, where $\Im$ is the identity matrix. In these terms, the main scattering operator can be written as

$$S = \Delta_{mm}(m_0 \Delta_{mn} - p_0 \Delta_{mm}) \Delta_{nn}.$$

The Scattering Matrix (continued)

Then using the properties of our projection operator, we find easily that

$$S = m_0 \Delta_{mn} - p_0 \Delta_{mm} \Delta_{nn}.$$

The result is that we can write the key matrix as

$$\begin{pmatrix} K_{mn}^{HH} & K_{mn}^{HV} \\ K_{mn}^{VH} & K_{mn}^{VV} \end{pmatrix} = \frac{ik^3 e^{ik(r_m + r_n)}}{4\pi\epsilon_0 c r_m r_n} \begin{pmatrix} d_H \hat{e}_x^T \\ d_V \hat{e}_y^T \end{pmatrix} S \begin{pmatrix} d_H \hat{e}_x & d_V \hat{e}_y \end{pmatrix}.$$

The superscripts H and V refer to the horizontal and vertical dipoles in each array element and their corresponding polarizations.

The Scattering Matrix (concluded)

Then the final result is

$$\begin{pmatrix} V_m^H \\ V_m^V \end{pmatrix} = \begin{pmatrix} K_{mn}^{HH} & K_{mn}^{HV} \\ K_{mn}^{VH} & K_{mn}^{VV} \end{pmatrix} \begin{pmatrix} I_n^H \\ I_n^V \end{pmatrix}.$$

The Coupling Matrix

The 2×2 matrix K_{mn} can be written as

$$K_{mn} = \frac{ik^3 q e^{ik(r_m + r_n)}}{4\pi\epsilon_0 c r_m r_n} \hat{K}_{mn},$$

where the elements of $\hat{K}_{mn}$ were given before, and $q \equiv \sqrt{|m_0|^2 + |p_0|^2}$. Note that $\hat{K}_{mn} = \hat{K}_{mn}^T$ by reciprocity. Note also that all combinations of polarization coupling are represented in K_{mn}.

Data and SVD

Our array has N crossed-dipole elements lying in a plane.

Let V be the vector of received voltages and I the vector of transmitted currents (both of length 2N). Then,

$$V = TI,$$

where

$$V = (V_1^H, V_1^V, \ldots, V_N^H, V_N^V)^T$$
$$I = (I_1^H, I_1^V, \ldots, I_N^H, I_N^V)^T,$$

and T is the transfer matrix.

Transfer Matrix

The response or transfer matrix for this problem is

$$T = \begin{pmatrix} K_{11} & K_{12} & \cdots & K_{1N} \\ K_{21} & K_{22} & \cdots & K_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ K_{N1} & K_{N2} & \cdots & K_{NN} \end{pmatrix}.$$

The matrices K_{mn} are 2×2 matrices connecting horizontal and vertical dipole sources to horizontal and vertical dipole receivers in all four possible combinations:

$$K_{mn} = \begin{pmatrix} K_{mn}^{HH} & K_{mn}^{HV} \\ K_{mn}^{VH} & K_{mn}^{VV} \end{pmatrix}.$$

Singular Value Decomposition (SVD)

We could compose the full time-reversal operator for this problem, which is T^*T. This matrix is square and Hermitian. Eigenvectors and eigenvalues can be found in a straightforward way. But this is actually somewhat more difficult (unwieldy) than performing the singular value decomposition on the matrix T itself. In this case,

$$T\Phi = \Lambda\Phi^*,$$

where the singular values Λ are real, non-negative, and also the square roots of the eigenvalues for the corresponding eigenvectors of T^*T.

Normalizing the Equations

We can simplify the problem somewhat more by normalizing the equations, and eliminating various common factors. Letting $z_j = e^{-ikr_j}$, for $j = 1, \ldots, N$, we define ϕ_1, …, ϕ_{2N} by

$$\Phi = \frac{1}{\sqrt{i}} \left(\phi_1 z_1, \phi_2, z_1, \ldots, \phi_{2N-1} z_N, \phi_{2N} z_N \right)^T,$$

and

$$\Lambda = \frac{k^3 q}{4\pi\epsilon_0 c} \lambda.$$

Then, the SVD reduces to

$$\hat{T}\phi = \lambda\phi^*,$$

Normalizing the Equations (continued)

where now

$$\hat{T} = \begin{pmatrix} \hat{K}_{11} & \hat{K}_{12} & \cdots & \hat{K}_{1N} \\ \hat{K}_{21} & \hat{K}_{22} & \cdots & \hat{K}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{K}_{N1} & \hat{K}_{N2} & \cdots & \hat{K}_{NN} \end{pmatrix}.$$

By factoring out the complex exponential from the original singular vectors Φ, the part of the phase responsible for focusing the transmitted field on the sphere is eliminated.

Normalizing the Equations (concluded)

This result is common to all eigenvectors of the TRO in the presence of a single scatterer.

The remaining vector ϕ represents the (signed) amplitude distribution over the array, which may have a pattern of nulls depending on the nature of the scattering from the sphere.

Deconstructing the Transfer Matrix

The transfer matrix can now be easily (!) deconstructed into its two main components, $\hat{T} = \hat{T}_p + \hat{T}_m$. These are terms for the dielectric and conducting contributions to the scattering:

$$\hat{T}_p = -e^{i\theta_p}(g_1 g_1^T + g_2 g_2^T + g_3 g_3^T)$$
$$\hat{T}_m = e^{i\theta_m}(g_4 g_4^T + g_5 g_5^T + g_6 g_6^T).$$

The vectors g_j, for $j = 1, \ldots, 6$ are known explicitly from the analysis. The singular vectors for a matrix of this form can be expressed as linear combinations of the same vectors:

$$\phi = \sum_{j=1}^{6} \gamma_j g_j.$$

The Reduced SVD

These results reduce the SVD for the $2N \times 2N$ matrix $\hat{T}$ to an SVD instead of a 6×6 matrix G. This reduction is obviously substantial if N is much greater than 3. The matrix elements of G are given by $G_{jl} = g_j^T \cdot g_l$ and the SVD takes the form:

$$-e^{i\theta_p} \sum_{l=1}^{6} G_{jl}\gamma_l = \lambda\gamma_j^* \text{ for } j = 1, 2, 3$$

$$e^{i\theta_m} \sum_{l=1}^{6} G_{jl}\gamma_l = \lambda\gamma_j^* \text{ for } j = 4, 5, 6.$$

This reduction follows from the fact that there are only a small number of terms used in the partial wave expansion for the scattered field.

The Reduced SVD (continued)

In particular, the field is generated by an electric dipole moment and a magnetic dipole moment, each of which can be oriented in three mutually orthogonal directions. Thus, for small ka, there are at most six eigenvectors associated with any small scattering object such as a conducting sphere.

CONCLUSIONS

- Six significant modes can be associated with a small spherical scatterer: three for the dielectric interaction are always present, and another three for the conductive interaction if the scatterer is highly conductive/metallic.
- Characterization using detected presence or absence of metallic/conductive properties should be relatively straightforward with this approach.
- The two modes corresponding to endfire dipoles can normally only be seen in the relatively near field.

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