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ALIGNED VERTICAL FRACTURES, HTI SYMMETRY, AND THOMSEN PARAMETERS

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OUTLINE

- Review Standard Seismic Results for TI Media

- Anisotropy Due to Low Density Fractures
  - Seismic velocities in TI media
  - Sayers-Kachanov method

- Higher Crack Densities
  - Higher order Sayers-Kachanov η's
  - Thomsen's weak anisotropy formulation

- Discussion and Conclusions

Standard results for
PHASE VELOCITIES IN TI MEDIA (1)

SH-wave velocity:

$$v_{sh}^2(\theta) = \frac{1}{\rho}\left[c_{44} + (c_{66} - c_{44})\, sin^2\,\theta\right].$$

PHASE VELOCITIES IN TI MEDIA (2)

Quasi-P wave velocity:

$$v_p^2(\theta) = \frac{1}{2\rho}\left\{\left[(c_{11}+c_{44})\,sin^2\theta + (c_{33}+c_{44})\,cos^2\theta\right] + R(\theta)\right\}$$

Quasi-SV wave velocity:

$$v_{sv}^2(\theta) = \frac{1}{2\rho}\left\{\left[(c_{11}+c_{44})\,sin^2\theta + (c_{33}+c_{44})\,cos^2\theta\right] - R(\theta)\right\}$$

where $R(\theta)$ is determined by

$$R^2(\theta) = \left[(c_{11}-c_{44})\,sin^2\theta - (c_{33}-c_{44})\,cos^2\theta\right]^2$$
$$+4(c_{13}+c_{44})^2\,sin^2\theta\,cos^2\theta$$

STIFFNESS VERSUS COMPLIANCE

The isotropic stiffness matrix (inverse of the compliance matrix) for an elastic material is often written as $C =$

$$\begin{pmatrix} \lambda + 2G & \lambda & \lambda & & & \\ \lambda & \lambda + 2G & \lambda & & & \\ \lambda & \lambda & \lambda + 2G & & & \\ & & & G & & \\ & & & & G & \\ & & & & & G \end{pmatrix}$$

where λ and G are the two Lamé parameters,

G is shear modulus, and $K = \lambda + 2G/3$ is bulk modulus.

ANISOTROPY DUE TO FRACTURES (1)

The isotropic compliance matrix (inverse of the stiffness matrix) for an elastic material is often written as $S =$

$$\begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & & & \\ & & & \frac{1}{G} & & \\ & & & & \frac{1}{G} & \\ & & & & & \frac{1}{G} \end{pmatrix}$$

where E is Young's modulus, ν is Poisson's ratio, and $G = E/2(1+\nu)$ is the shear modulus.

ANISOTROPY DUE TO FRACTURES (2)

We assume all the fractures/cracks are in the form of penny-shaped cracks. These are defined as oblate spheroids, being round/flat holes, circular in cross-section with area $A = \pi b^2$, and volume $V_c = 4\pi a b^2/3$, or $V_c = 4\pi\alpha b^3/3$, where $\alpha \equiv a/b$ is the aspect ratio. We assume $\alpha = a/b << 1$, so the crack porosity $\phi = V_c/V << 1$.

The crack density parameter $\rho = \rho_c = N b^3/V$, where N/V is the number of cracks per unit volume, and b is the radius of the (assumed) penny-shaped crack.

ANISOTROPY DUE TO FRACTURES (3)

Sayers and Kachanov (1991) show that corrections to the isotropic matrix S_{ij}, caused by low crack densities ($\rho << 1$), can be written as

$$\Delta\left(\frac{1}{G}\right) = 4\eta_2\rho/3,$$

$$\Delta\left(-\frac{\nu}{E}\right) = 2\eta_1\rho/3,$$

$$\Delta\left(\frac{1}{E}\right) = 2(\eta_1 + \eta_2)\rho/3,$$

where η_1 and η_2 are parameters to be found from EMT.

ANISOTROPY DUE TO FRACTURES (4)

Thus, in the isotropic case, we have

$$\Delta S_{ij} = (2\rho/3)\times$$

$$\begin{pmatrix} (\eta_1+\eta_2) & \eta_1 & \eta_1 & & & \\ \eta_1 & (\eta_1+\eta_2) & \eta_1 & & & \\ \eta_1 & \eta_1 & (\eta_1+\eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 2\eta_2 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (5)

For horizontal cracks, we get an anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} 0 & 0 & \eta_1 & & & \\ 0 & 0 & \eta_1 & & & \\ \eta_1 & \eta_1 & 2(\eta_1 + \eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 0 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (6)

For vertical cracks whose axis of symmetry is randomly oriented in the xy-plane, we have another anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} (\eta_1+\eta_2) & \eta_1 & \eta_1/2 & & & \\ \eta_1 & (\eta_1+\eta_2) & \eta_1/2 & & & \\ \eta_1/2 & \eta_1/2 & 0 & & & \\ & & & \eta_2 & & \\ & & & & \eta_2 & \\ & & & & & 2\eta_2 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (7)

Examples of the values of the η's found from various effective medium theories are:

EMT	η_1	η_2
NI	-0.000216	0.0287
DS	-0.000216	0.0290
CPA	-0.000258	0.0290
SC	$-.0000207$	0.0290

THOMSEN PARAMETERS (1)

For the case of randomly oriented vertical fractures,
two of the Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{C_{66}-C_{44}}{2C_{44}} = -\eta_2\rho\frac{E}{4(1+\nu)} = -\eta_2\rho\frac{G}{2}$$

$$\epsilon \equiv \frac{C_{11}-C_{33}}{2C_{33}} \simeq -\eta_2\rho\frac{G}{1-\nu}.$$

The remaining Thomsen parameter δ, which is the one that determines the degree of anellipticity in angular dependence of the wave speeds is given exactly by $\delta = \epsilon$,
which means there is no deviation from ellipticity.

THOMSEN PARAMETERS (2)

For the case of horizontal fractures, the same two Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{C_{66}-C_{44}}{2C_{44}} = \eta_2 \rho G$$

$$\epsilon \equiv \frac{C_{11}-C_{33}}{2C_{33}} \simeq \eta_2 \rho \frac{2G}{1-\nu}.$$

Note that these results both differ exactly by a factor of -2 from the previous results for randomly oriented vertical fractures. This fact can be easily understood in terms of the Sayers and Kachanov style of analysis.

ANISOTROPY DUE TO HIGHER CRACK DENSITY

For horizontal cracks, we get an anisotropic medium whose second order compliance correction matrix is

$$\Delta^{(2)} S_{ij} = \rho^2 \times$$

$$\begin{pmatrix} 0 & 0 & \eta_4 & & & \\ 0 & 0 & \eta_4 & & & \\ \eta_4 & \eta_4 & 2(\eta_3 + \eta_4 + \eta_5) & & & \\ & & & 2\eta_5 & & \\ & & & & 2\eta_5 & \\ & & & & & 0 \end{pmatrix}.$$

CRACK-INFLUENCE PARAMETERS

Examples of the values of the η's found from simulation results supplied by V. Grechka are:

Parameter	$\nu_0 = 0.00$	$\nu_0 = 0.4375$
η_1	0.0000	-0.0192
η_2	0.1941	0.3994
η_3	-0.3666	-1.3750
η_4	0.0000	0.0000
η_5	-0.0917	0.5500

THOMSEN WEAK ANISOTROPY FORMULAS

$$v_p(\theta) = v_p(0)\left(1 + \delta\, sin^2\,\theta\, cos^2\,\theta + \epsilon\, sin^4\,\theta\right),$$

$$v_{sv}(\theta) = v_s(0)\left(1 + (c_{33}/c_{44})(\epsilon - \delta)\, sin^2\,\theta\, cos^2\,\theta\right),$$

$$v_{sh}(\theta) = v_s(0)\left(1 + \gamma\, sin^2\,\theta\right).$$

where

$$\delta = \left(\frac{c_{13}+c_{33}}{2c_{33}}\right)\left(\frac{c_{13}+2c_{44}-c_{33}}{c_{33}-c_{44}}\right)$$

and

$$c_{33}/c_{44} = v_p^2(0)/v_s^2(0)$$

EXAMPLES OF SEISMIC VELOCITIES

Series of examples have been computed using the "empirical" crack influence parameters.

CONCLUSIONS

• The Sayers and Kachanov (1991) approach has some powerful advantages for both forward and inverse modeling in fractured systems, especially when used in conjunction with measured Thomsen parameters.

• For a given material (*e.g.*, quartz) and a given crack shape (*e.g.*, penny-shaped), η_1 and η_2 can be computed once and for all.

• Measured Thomsen parameters can then be inverted for crack density.

• Incorporating fluid dependence rigorously into this problem is easy using the Sayers-Kachanov parameters.

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Extending Thomsen's Approach to Stronger Anisotropies (1)

Reconsider

$$R^2(\theta) = \left[(c_{11} - c_{44})\, sin^2\theta - (c_{33} - c_{44})\, cos^2\theta\right]^2$$
$$+4(c_{13} + c_{44})^2\, sin^2\theta\, cos^2\theta$$

and rewrite this way:

$$R(\theta) = \left[(c_{11} - c_{44})\, sin^2\theta + (c_{33} - c_{44})\, cos^2\theta\right]\sqrt{1 - \zeta(\theta)}$$

where

$$\zeta(\theta) \equiv 4\frac{[(c_{11}-c_{44})(c_{33}-c_{44})-(c_{13}+c_{44})^2]\, sin^2\theta\, cos^2\theta}{[(c_{11}-c_{44})\, sin^2\theta+(c_{33}-c_{44})\, cos^2\theta]^2}$$

This probably does not look like progress to you yet, but the next step shows the usefulness of this rearrangement:

$$\zeta(\theta) = \frac{\zeta_m\, sin^2 2\theta_m\, sin^2 2\theta}{[1-cos\, 2\theta_m\, cos\, 2\theta]^2}.$$

Extending Thomsen's Approach to Stronger Anisotropies (2)

The maximum/minimum value of $\zeta(\theta)$ over all θ's is

$$\zeta_m = \frac{2(\epsilon-\delta)v_p^2(0)}{v_p^2(0)(1+2\epsilon)-v_s^2(0)},$$

while the location of this extreme value of $\zeta(\theta)$ is determined by

$$tan^2(\theta_m) = (c_{33} - c_{44})/(c_{11} - c_{44}),$$

which can also be rewritten in terms of Thomsen parameters as

$$tan^2\theta_m = \frac{v_p^2(0)-v_s^2(0)}{(1+2\epsilon)v_p^2(0)-v_s^2(0)}.$$