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ANISOTROPIC SEISMIC WAVES IN SYSTEMS WITH FLUID-SATURATED FRACTURES

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OUTLINE

- Anisotropy Due to Fractures
  - Sayers-Kachanov method
  - Thomsen parameters and Rayleigh waves
- Inverting for Crack Density $\rho = na^3$
  - Seismic reflection data
  - Surface wave data
- Fluid Effects, Gassmann's Equations, and Mavko-Jizba
- Discussion and Conclusions
- References

DEFINITION OF CRACK DENSITY ρ

A key parameter is crack density $\rho = na^3$, where a is the radius of a penny-shaped crack and $n = N/V$ is the number (N) of cracks per unit volume (V).

A penny-shaped crack is an oblate spheroidal void *i.e.*, an ellipsoid having principal axis dimensions $a \geq b \geq c$ such that $a = b$ and $c = \alpha a$, where $\alpha < 1$ is the aspect ratio. For flat cracks, α can be very small. Crack density does not depend on α.

ANISOTROPY DUE TO FRACTURES (1)

The isotropic compliance matrix (inverse of the stiffness matrix) for an elastic material is often written as $S =$

$$\begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & & & \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & & & \\ & & & \frac{1}{G} & & \\ & & & & \frac{1}{G} & \\ & & & & & \frac{1}{G} \end{pmatrix}$$

where E is Young's modulus, ν is Poisson's ratio, and $G = E/2(1+\nu)$ is the shear modulus.

ANISOTROPY DUE TO FRACTURES (2)

Sayers and Kachanov (1991) show that corrections Δ to the isotropic matrix S_{ij}, induced by low crack densities ($\rho = na^3 << 1$), can be written as

$$\Delta \left(\frac{1}{G} \right) = 4\eta_2 \rho / 3,$$

$$\Delta \left(-\frac{\nu}{E} \right) = 2\eta_1 \rho / 3,$$

$$\Delta \left(\frac{1}{E} \right) = 2(\eta_1 + \eta_2) \rho / 3,$$

where η_1 and η_2 are parameters to be found from EMT.

ANISOTROPY DUE TO FRACTURES (3)

Thus, for isotropic crack distributions, we have

$$\Delta S_{ij} = (2\rho/3)\times$$

$$\begin{pmatrix} (\eta_1+\eta_2) & \eta_1 & \eta_1 & & & \\ \eta_1 & (\eta_1+\eta_2) & \eta_1 & & & \\ \eta_1 & \eta_1 & (\eta_1+\eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 2\eta_2 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (4)

For horizontal cracks, we get an anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} 0 & 0 & \eta_1 & & & \\ 0 & 0 & \eta_1 & & & \\ \eta_1 & \eta_1 & 2(\eta_1 + \eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 0 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (5)

For vertical cracks whose axis of symmetry is randomly oriented in the xy-plane, we have another anisotropic medium whose correction matrix is

$$\Delta S_{ij} = \rho \times$$

$$\begin{pmatrix} (\eta_1+\eta_2) & \eta_1 & \eta_1/2 & & & \\ \eta_1 & (\eta_1+\eta_2) & \eta_1/2 & & & \\ \eta_1/2 & \eta_1/2 & 0 & & & \\ & & & \eta_2 & & \\ & & & & \eta_2 & \\ & & & & & 2\eta_2 \end{pmatrix}.$$

ANISOTROPY DUE TO FRACTURES (6)

Examples of the values of the η's for isotropic quartz found using various effective medium theories are:

$\underline{EMT}$	$\underline{\eta_1}$	$\underline{\eta_2}$
NI	-0.000216	0.0287
DS	-0.000216	0.0290
CPA	-0.000258	0.0290
SC	$-.0000207$	0.0290

Note that $|\eta_1|/\eta_2 < 0.01$ in all these cases. So only one parameter is important for quartz-like host media.

NIA EXAMPLES (1)

Since it makes little difference at low crack densities which theoretical method we use, we might as well consider the simplest one which is surely the non-interaction approximation (NIA).

It is well-known (e.g., see Zimmerman's book) that:

$$\frac{1}{K_{NI}} - \frac{1}{K} = \frac{\rho}{K}\frac{16(1-\nu^2)}{9(1-2\nu)}$$

$$\frac{1}{G_{NI}} - \frac{1}{G} = \frac{\rho}{G}\frac{32(1-\nu)(5-\nu)}{45(2-\nu)},$$

both obviously linear expressions in the crack density ρ.

NIA EXAMPLES (2)

But we also have

$$\eta_2 = \frac{3}{4\rho}\left(\frac{1}{G^*} - \frac{1}{G}\right)$$

and

$$\eta_1 = \frac{1}{6\rho}\left(\frac{1}{K^*} - \frac{1}{K}\right) - \frac{1}{4\rho}\left(\frac{1}{G^*} - \frac{1}{G}\right).$$

NIA EXAMPLES (3)

Thus, we find directly that

$$\eta_2 = \tfrac{1}{G}\tfrac{8(1-\nu)(5-\nu)}{15(2-\nu)}$$

and

$$\eta_1 = -\tfrac{1}{G}\tfrac{4\nu(1-\nu)}{15(2-\nu)},$$

where we also used the fact that

$$1/2G = (1+\nu)/3K(1-2\nu).$$

Especially note that, in the NIA, η_1 is negative and directly proportional to Poisson's ratio ν, and that

$$\eta_1/\eta_2 = -\nu/2(5-\nu).$$

POISSON'S RATIO FROM SEISMIC DATA

Standard expressions for shear and compressional wave speeds in isotropic media of mass density ρ_m are:

$$v_s^2 = G/\rho_m,$$

$$v_p^2 = (K + 4G/3)/\rho_m.$$

Poisson's ratio can be expressed then as

$$\nu = \frac{1}{2}\left(1 - \frac{v_s^2}{v_p^2 - v_s^2}\right).$$

So seismic data in the background medium for regions where there are no cracks would be useful for determining the parameters needed in the theory.

THOMSEN PARAMETERS (1)

For the case of randomly oriented vertical fractures, two of the Thomsen parameters can be expressed as:

$$\gamma \equiv \tfrac{C_{66}-C_{44}}{2C_{44}} = -\eta_2\rho\tfrac{G}{2},$$
$$\epsilon \equiv \tfrac{C_{11}-C_{33}}{2C_{33}} \simeq -\eta_2\rho\tfrac{G}{1-\nu}.$$

The remaining Thomsen parameter δ, which determines degree of anellipticity in angular dependence of the wave speeds is given exactly by $\delta = \epsilon$, showing there is no deviation from ellipticity.

THOMSEN PARAMETERS (2)

For the case of horizontal fractures, the same two Thomsen parameters can be expressed as:

$$\gamma \equiv \frac{C_{66}-C_{44}}{2C_{44}} = \eta_2 \rho G$$

$$\epsilon \equiv \frac{C_{11}-C_{33}}{2C_{33}} \simeq \eta_2 \rho \frac{2G}{1-\nu}.$$

Note: these results both differ exactly by a factor of -2 from previous results for randomly oriented vertical fractures. This fact can be easily understood in terms of the Sayers and Kachanov style of analysis.

THOMSEN PARAMETERS (3)

Inverting the expressions for vertical fractures gives two formulas for the crack density ρ:

$$\rho = -2\gamma/\eta_2 G = -\frac{C_{66}-C_{44}}{\eta_2 G C_{44}},$$

and

$$\rho = -\epsilon(1-\nu)/2\eta_2 G = -\frac{(C_{11}-C_{33})(1-\nu)}{2\eta_2 G C_{33}},$$

providing some redundancy if all the required data are available. If shear wave data in the fractured region are not available, then the second formula is the pertinent one – but values of G and ν seem to be needed. However, $\eta_2 \propto 1/G$, so only ν is required.

THOMSEN PARAMETERS (4)

And similarly, inverting the expressions for horizontal fractures gives two formulas for the crack density ρ:

$$\rho = \gamma/\eta_2 G = \frac{C_{66}-C_{44}}{2\eta_2 G C_{44}}$$

and

$$\rho = \epsilon(1-\nu)/2\eta_2 G = \frac{(C_{11}-C_{33})(1-\nu)}{4\eta_2 G C_{33}},$$

providing some redundancy if all the desired data are available. If shear wave data in the fractured region are not available, then the second formula is again the pertinent one – again values of G and ν seem to be needed. However, $\eta_2 \propto 1/G$, so only ν is required.

RAYLEIGH WAVE SPEED

For a transversely isotropic medium with vertical axis of symmetry (which is true of both the cases described so far), the Rayleigh surface wave has a speed determined by the following equation:

$$\frac{1}{16}q^3 - \frac{1}{2}q^2 + \left(\frac{3}{2} - \frac{C_{66}}{C_{11}}\right)q + \left(\frac{C_{66}}{C_{11}} - 1\right) = 0,$$

where $q \equiv v_R^2/v_s^2$ and $v_s^2 = C_{66}/\rho_0$,
with ρ_0 being the inertial mass density of the medium.
Recall that $C_{66} = C_{44}(1+2\gamma)$ and that $C_{11} = C_{33}(1+2\epsilon)$ in terms of Thomsen parameters.

GASSMANN'S EQUATIONS (1)

$$K_u = K_d + \frac{\alpha^2}{(\alpha-\phi)/K_m + \phi/K_f},$$

where K_u is the undrained bulk modulus, K_d is the drained bulk modulus, K_m is the mineral (or solid) modulus, K_f is the pore fluid bulk modulus, ϕ is the porosity, and $\alpha = 1 - K_d/K_m$. Rearranging into compliance form, we have

$$\frac{1}{K_u} - \frac{1}{K_d} = -\frac{\alpha}{K_d} \times$$

$$\left[1 + \frac{K_d \phi}{K_f \alpha}\left(1 - \frac{K_f}{K_m}\right)\right]^{-1}.$$

Also, porosity $\phi = \frac{4\pi a}{3b}\frac{Nb^3}{V} = \frac{4\pi a}{3b}\rho$.

GASSMANN'S EQUATIONS (2)

Compliance correction matrix for fluid inclusions:

$$\Delta S_{ij} = -\gamma^{-1} \begin{pmatrix} \beta_1^2 & \beta_1\beta_2 & \beta_1\beta_3 & & & \\ \beta_1\beta_2 & \beta_2^2 & \beta_2\beta_3 & & & \\ \beta_1\beta_3 & \beta_2\beta_3 & \beta_3^2 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$$

The fluid effects (K_f) appear only in the overall factor γ. The coefficients β_i, $i = 1, 2, 3$, satisfy a sumrule of the form $\beta_1 + \beta_2 + \beta_3 = 1/K_d - 1/K_m \equiv \alpha/K_d$. K_d is the drained bulk modulus. K_m is the mineral (i.e., solid) modulus.

GASSMANN CORRECTIONS TO SAYERS-KACHANOV FORMULAS

With $\rho = Nr^3/V$ being the crack density parameter, and η_2 being the Sayers-Kachanov parameter that perturbs the shear compliance, we have

$$\Delta S_{ij} \simeq \tfrac{2\eta_2\rho}{3}\times$$
$$\begin{pmatrix} (1-K_f/K_m) & 0 & 0 & & & \\ 0 & (1-K_f/K_m) & 0 & & & \\ 0 & 0 & (1-K_f/K_m) & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$$

MAVKO-JIZBA REDERIVED

It is now easy to show that

$$\frac{1}{K_u} - \frac{1}{K_d} \equiv \Delta\left(\frac{1}{K_u}\right) = 2\eta_2\rho(1 - K_f/K_m)$$

and

$$\frac{1}{G_u} - \frac{1}{G_d} \equiv \Delta\left(\frac{1}{G_u}\right) = \frac{2}{5}\left(\frac{1}{G^r_{eff}}\right) + \frac{3}{5} \times 0$$

where

$$\Delta\left(\frac{1}{G^r_{eff}}\right) = 4\eta_2\rho(1 - K_f/K_m)/3$$

So

$$\Delta\left(\frac{1}{G_u}\right) = \frac{4}{15}\Delta\left(\frac{1}{K_u}\right).$$

CONCLUSIONS

• The Sayers and Kachanov (1991) approach has some powerful advantages for both forward and inverse modeling in fractured systems, especially when used in conjunction with measured Thomsen parameters.

• For a given material (*e.g.*, quartz) and a given crack shape (*e.g.*, penny-shaped), η_1 and η_2 can be computed once and for all.

• Measured Thomsen parameters can then be inverted for crack density.

• Incorporating fluid dependence rigorously into this problem is easy using the Sayers-Kachanov parameters.

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