Stanford Exploration Project, Report 73, November 18, 1997, pages 41–47

Stanford Exploration Project, Report 73, November 18, 1997, pages 41–47

Short Note

Robust wavelet deconvolution

Lewei Mo [1]

keywords: *wavelet, Fourier transform, deconvolution*

INTRODUCTION

Conventional wavelet deconvolution "flattens" the signal frequency band and amplifies random noise. We present a method in this note that "flattens" the wavelet (signal) frequency band without amplifying random noise. It is a modified version of conventional wavelet decon. The method uses a decimated autocorrelation to compute the inverse wavelet at a coarser sampling interval. The order of decimation depends on the signal frequency range, relative to the full Nyquist frequency. After decimating, the algorithm fills with zeroes in the computed inverse wavelet to mimic the original sampling interval to obtain a new inverse wavelet. The inverse wavelet as computed has the desired inverse spectrum in the original signal frequency band. However, this zero-filled inverse wavelet carries a duplicate spectrum. Then we do a lowpass filtering on the zero-filled inverse wavelet to remove its duplicate spectrum, obtaining the final inverse wavelet. Convolving this final inverse wavelet with the original seismogram "flattens" the signal frequency band without amplifying random noise.

ROBUST WAVELET DECON

The source wavelet emitted from a near surface source travels a great distance into the earth and reflects back to receivers at the surface. It suffers frequency absorption so that it changes shape from a possibly sharp wavelet into a broad one. To have a good resolution reflection seismic section, the wavelet should be a spike, which has a flat spectrum from DC to Nyquist. Textbooks tell us to sharpen the wavelet. From a wavelet, we compute the inverse wavelet which has a spectrum that is the inverse of the original and apply it to the input. The resulting output is a "spike." We compute the minimum phase inverse wavelet from the autocorrelation of the wavelet (Claerbout, 1976). Wavelet decon is equivalent to ungapped predictive error

[1] **email:** not available

filtering (PEF). *The output of PEF is white* . This processing is both correct and incorrect: correct because it is logical and mathematically correct, and not correct for the following reasons. Look at the spectrum of a reflection seismogram. There is not energy all over the spectrum from DC to Nyquist. All recordings have energy up to the halfway point, at best. Conventional wavelet deconvolution has done too much. It “flattens” the low frequency signal band and at the same time also “flattens” the high frequency random noise band, so that the output is white.

Our goal is to obtain an inverse wavelet filter that has a spectrum that is the inverse of the wavelet's spectrum at low frequencies only, with zero values beyond. Like other methods, we propose to compute the inverse wavelet from autocorrelation of the wavelet. This paper assumes that *the reflectivity series is white and the autocorrelation of the seismogram is a scaled version of that of the seismic wavelet* (Yilmaz, 1987), and *the spectrum of the seismogram is a jagged version of that of the wavelet* (Claerbout, 1991). Since the spectrum of the wavelet can be computed from its autocorrelation, and in our seismic case the wavelet spectrum band occupies only the low frequency end of the full spectrum, proper subsampling of the autocorrelation does not incur loss of information. The idea is to compute the inverse wavelet with a lower Nyquist frequency (coarser sampling interval) that contains just the wavelet spectrum. Using a subsampled autocorrelation to compute the inverse wavelet at a coarser sampling interval confines its effective frequency range to the wavelet frequency band.

Suppose we have decided that there is no energy beyond half Nyquist. Then the effective frequency range of the inverse wavelet should be the first half Nyquist interval. Then the inverse wavelet should be computed at a sampling interval that is twice the original sampling interval. This means that we need compute only every other point of the autocorrelation $(r0, r2, r4, \ldots)$. Next we put these values into a conventional wavelet decon operator design program (Claerbout, 1976, p. 57) to compute the inverse wavelet $(1, a1, a2, \ldots)$. The resulting inverse wavelet has a spectrum that is the inverse of the wavelet spectrum, and has a Nyquist frequency that is half of the original Nyquist. We then spread these points out to mimic the original sampling interval, yielding $(1, 0, a1, 0, a2, \ldots)$, and use this as the wavelet decon operator by convolving it with the seismogram. As the entries of the filter are periodically zero, the increment step of the convolution calculation can be made larger than one. This has two important effects:

1. Effort is not wasted to build up the missing part of the spectrum, which contains only random noise.
2. Estimating autocorrelation and processing costs are halved, as are the degrees of freedom.

Now we can generalize. Whether there is energy all over the spectrum, or only up to half Nyquist, one-third Nyquist, or one-quarter Nyquist will determine whether we use every point, every other point, every third point, or every fourth point of the autocorrelation, and the corresponding order of zero filling of the computed inverse wavelet. The next important aspect is operator length. We try 2,4,8,16,32,64 non-zero points, compare their results, and select the one that best flattens the wavelet spectrum and does not enhances noise much.

Figure 1 shows (on the left) a common shot gather from a Geco marine dataset, after muting to remove direct wave and head waves, and after $tpow = 1$ spherical divergence correction[2]. The water bottom reflections are so strong that even when we use $tpow = 2$, the deep parts are still not visible; its use only causes the waterbottom multiples to be stronger than its primary. So we stay at $tpow = 1$ correction. On the right of Figure 1 is the output of wavelet decon. On this section, the wavelet is compressed, but the noise is not enhanced much. Figure 2 shows the amplitude spectrum of the 12th trace before and after wavelet decon.

Because there is energy only up to half Nyquist frequency, we compute only every other point of the autocorrelation. We tried and finally made the choice of 16 non-zero points in the autocorrelation and the inverse wavelet filter. The inverse wavelet filter for the 12th trace is shown on the left of Figure 3. For comparison, on the right of Figure 3 is the conventional Burg inverse wavelet. Figure 4 shows the spectra of the two different wavelets. Convolving our inverse wavelet with the seismogram better flattens the wavelet spectrum. While the amplitude spectrum of the noise is smaller than that of the signal, this processing does not significantly enhance random noise.

Here we use time-invariant decon because we concern ourselves only with the hard waterbottom reflections that have a consistent waveform. Other events are too weak to be seen.

MORE ROBUST WAVELET DECON

Our inverse wavelet is computed at a coarser sampling interval. After being filled with zeroes to mimic the original sampling interval, we have a higher Nyquist frequency. Frequencies higher than the Nyquist frequency of the inverse wavelet are computed, showing the duplicate spectrum. The order of duplication is the order of the subsampling of the autocorrelation. When the inverse wavelet is convolved with the original trace, the duplicate spectrum amplifies random noise. The cure is to do lowpass filtering of the inverse wavelet to remove the duplicate spectrum. Figure 5 presents the lowpass filtered version of the inverse wavelet of Figure 3, and its spectrum. Convolving this inverse wavelet with the seismogram will not generate the energy near Nyquist that was present in the spectrum shown in Figure 2. The output section of this processing looks roughly the same as the previous deconvolved section; we do not display it here.

Lowpass filtering increases the number of non-zero points in the inverse wavelet filter. Thus it increases the processing cost, while only slightly improving the results.

[2]Tpow - Selects a power of time t for seismogram scaling.

Therefore, we propose not doing the lowpass filtering, leaving the slightly enhanced noise to be suppressed by CDP stacking, which has the effect of lowpass filtering. CDP stacking also attenuates random noise that is buried in the wavelet spectrum band, which could have been enhanced by the wavelet decon.

Gapped predictive error decon (Peacock and Treitel, 1969) does not have good control between wavelet compression and random noise enhancement. For this reason, robust wavelet decon may be a better approach. Interested readers can find classic papers describing in detail the pros and cons of various deconvolution schemes in a monograph "Deconvolution" edited by Webster.

CONCLUSIONS

Using decimated autocorrelations to compute the inverse wavelet at a coarser sampling interval and zero-filling the inverse wavelet to return to the original sampling interval is an effective method to sharpen a wavelet, thus increasing resolution and reducing computation cost. Doing lowpass filtering on the zero-filled inverse wavelet to remove its duplicate spectrum gives an even better decon result at the expense of increasing computation cost. It remains for further research how to extract the *white reflectivity series* from the data after wavelet deconvolution.

ACKNOWLEDGMENTS

My thanks to the sponsors of SEP and its director, Jon Claerbout for their generous support. I would like to thank Francis Muir for suggesting to me his ideas of decimating autocorrelations before computing inverse wavelets, and for helping me to understand these concepts. His constant guidance and ideas were invaluable. I thank Steve Cole, Dave Nichols, and Lin Zhang for many illuminating discussions.

REFERENCES

Claerbout, J. F., 1976, Fundamentals of Geophysical Data Processing: McGraw-Hill.

Claerbout, J. F., 1991, Earth Soundings Analysis: Processing versus Inversion: Blackwell Scientific.

Peacock, K. L. and Treitel, S., 1969, Predictive deconvolution: Theory and practice: Geophysics, **34**, 155–169.

Webster, G. M., 1978, Deconvolution, Chapter III: Geophysics reprint by SEG.

Yilmaz, O., 1987, Seismic data processing: SEG monograph.

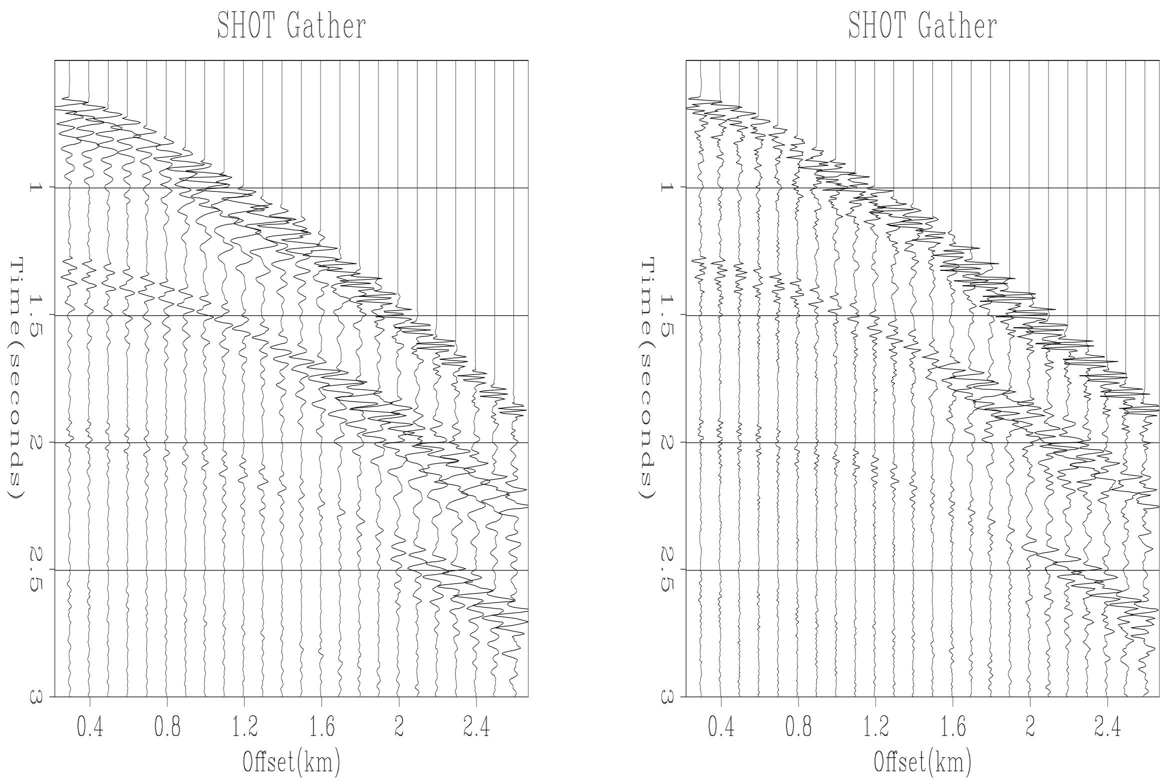

Figure 1: On the left is the original data; on the right is the data after wavelet decon, on this section the wavelet is compressed and the noise is not enhanced much. mo1-Gm1Gm1d [ER]

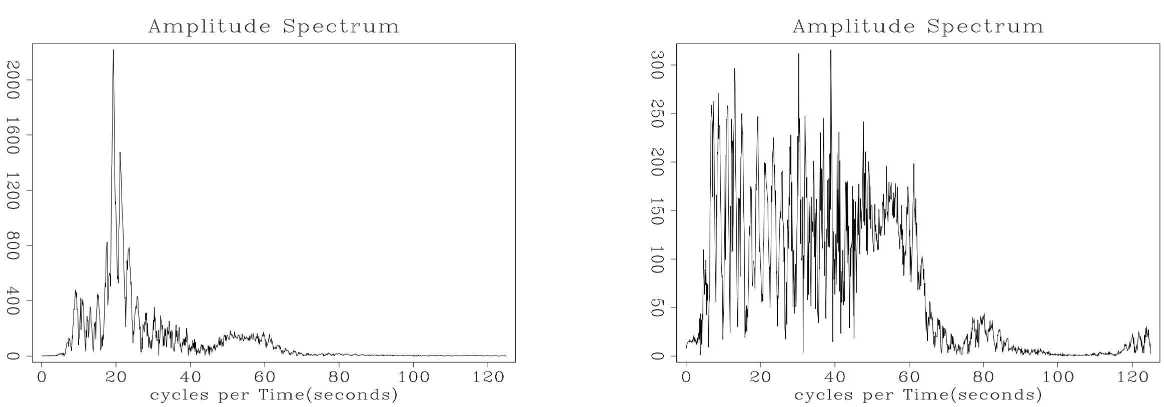

Figure 2: On the left is the spectrum of the 12th trace of the original data; on the right is the spectrum after wavelet decon. The wavelet spectrum has been flattened but there are two mounds beyond half Nyquist frequency. mo1-Gm1.12fdf [ER]

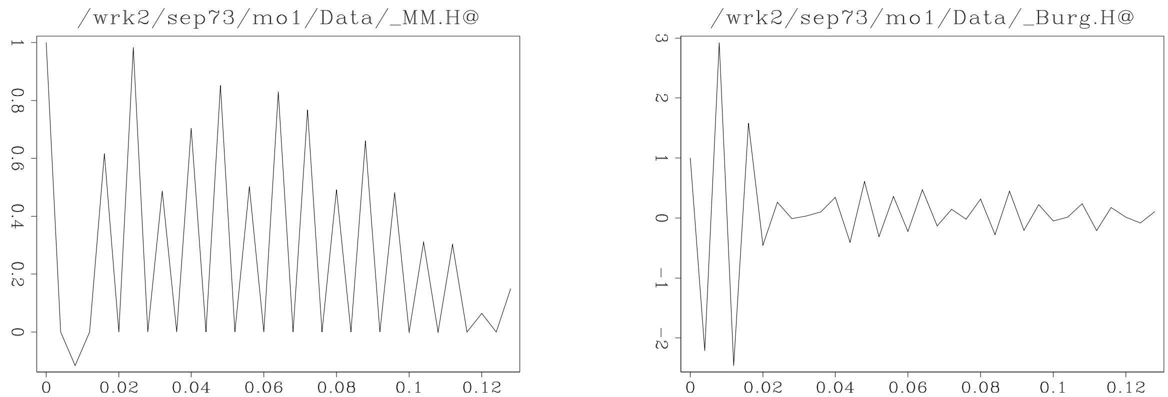

Figure 3: On the left is our inverse wavelet filter, it does not enhance noise significantly. On the right is the conventional Burg inverse wavelet filter, it enhances noise significantly. mo1-Infilters [ER]

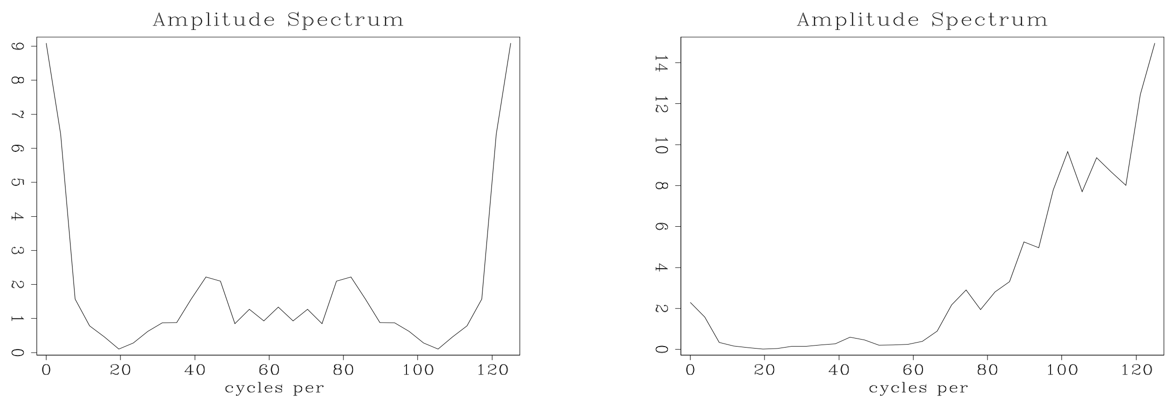

Figure 4: On the left is the spectrum of our inverse wavelet filter; on the right is the spectrum of Burg inverse wavelet. mo1-Inspectra [ER]

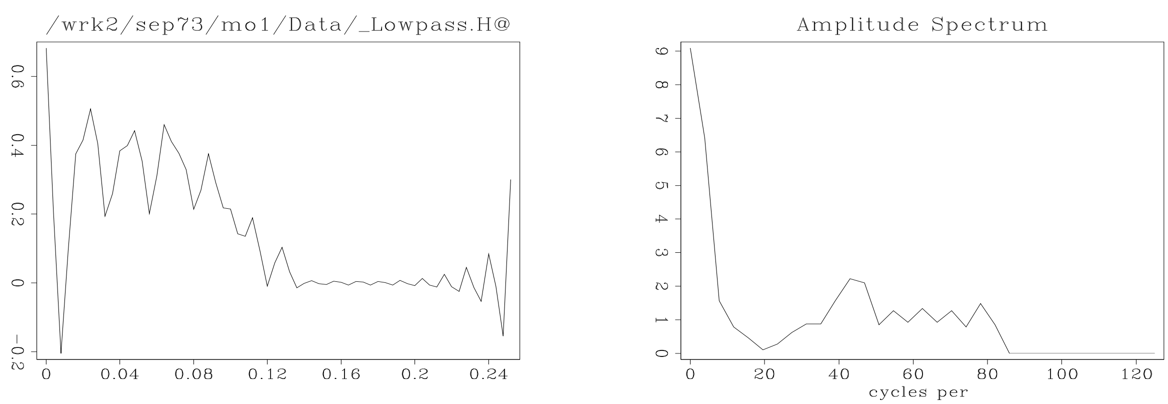

Figure 5: On the left is the lowpass filtered inverse wavelet filter; on the right is its spectrum. Convolving this inverse wavelet filter with the seismogram flattens the wavelet spectrum without enhancing noise. mo1-Lowpass [ER]