Attenuation of specular and diffracted 2D multiples in image space

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Abstract

In complex areas, attenuation of specular and diffracted multiples in image space is an attractive alternative to surface-related multiple elimination (SRME) and to data space Radon filtering. We present the equations that map, via wave-equation migration, 2D diffracted and specular water-bottom multiples from data space to image space. We show the equations for both subsurface-offset-domain common-image-gathers (SODCIGs) and angle-domain common-image-gathers (ADCIGs). We demonstrate that when migrated with sediment velocities, the overmigrated multiples map to predictable regions in both SODCIGs and ADCIGs. Specular multiples move as primaries whereas diffracted multiples do not. In particular, the apex of the residual moveout curve of diffracted multiples in ADCIGs is not located at zero aperture angle.

We use our equation of the residual moveout of the multiples in ADCIGs to design an apex-shifted Radon transform that maps the 2D ADCIGs into a 3D model space cube whose dimensions are depth, curvature and apex-shift distance. Well-corrected primaries map to or near the zero curvature plane and specularly-reflected multiples map to or near the zero apex-shift plane. Diffracted multiples map elsewhere in the cube according to their curvature and apex-shift distance. Thus, specularly reflected as well as diffracted multiples can be attenuated simultaneously. We show the application of our apex-shifted Radon transform to a 2D seismic line from the Gulf of Mexico. Diffracted multiples originate at the edges of the salt body and we
show that we can successfully attenuate them, along with the specular multiples, in the image
Radon domain.

INTRODUCTION

Surface-related multiple elimination (SRME) uses the recorded seismic data to predict and iteratively
subtract the multiple series (Verschuur et al., 1992). 2D SRME can deal with all kinds of surface-
related 2D multiples, provided all relevant data are recorded within the aperture and offset limitations
of the survey line. Diffracted multiples from scatterers with a cross-line component cannot be pre-
dicted by 2D SRME but in principle can be predicted by 3D SRME provided the acquisition is dense
enough and the apertures large enough in both in-line and cross-line directions. With standard marine
streamer acquisition, the sampling in the cross-line direction is too coarse and diffracted multiples
need to be removed by other methods (Hargreaves and Trad, 2003) or the data need to be interpolated
and extrapolated to a dense, large aperture grid (van Dedem and Verschuur, 1998; Nekut, 1998).

Radon demultiple in data space (Hampson, 1986; Foster and Mosher, 1992) has proven successful
in attenuating specular multiples if the subsurface is not very complex so that primaries are flat after
normal moveout (NMO) correction in common midpoint (CMP) gathers and the under-corrected
multiples can be approximated by parabolas or hyperbolas. In complex subsurface areas, such as
under salt, the hyperbolic NMO approximation breaks down. The NMO velocities are inaccurate and
therefore, after NMO, the primaries are unlikely to be flat and the residual moveout of the multiples
is unlikely to be well represented by parabolas or hyperbolas. The quality of the separation between
primaries and multiples in the Radon domain, therefore, deteriorates.

An alternative to SRME and data space Radon demultiple is the attenuation of multiples in the
image space. Prestack wave-equation depth migration accurately handles the complex wave propaga-
tion of primaries (Sava and Guitton, 2003), to the extent that the presence of the multiples allows an accurate estimation of the migration velocities. The residual moveout of primaries in angle-domain common-image gathers (ADCIGs), therefore, is likely to be flat. It is not immediately obvious, however, what the residual moveout of the over-migrated multiples is in ADCIGs. In order to maximize the separation of primaries and multiples in the Radon domain, the kernel of the Radon transform should approximate the functional dependency of the residual moveout of the multiples as a function of the aperture angle as much as possible. Sava and Guitton (2003) and Alvarez et. al. (2004) used the tangent-squared approximation of Biondi and Symes (2004) assuming that the residual moveout of the multiples is the same as that of primaries migrated with faster velocity. The tangent-squared approximation, however, is a straight ray approximation that is appropriate for primaries that are likely to be only moderately over-migrated. Multiples, on the other hand, given their large difference in velocity with respect to that of the primaries, are likely to be severely over-migrated and the straight ray approximation may not be appropriate for them.

In this paper we present the equations that describe the mapping of specular and diffracted 2D water-bottom multiples from data space to image space by wave-equation migration in both subsurface-offset-domain common-image-gathers (SODCIGs) and ADCIGs. Primaries are migrated to zero subsurface offsets in SODCIGs and with at moveout in ADCIGs. We show that specular water-bottom multiples are imaged as primaries. Hence, if migrated with constant water velocity, they too are mapped to zero subsurface-offset in SODCIGs and with flat moveout in ADCIGs (Biondi, 2005). In the usual case of migration with velocities faster than water velocity, however, specular water-bottom multiples are not mapped to zero subsurface offsets. They are mapped to subsurface offsets with the opposite sign to that of their surface offsets. We derive the moveout curve of these multiples in SODCIGs and ADCIGs. We then take the special case of the residual moveout of a specular multiple from flat water-bottom in ADCIGs and use it to design a Radon transform that
accounts for ray-bending of the multiple raypath at the multiple-generating interface. This Radon transform improves the separation of primaries and multiples in the Radon domain compared with a Radon transform based on the tangent-squared approximation.

Water-bottom diffracted multiples, even from a flat water-bottom, do not migrate as primary reflections (Alvarez, 2005). That is, they do not focus to zero subsurface offset even if migrated with constant water velocity. These multiples migrate to both positive and negative subsurface offsets in SODCIGs depending on the relative position of the diffractor with respect to the receiver (for receiver-side diffracted multiples). In ADCIGs, these multiples have their apex at non-zero aperture angle, similar to their behavior in data space (CMP gathers) (Alvarez, 2005). We propose to attenuate these multiples with an apex-shifted Radon transform similar to that used by Alvarez et. al. (2004) but replacing the tangent-squared Radon kernel with our new equation for the residual moveout of the multiples in ADCIGs. Apex-shifted transforms were introduced for data interpolation by Trad (2002) and for attenuation of diffracted multiples in data space by Hargreaves (2003).

The real impact of our method for attenuating diffracted multiples is likely to be in 3D rather than in 2D, though the results that we show in this paper are limited to 2D. Biondi and Tisserant (2004) have presented a method for computing 3D ADCIGs from full 3D prestack migration. These 3D ADCIGs are functions of both the aperture angle and the reflection azimuth. Simple ray tracing modeling shows that out-of-plane multiples map into events with shifted apexes (like the 2D diffracted multiples) and different reflection azimuth than the primaries. Attenuation of these multiples from 3D ADCIGs can be accomplished with a methodology similar to the one we present in this paper.

The next section presents the general formalism for the mapping of first order specular and diffracted water-bottom multiples. We give closed-form equations for the residual moveout of the multiples in SODCIGs and ADCIGs in the special case of flat water-bottom specular multiple. Para-
metric equations for diffracted and specular multiples from dipping water bottom can be found in Alvarez (2005). The following section discusses the design of an apex-shifted Radon transform that uses as kernel our equation for the residual moveout of the specular multiple derived in the previous section. The apex-shifted transform is then applied to a real 2D section from the Gulf of Mexico. The next section discusses some important practical issues of our approach and the last section gives our conclusions.

KINEMATICS OF 2D MULTIPLES IN IMAGE SPACE

In this section we give the equations that map first-order water-bottom multiple reflections from data space (CMP gathers) to image space (SODCIGs and ADCIGs). We study in detail the special case of a specular multiple from a flat water-bottom. The equation that we derive for the residual moveout of the multiples in ADCIGs for this special case will be the basis for the attenuation of the multiples in the Radon domain. Alvarez (2005) gives parametric equations for other simple cases: specular multiple from a dipping water-bottom and diffracted multiples from flat and dipping water-bottom.

General formulation

The propagation path of a first-order water-bottom multiple, as shown in Figure 1, consists of four segments, such that the total travel-time for the multiple is given by

$$ t_m = t_{s1} + t_{s2} + t_{r2} + t_{r1}, $$

where the subscript \( s \) refers to the source-side rays and the subscript \( r \) refers to the receiver-side rays. The data space coordinates are \((m_D, h_D, t_m)\) where \( m_D \) is the horizontal position of the common-midpoint (CMP) gather and \( h_D \) is the half-offset between the source and the receiver.
Wave-equation migration maps the CMP gathers to SODCIGs with coordinates \((m_\xi, h_\xi, z_\xi)\) where \(m_\xi\) is the horizontal position of the image gather, and \(h_\xi\) and \(z_\xi\) are the half subsurface-offset and the depth of the image, respectively.

As illustrated in the sketch of Figure 2, at any given depth the spatial coordinates of the source and receiver rays are given by:

\[
x_{s\xi} = m_D - h_D + V_1 (t_{s1} \sin \alpha_s + \rho \tilde{t}_{s2} \sin \beta_s),
\]

\[
x_{r\xi} = m_D + h_D - V_1 (t_{r1} \sin \alpha_r + \rho \tilde{t}_{r2} \sin \beta_r),
\]

where \(V_1\) is the water velocity, \(\rho = V_2 / V_1\) with \(V_2\) the sediment velocity, \(\alpha_s, \alpha_r\) are the acute takeoff angles of the source and receiver rays with respect to the vertical and \(\beta_s, \beta_r\) are the angles of the refracted source and receiver rays, respectively. The coordinates of the migrated multiple in the image space are given by:

\[
h_\xi = \frac{x_{r\xi} - x_{s\xi}}{2} = h_D - \frac{V_1}{2} [t_{s1} \sin \alpha_s + t_{r1} \sin \alpha_r + \rho (\tilde{t}_{s2} \sin \beta_s + \tilde{t}_{r2} \sin \beta_r)],
\]

\[
z_\xi = V_1 (t_{s1} \cos \alpha_s + \rho \tilde{t}_{s2} \cos \beta_s) = V_1 (t_{r1} \cos \alpha_r + \rho \tilde{t}_{r2} \cos \beta_r),
\]

\[
m_\xi = \frac{x_{r\xi} + x_{s\xi}}{2} = m_D + \frac{V_1}{2} (t_{s1} \sin \alpha_s - t_{r1} \sin \alpha_r + \rho (\tilde{t}_{s2} \sin \beta_s - \tilde{t}_{r2} \sin \beta_r)),
\]

The traveltime of the refracted ray segments \(\tilde{t}_{s2}\) and \(\tilde{t}_{r2}\) can be computed from the two imaging conditions: (1) at the image point the depth of both rays has to be the same (since we are computing horizontal subsurface offset gathers) and (2) \(t_{s2} + t_{r2} = \tilde{t}_{s2} + \tilde{t}_{r2}\) which follows immediately from equation 1 since at the image point the total extrapolated time equals the traveltime of the multiple.

As shown in Appendix A, the traveltimes of the refracted rays are given by

\[
\tilde{t}_{s2} = \frac{t_{s1} \cos \alpha_r - t_{s1} \cos \alpha_s + \rho (t_{s2} + t_{r2}) \cos \beta_r}{\rho (\cos \beta_s + \cos \beta_r)},
\]

\[
\tilde{t}_{r2} = \frac{t_{s1} \cos \alpha_r - t_{r1} \cos \alpha_s + \rho (t_{s2} + t_{r2}) \cos \beta_s}{\rho (\cos \beta_s + \cos \beta_r)}.
\]
The refracted angles are related to the takeoff angles by Snell’s law: \( \sin(\beta + \varphi) = \rho \sin(\alpha + \varphi) \) and \( \sin(\beta - \varphi) = \rho \sin(\alpha - \varphi) \), from which we get

\[
\sin \beta_s = \rho \sin(\alpha + \varphi) \cos \varphi - \sqrt{1 - \rho^2 \sin^2(\alpha + \varphi)} \sin \varphi, \quad (9)
\]

\[
\sin \beta_r = \rho \sin(\alpha - \varphi) \cos \varphi + \sqrt{1 - \rho^2 \sin^2(\alpha - \varphi)} \sin \varphi, \quad (10)
\]

\[
\cos \beta_s = \sqrt{1 - \rho^2 \sin^2(\alpha + \varphi)} \cos \varphi + \rho \sin(\alpha + \varphi) \sin \varphi, \quad (11)
\]

\[
\cos \beta_r = \sqrt{1 - \rho^2 \sin^2(\alpha - \varphi)} \cos \varphi - \rho \sin(\alpha - \varphi) \sin \varphi. \quad (12)
\]

**Figure 2 about here**

In ADCIGs, the mapping of the multiples can be directly related to the previous equations by the geometry shown in Figure 2. The half-aperture angle is given by

\[
\gamma = \frac{\beta_r + \beta_s}{2}, \quad (13)
\]

which is the same equation derived for converted waves by Rosales and Biondi (2005). The depth of the image point in ADCIGs \( z_{\xi_r} \) is given by (Appendix B)

\[
z_{\xi_r} = z_{\xi} - h_{\xi} \tan \gamma. \quad (14)
\]

Equations 4–6 describe the transformation performed by wave-equation migration between CMP gathers \( (m_D, h_D, t) \) and SODCIGs \( (m_{\xi}, h_{\xi}, z_{\xi}) \). Equations 7–12 relate the traveltimes and angles of the refracted segments to parameters that can in principle be computed from the data (traveltimes, takeoff angles, reflector dips and velocities). Equations 13 and 14 provide the transformation from SODCIGs to ADCIGs. These equations are valid for any first-order water-bottom multiple, whether from a flat or dipping water-bottom. They even describe the migration of source- or receiver-side diffracted multiples with the diffractor at the water bottom, since no assumption has been made relating \( \alpha_r \) and \( \alpha_s \) or the individual travelt ime segments. They are, however, of little practical use.
unless we can relate the individual traveltime segments \((t_{s1}, t_{s2}, t_{r2}, t_{r1})\), and the angles \(\alpha_s\) and \(\alpha_r\) to the known data space coordinates \((m_D, h_D, t_m)\) and the model parameters \((V_1, \varphi \text{ and } \rho)\). This may not be easy or even possible analytically for all situations, but it is for the simple but important case of a specular multiple from a flat water-bottom.

**Specular multiple from flat water-bottom**

The traveltime of the first-order water-bottom multiple is given by

\[
t_m = \frac{4}{V_1} \sqrt{\left(\frac{h_D}{2}\right)^2 + Z_{wb}^2} = \sqrt{t_m^2(0) + \left(\frac{2h_D}{V_1}\right)^2},
\]  
which is simply the traveltime of a primary at twice the depth of the water-bottom \(Z_{wb} = \frac{V_{1m}(0)}{4}\).

From the symmetry of the problem, \(t_{s1} = t_{s2} = t_{r1} = t_{r2} = t_m/4\) and \(\alpha_s = \alpha_r\), which in turn means \(\beta_s = \beta_r\). Furthermore, from Equations 7 and 8 it immediately follows that \(\tilde{t}_{s2} = t_{s2}\) and \(\tilde{t}_{r2} = t_{r2}\) which says that the traveltimes of the refracted rays are equal to the traveltimes of the corresponding segments of the multiple. Equation 4 thus simplifies to

\[
h_{\xi} = \frac{h_D}{2}(1 - \rho^2),
\]  
which indicates that the subsurface offset at the image point of a trace with half surface offset \(h_D\) depends only on the velocity contrast between the water and the sediments. In particular, if the trace is migrated with the water velocity, \(i.e. \rho = 1\), then \(h_{\xi} = 0\) which proves our claim that the multiple is imaged exactly as a primary. It should also be noted that, since usually sediment velocity is faster than water velocity, then \(\rho^2 > 1\) and therefore the multiples are mapped to subsurface offsets with the opposite sign to that of the surface offset \(h_D\) when migrated with sediment velocity.

From Equation 5, the depth of the image point can be easily computed as

\[
z_{\xi} = Z_{wb} + \frac{\rho}{2} \sqrt{h_D^2(1 - \rho^2) + 4Z_{wb}^2},
\]  
8
which for migration with the water velocity reduces to \( z_\xi = 2Z_{wb} \), showing that the multiple is migrated as a primary at twice the water depth as is intuitively obvious. Finally, from Equation 6, the horizontal position of the image point reduces to

\[
m_\xi = m_D.
\]  

This result shows that the multiple is mapped in the image space to the same horizontal position as the corresponding CMP even if migrated with sediment velocity. This result is a direct consequence of the symmetry of the raypaths of the multiple reflection in this case. For dipping water bottom or for diffracted multiples this is not the case (Alvarez, 2005).

Equations 16–18 give the image space coordinates in terms of the data space coordinates. An important issue is the functional relationship between the subsurface offset and the image depth, since it determines the moveout of the multiples in the subsurface-offset-domain common-image-gathers (SODCIGs). Replacing \( h = 2h_\xi/(1 - \rho^2) \) and \( Z_{wb} = z_\xi(0)/(1 + \rho) \) in Equation 17 we get

\[
z_\xi = \frac{z_\xi(0)}{1 + \rho} + \rho \sqrt{\left( \frac{z_\xi(0)}{1 + \rho} \right)^2 + \frac{h_\xi^2}{1 - \rho^2}} \quad (\rho \neq 1)
\]  

which shows that the moveout is an hyperbola (actually half of an hyperbola since we already established that \( h_\xi \leq 0 \) if \( h_D \geq 0 \)).

Figure 3 shows an SODCIG for a specular water-bottom multiple from a flat water-bottom 500 m deep. The data was migrated with a two-layer velocity model: the water layer of 1500 m/s and a sediment layer of velocity 2500 m/s. Larger subsurface offsets (which according to Equation 16 correspond to larger surface offsets) map to shallower depths for the usual situation of \( \rho > 1 \), as we should expect since the rays are refracted to increasingly larger angles until the critical reflection
angle is reached. Also notice that the hyperbola is shifted down by a factor \((1 + \rho)\) with respect to its
image point when migrated with water velocity.

In angle-domain common-image-gathers (ADCIGs), the half-aperture angle reduces to \(\gamma = \beta_s = \beta_r\), which in terms of the data space coordinates is given by

\[
\gamma = \sin^{-1}\left(\frac{2\rho h_D}{V_1 t_m}\right). \tag{20}
\]

The depth of the image can be easily computed from Equation 14. In particular, if the data are mi-
grated with the velocity of the water, then \(\rho = 1\), and therefore \(z_{\xi,y} = 2Z_{wb}\), which means a horizontal
line in the \((z_{\xi,y}, \gamma)\) plane. Equivalently, we can say that the residual moveout in the \((z_{\xi,y}, \gamma)\) plane is
zero, once again corroborating that the water-bottom multiple is migrated as a primary if \(\rho = 1\).

Equation 14 can be expressed in terms of the data space coordinates using Equations 16 and 17 and
noting that

\[
\tan \gamma = \tan \beta_s = \frac{\rho \sin \alpha_s}{\sqrt{1 - \rho^2 \sin^2 \alpha_s}} = \frac{\rho h_D}{\sqrt{4Z_{wb}^2 + h_D^2(1 - \rho^2)}} \tag{21}
\]

If \(\rho = 1\) this expression simplifies to \(\tan \gamma = \frac{h_D}{2Z_{wb}}\), which is the aperture angle of a primary at twice
the water-bottom depth.

As we did with the SODCIG, it is important to find the functional relationship between \(z_{\xi,y}\) and
\(\gamma\) since it dictates the residual moveout of the multiple in the ADCIG. Plugging Equations 16 and 17
into equation 14, using Equations 15, and 20 to eliminate \(h_D\) and simplifying we get

\[
z_{\xi,y} = Z_{wb} \left[1 + \frac{\cos \gamma (\rho^2 - \tan^2 \gamma (1 - \rho^2))}{\sqrt{\rho^2 - \sin^2 \gamma}}\right] \tag{22}
\]

\[
= \frac{z_{\xi,y}(0)}{1 + \rho} \left[1 + \frac{\cos \gamma (\rho^2 - \tan^2 \gamma (1 - \rho^2))}{\sqrt{\rho^2 - \sin^2 \gamma}}\right]. \tag{23}
\]

Once again, when the multiple is migrated with the water velocity \((\rho = 1)\) we get the expected result
\(z_{\xi,y} = z_{\xi,y}(0)\), that is, flat moveout (no angular dependence). The residual moveout in ADCIGs is
therefore given by

\[ \Delta n_{\text{RMO}} = z_{\xi_\gamma}(0) - z_{\xi_\gamma} = \left[ \rho - \frac{\cos \gamma(\rho^2 - (1 - \rho^2) \tan^2 \gamma)}{\sqrt{\rho^2 - \sin^2 \gamma}} \right] \frac{z_0}{1 + \rho}. \]  

(24)

This equation reduces to that of Biondi and Symes (2004) when \( \gamma \) is small (Appendix C), which is when we can neglect ray bending at the multiple-generating interface. Figure 4 shows the ADCIG corresponding to the SODCIG shown in Figure 3. Notice that the migrated depth at zero aperture angle is the same as that for the zero sub-surface offset in Figure 3. For larger aperture angles, however, the migrated depth increases as indicated in equation 23. The continuous line corresponds to Equation 24 whereas the dotted line corresponds to the tangent-squared of Biondi and Symes (2004). For large aperture angles the departure of the straight ray approximation can be significant.

**Figure 4 about here**

**RADON TRANSFORM**

In this section we show how to exploit the difference in residual moveout between primaries and multiples in ADCIG’s given by Equation 24 to design a Radon transform that focuses the primaries and multiples to separate regions of the Radon domain. The general expression for the Radon transform in the angle domain is (Sava and Guitton, 2003)

\[ z(q, \gamma) = z_0 + q \ g(\gamma). \]  

(25)

where \( q \) is a measure of curvature and \( g(\gamma) \) is the function that approximates the residual moveout of the multiples as a function of the aperture angle \( \gamma \). Sava and Guitton (2005) and Alvarez et. al. (2004) used the tangent-squared approximation of Biondi and Symes (2004)

\[ g(\gamma) = \tan^2 \gamma. \]  

(26)
but for the focusing of the multiples a better approximation is given by Equation 24:

\[ g(\gamma) = \frac{1}{1 + \rho} \left[ \frac{\cos \gamma (\rho^2 - (1 - \rho^2) \tan^2 \gamma)}{\sqrt{\rho^2 - \sin^2 \gamma}} - \rho \right]. \]  

(27)

This approximation is better because it takes into account ray bending at the multiple-generating interface. This can be seen in Figure 5 which shows a comparison of the Radon transforms defined by equations 24 and A-7 applied to a synthetic ADCIG. Notice that the focusing of the primaries does not change since their moveout is zero. The multiples, on the other hand, are better focused with the new transform since the curvature more closely follows their residual moveout in the ADCIGs.

Figure 5 about here

Apex-shifted Radon Transform

The apex of the residual moveout curve of the diffracted multiples in ADCIGs is shifted away from zero aperture angle (Alvarez, 2005). Therefore, to attenuate the diffracted multiples, we define the transformation from “data” space (ADCIGs) to model space (Radon-transformed domain) as:

\[ m(h, q, z') = \sum_{\gamma} d(\gamma, z = z' + q g(\gamma - h)), \]

and from model space to data space as

\[ d(\gamma, z) = \sum_{q} \sum_{h} m(h, q, z' = z - q g(\gamma - h)), \]

where this time \( g(\gamma) \) is given by Equation 27 and \( h \) is the lateral apex shift (in units of aperture angle). In this way, we transform the two-dimensional data space of ADCIGs, \( d(z, \gamma) \), into a three-dimensional model space, \( m(z', q, h) \).

In the ideal case, primaries would be perfectly horizontal in the ADCIGs and would thus map in the model space to the zero-curvature \( (q = 0) \) plane, \( i.e., \) a plane of dimensions depth and apex-shift
distance \((h, z')\). Specular multiples would map to the zero apex-shift distance \((h = 0)\) plane, \(i.e.,\) a plane of dimensions depth and curvature \((q, z')\). Diffracted multiples would map elsewhere in the cube depending on their curvature and apex-shift distance.

**Sparsity Constraint**

As a linear transformation, the apex-shifted Radon transform can be represented simply as

\[
d = Lm,
\]

where \(d\) is the (migrated) data in the angle domain, \(m\) is the model in the Radon domain and \(L\) is the forward apex-shifted Radon transform operator. To find the model \(m\) that best fits the data in a least-squares sense, we minimize the objective function:

\[
f(m) = \|Lm - d\|^2 + \epsilon^2 b^2 \sum_{i=1}^{n} \ln \left(1 + \frac{m_i^2}{b^2}\right),
\]

where the second term is a Cauchy regularization that enforces sparseness in the model space. Here \(n\) is the size of the model space, and \(\epsilon\) and \(b\) are two constants chosen a-priori: \(\epsilon\) which controls the amount of sparseness in the model space and \(b\) which controls the minimum value below which everything in the Radon domain should be zeroed. The least-squares inverse of \(m\) is given by

\[
\hat{m} = \left[L'L + \epsilon^2 \text{diag}\left(\frac{1}{1 + \frac{m_i^2}{b^2}}\right)\right]^{-1} L'd,
\]

where \text{diag} defines a diagonal operator. Because the model space can be large, we estimate \(m\) iteratively. Notice that the objective function in Equation (29) is non-linear because the model appears in the definition of the regularization term. Therefore, we use a limited-memory quasi-Newton method (Guitton and Symes, 2003) to find the minimum of \(f(m)\).
In this section we introduce the seismic line we use to test our apex-shifted Radon transform. The line is from the Gulf of Mexico and was shot over a large salt body. The presence of the salt creates a host of multiples that obscure any genuine subsalt reflections, as shown in the angle stack of Figure 6. Most multiples are surface-related peg-legs with a leg related to the water bottom, shallow reflectors or the top of salt. Below the edges of the salt we also encounter diffracted multiples (e.g., CMP position 6000 m below 4000 m depth in Figure 6).

Figure 6 about here

Figure 7 shows four ADCIGs obtained with wave-equation migration as described by Sava and Fomel (2003). Notice that although the data is marine, the ADCIGs show positive and negative aperture angles. We used reciprocity to simulate negative offsets and interpolation to compute the two shortest-offset traces not present in the original data. The CMP gathers were then migrated to angle gathers. The purpose of having both positive and negative aperture angles is to see more clearly the position of the apexes of the diffracted multiples. The top two ADCIGs correspond to lateral positions directly below the edges of the salt body (CMP positions 6744 m and 22056 m in Figure 6). Notice how the apexes of the diffracted multiples are shifted away from zero aperture angle (e.g., the seagull-looking event at about 4600 m in panel (a)). For comparison, the bottom panels in Figure 7 show two ADCIGs that do not have diffracted multiples. Figure 7(c) corresponds to an ADCIG below the sedimentary section (CMP 3040 m in Figure 6) and Figure 7(d) to an ADCIG below the salt body (CMP position 12000 m in Figure 6). In these ADCIGs all the multiples are specularly-reflected and thus have their apexes at zero aperture angle.

Figure 7 about here
MULTIPLE ATTENUATION RESULTS

With ideal data, attenuating both specular and diffracted multiples could, in principle, be accomplished simply by zeroing out (with a suitable taper) all the $q$-planes except $q = 0$ in the model cube $m(z', q, h)$ and taking the inverse apex-shifted Radon transform. In practice, however, the primaries may not be well-corrected and primary energy may map to a few other $q$-planes. Energy from the multiples may also map to those planes and so we have the usual trade-off of primary preservation versus multiple attenuation. The advantage now is that the diffracted multiples are well focused to their corresponding $h$-planes and can therefore be easily attenuated.

To illustrate the mapping of the primaries, the specular multiples and the diffracted multiples, between the image space $(z, \gamma)$ and the apex-shifted Radon space $(z', q, h)$, we chose the ADCIG in Figure 7(a). Although this ADCIG shows no discernible primaries below the salt, it nicely shows the apex-shifted moveout of the diffracted multiples. This ADCIG was transformed to the Radon domain with the apex-shifted transform presented before section. Figure 8(a) shows the $h = 0$ plane from the $(z', q, h)$ volume. This plane corresponds to zero apex-shift and therefore this is where the majority of the specular multiples should map. Figure 8(b) shows the zero-curvature $q = 0$ plane, that is, the plane where the primaries should map. Notice that since the primaries are flat, they are independent of the apex-shift $h$ and therefore map as flat lines on this plane. For comparison, Figure 8(c) shows the $h = 8$ plane. This corresponds to the apex-shift of the most obvious diffracted multiple and we see its energy mapped on this plane at about 4000 m. Finally, Figure 8(d) shows a plane at a large curvature, $q = 7200$. Notice the energy from the diffracted multiple at $h = 10$ approximately.

Figure 8 about here

It is important to emphasize the difference between the standard transform and the apex-shifted
transform. While the $h = 0$ plane of the apex-shifted transform is similar to the standard transform, they are not the same, as shown in Figure 9. Both panels in this figure are plotted with the exact same clip. Primaries are mapped near the $q = 0$ line in both planes while specular multiples are mapped to other $q$ values. Notice how in the standard transform Figure 9(a), the diffracted-multiple energy is mapped as background noise, especially at the largest positive and negative $q$ values. In the $h = 0$ plane of the apex-shifted transform, however, the diffracted multiples are not present since their moveout apex is not zero, and so do not obscure the mapping of the specular multiples. Notice also that the primary energy is much higher than in Figure 9(a) since in the apex-shifted transform the primary energy is mapped not only to the $h = 0$ plane but to other $h$ planes as well as illustrated previously in Figure 8(b).

**Figure 9 about here**

Rather than suppressing the multiples in the model domain, we chose to suppress the primaries and inverse transform the multiples to the “data” space (ADCIGs). The primaries were then recovered by subtracting the multiples from the data. Figure 10 shows a close-up comparison of the primaries extracted with the standard 2D transform (Sava and Guitton, 2003) and with the apex-shifted Radon transform for the two ADCIGs at the top in Figure 7. The standard transform (Figures 10a and 10c) was effective in attenuating the specular multiples, but failed at attenuating the diffracted multiples (below 4000 m), which are left as residual multiple energy in the primary data. Again, this is a consequence of the apex shift of these multiples. There appears not to be any subsalt primary in Figures 10a and 10b and only one clearly visible subsalt primary in Figures 10c and 10d (just above 4400 m). This primary was well preserved with both transformations.

Figure 11 shows a similar comparison for the extracted multiples. Notice how the diffracted multiples were correctly identified and extracted by the apex-shifted Radon transform, in particular
in Figure 11b. In contrast, the standard 2D transform misrepresent the diffracted multiples as though they are specular multiples as seen in Figure 11a. We can take advantage of the three dimensions in the model space of the apex-shifted transform to separate the diffracted multiples from the specular ones. This is shown in Figure 12. The diffracted multiples are clearly seen in Figure 12c.

Figure 10 about here

Figure 11 about here

Figure 12 about here

In order to assess the effect of the improved attenuation of the diffracted multiples on the angle stack, we processed all ADCIGs. Figure 13 shows a close-up view of the stack of the primaries extracted with the standard Radon transform, the stack of the primaries extracted with the apex-shifted Radon transform, and their difference. All panels are plotted with the exact same clip value. Notice that the diffracted multiple energy below the edge of the salt (5000 m to 7000 m) that appears as steeply-dipping noise with the standard transform, has been attenuated with the apex-shifted transform. This is shown in detail in Figure 13c. It is very difficult to identify any primary reflections below the edge of the salt, so it is hard to assess if the primaries have been equally preserved with both transforms. It is known, however, that for this dataset, there are no multiples above a depth of about 3600 m, between CMP positions 3000 m to 5000 m. The fact that the difference panel appears nearly white in that zone shows that the attenuation of the diffracted multiples did not affect the primaries. Of course, this is only true for those primaries that were correctly imaged, so that their moveout in the ADCIGs was nearly flat. Weak subsalt primaries may not have been well-imaged due to inaccuracies in the migration velocity field and may therefore have been attenuated with both the standard and the apex-shifted Radon transforms.
For the sake of completeness, Figure 14 shows the extracted multiples with the standard and the apex-shifted Radon transforms and their difference. Again, the main difference is largely in the diffracted multiples.

DISCUSSION

For the specular multiple from a flat water-bottom, the mapping between the image-space coordinates and the data-space coordinates is essentially 2D since $m_D = m_\xi$, which allowed the computation of closed-form expressions for the residual moveout of the multiples in both SODCIGs and ADCIGs. For diffracted multiples in particular, it is not easy to compute equivalent closed-form expressions, but we can compute numerically the residual moveout curves given the expressions for $(h_\xi, z_\xi, m_\xi)$ in terms of the data-space coordinates $(t_m, h_D, m_D)$, and the model parameters $(V_1, \varphi, \rho)$ (Alvarez, 2005). For dipping water-bottom, the dip can be estimated from the data. For diffracted multiples, the position of the diffractor corresponds to the lateral position of the apex of the multiple diffraction in a shot gather and can also be estimated from the data.

specularly-reflected water-bottom multiples, whether from a flat or dipping water-bottom, map to zero or negative subsurface offsets when migrated with the velocity of the sediments $\rho > 1$ for $h_D \geq 0$. On the other hand, primaries migrated with slower velocities map to positive subsurface offsets. It may be possible to exploit this fact to attenuate these multiples by migrating the data with a velocity faster than that of the multiples but slower than that of the primaries.

Working in image space (ADCIGs in this case) is convenient because the migration takes care
of the complexity of the wavefield propagation. Attenuating the multiples after migration, however, does not come without a price. The estimation of the migration velocities may be more difficult and less accurate because of the presence of the multiples. There is therefore an inherent trade-off when choosing to work in image space. Good migration velocities for weak subsalt primaries may be particularly difficult to estimate in the presence of the multiples. On the other hand, the parabolic or hyperbolic assumption for the moveout of the multiples in data space may not be appropriate at all in complex media. An alternative could be to do a standard Radon demultiple before prestack migration to facilitate the choice of the migration velocities and an apex-shifted Radon demultiple on the ADCIGs to attenuate residual multiples, in particular diffracted multiples.

The results shown in the previous section demonstrate that with the apex-shifted Radon transform it is possible to attenuate, although not completely remove, the diffracted multiples. It should be noted, however, that in our seismic section it is very difficult to find a legitimate primary reflection below the salt and in particular below the edge of the salt, where the contamination by the diffracted multiples is stronger. It is somewhat disappointing that the attenuation of the diffracted multiples didn’t help in uncovering any meaningful primary reflections in this case. We expect the situation to be different with other datasets.

We should also emphasize that adding the extra dimension to deal with the diffracted multiples does not in itself resolve the usual trade-off between primary preservation and multiple attenuation. We saw this limitation in this case, which forced us to let some residual multiple energy leak into the extracted primaries. The flatter the primaries are in the ADCIGs, and the more accurately the kernel of the Radon transform approximates the residual moveout of the multiples, the better are our chances to reduce the residual multiple energy.

With 3D data, the ADCIGs are function of the reflection azimuth as well as the aperture angle
(Biondi and Tisserant, 2004). The ADCIGs therefore are three dimensional and even for the specular multiples the Radon transform will also be three-dimensional. For the diffracted multiples, the apex shift is a function of both the aperture angle and the reflection azimuth and the Radon transform would be more complicated. This is the subject of current research.

**CONCLUSIONS**

To attenuate multiples in the image space via filtering in the Radon domain, we need an accurate representation of their residual moveouts in either SODCIGs or ADCIGs. Accounting for ray-bending at the multiple-generating interface increases the focusing power of the Radon transform and therefore the separation between primaries and multiples.

The apex-shifted Radon transform in ADCIGs to map from \((z, \gamma)\) to \((z', q, h)\) has proven to be effective in attenuating specular and diffracted multiples in 2D marine data. The residual moveout of both multiples in ADCIGs is well-behaved and the extra dimension provided by the apex-shift allows the attenuation of the multiples without compromising the integrity of the primaries.

**ACKNOWLEDGMENTS**

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**REFERENCES**


TRAVELTIME OF REFRACTED RAYS

In this Appendix we derive equations 7 and 8. From equation 5 we have:

\[ t_s \cos \alpha_s + \rho \tilde{t}_s \cos \beta_s = t_r \cos \alpha_r + \rho \tilde{t}_r \cos \beta_r, \]  

(A-1)

and, from the imaging condition (the sum of the traveltime of the extrapolated rays at the image point has to be equal to the traveltime of the multiple) we have

\[ t_s + t_r = \tilde{t}_s + \tilde{t}_r. \]  

(A-2)

Solving those two equations for \( \tilde{t}_s \) and \( \tilde{t}_r \) we get

\[ \tilde{t}_s = \frac{t_r \cos \alpha_r - t_s \cos \alpha_s + \rho(t_s + t_r) \cos \beta_r}{\rho(\cos \beta_s + \cos \beta_r)}, \]  

(A-3)

\[ \tilde{t}_r = \frac{t_s \cos \alpha_s - t_r \cos \alpha_r + \rho(t_s + t_r) \cos \beta_s}{\rho(\cos \beta_s + \cos \beta_r)}. \]  

(A-4)
It is interesting to check these equations in two particular cases. For a specular multiple from a flat water-bottom, we have $\alpha_s = \alpha_r$, $\beta_s = \beta_r$, $t_{s1} = t_{s2} = t_{r1}$ and therefore we get $\tilde{t}_{s2} = t_{s2}$ and $\tilde{t}_{r2} = t_{r2}$ as the geometry of the problem requires. Notice that this is true for any $\rho$. The second case is for a specular water-bottom multiple migrated with water velocity ($\rho = 1$). In that case, $\beta_s = \alpha_s$ and $\beta_r = \alpha_r$. Furthermore, since the multiple behaves as a primary, $(t_{s1} + t_{s2}) \cos \alpha_s = (t_{r1} + t_{r2}) \cos \alpha_r$ and we again get $\tilde{t}_{s2} = t_{s2}$ and $\tilde{t}_{r2} = t_{r2}$.

**IMAGE DEPTH IN ADCIGS**

**Figure 15 about here**

Figure 15 shows the basic construction to compute the image depth in ADCIGs based on the image depth in SOCIGs. Triangles $ABD$ and $CBD$ are congruent since they have one side common and the other equal because $|AB| = |BC| = h_\xi$. Therefore, $\theta = \pi/2 - \beta_r + \delta$. Also, triangles $AED$ and $FCD$ are congruent because $|AD| = |CD|$ and also $|AE| = |CF|$ (Biondi and Symes, 2004). Therefore, the angle $\delta$ in triangle $DCF$ is the same as in triangle $AED$. We can compute $\delta$ from the condition

$$
\theta + \delta + \beta_s = \frac{\pi}{2},
$$

$$
\frac{\pi}{2} - \beta_r + \delta + \beta_s = \frac{\pi}{2},
$$

$$
\delta = \frac{\beta_r - \beta_s}{2}.
$$

The depth of the image point in the ADCIG, from triangle $ABC$, is therefore

$$
z_{\xi'} = z_\xi + z^a = z_\xi + (\text{sign}(h_\xi))h_\xi \cot \left(\frac{\pi}{2} - \beta_r + \delta\right).
$$

(A-5)

Replacing the expression for $\delta$ we get, after some simplification (and taking $\text{sign}(h_\xi) = -1$)

$$
z_{\xi'} = z_\xi + z^a = z_\xi - h_\xi \tan \left(\frac{\beta_r + \beta_s}{2}\right) = z_\xi - h_\xi \tan(\gamma).
$$

(A-6)
RESIDUAL MOVEOUT IN ADCIGS

In this appendix we show that, for a flat reflector, the residual moveout of the multiples in ADCIGs reduces to the tangent-squared expression derived by Biondi and Symes (2004) for the residual moveout of under-migrated primaries:

\[ \Delta n_{\text{RMO}} = (\rho - 1) \tan^2 \gamma z_0 n. \]  
(A-7)

Start with Equation 23

\[ z_{\xi_r} = z_{\xi_r}(0) \frac{1 + \cos \gamma (\rho^2 - (1 - \rho^2) \tan^2 \gamma)}{\rho^2 - \sin^2 \gamma}, \]  
(A-8)

where \( z_{\xi_r}(0) \) is the normal-incidence migrated-depth, (i.e. \( z_0 \)) in the previous equations.

There is an important and unfortunate difference in notation here, however, because \( \rho \) in equation A-7 is the ratio of the migration to the true slowness whereas \( \rho \) in equation A-8 is the ratio of the migration to the true velocity. Therefore, in order to get a better idea of how the approximation for the RMO of the multiples (accounting for ray bending at the reflector interface) relates to that of the primaries (neglecting ray bending), we rewrite equation A-8 replacing \( \rho \) by \( 1/\rho \) and \( z_{\xi_r}(0) \) with \( z_0 \) to get:

\[ z_{\xi_r} = \left[ \rho + \frac{\cos \gamma (1 - (\rho^2 - 1) \tan^2 \gamma)}{\sqrt{1 - \rho^2 \sin^2 \gamma}} \right] \frac{z_0}{1 + \rho}. \]  
(A-9)

Since \( \Delta n_{\text{RMO}} = z_0 - z_{\xi_r} \) we get:

\[ \Delta n_{\text{RMO}} = \left[ 1 - \frac{\cos \gamma (1 - (\rho^2 - 1) \tan^2 \gamma)}{\sqrt{1 - \rho^2 \sin^2 \gamma}} \right] \frac{z_0}{1 + \rho}. \]  
(A-10)

For small \( \gamma \), \( \sin \gamma \approx 0 \) and \( \cos \gamma \approx 1 \), therefore

\[ \Delta n_{\text{RMO}} = (\rho^2 - 1) \tan^2 \gamma \frac{z_0}{1 + \rho} = (\rho - 1) \tan^2 \gamma z_0. \]  
(A-11)

This is the same as equation A-7 save for the unit vector \( n \). This result is intuitively appealing because
it shows that the approximation of neglecting ray bending at the reflecting interface deteriorates as the aperture angle increases which is when the ray bending is larger.
1 Water-bottom multiple. The subscript $s$ refers to the source and the subscript $r$ to the receiver.

2 Imaging of water-bottom multiple in SODCIG and ADCIG. The subscript $\xi$ refers to the image point. The line AB represents the apparent reflector at the image point.

3 Subsurface offset domain common image gather of a water-bottom multiple from a flat water-bottom. Water velocity is 1500 m/s, water depth 500 m, sediment velocity 2500 m/s and surface offsets from 0 to 2000 m. Overlaid is the residual moveout curve computed with Equation 19.

4 ADCIG for a water-bottom multiple from a two flat-layer model. The dotted curve corresponds to the straight ray approximation whereas the solid curve corresponds to the ray-bending approximation.

5 Comparison of Radon transforms for a synthetic ADCIG. Panel (a) shows the ADCIG. Panel (b) corresponds to the straight-ray approximation whereas panel (c) corresponds to the ray-bending approximation.

6 Angle stack of migrated ADCIGs of 2D seismic line in the Gulf of Mexico. Notice that multiples below the salt obscure any primary reflections.

7 Angle domain common image gathers. (a) under the left edge of the salt, CMP at 6744 m; (b) under the right edge of the salt, CMP at 22056 m; (c) below the sedimentary section, CMP at 3040; (d) below the salt body, CMP at 12000 m.

8 Different views from the cube of the apex-shifted transform for the ADCIG at 6744 m. (a): zero apex-shift plane. (b) zero curvature plane. (c): plane at apex shift $h = 6$ and (d): plane at curvature $q = 72000$.

9 Radon transforms of the ADCIG in Figure 7b. (a): standard 2D transform. (b): $h = 0$ plane
of the apex-shifted 3D transform. Both panels plotted at the exact same clip value.

10 Comparison of primaries extracted with the 2D Radon transform (a) and (c) and with the apex-shifted Radon transform (b) and (d). Notice that some of the diffracted multiples remain in the result with the 2D transform.

11 Comparison of multiples extracted with the 2D Radon transform (a) and (c) and with the apex-shifted Radon transform (b) and (d).

12 Comparison of (a) diffracted and (b) specular multiples for the ADCIG in Figure 7a. Notice the lateral shifts in the apexes of the diffracted multiples.

13 Comparison of angle stacks for primaries.

14 Comparison of angle stacks for multiples.

15 Sketch to show the computation of the image depth in an ADCIG.
Figure 1.

file name: //mul_sketch1.ps  dimensions: width=8cm
Figure 3.

file name: //odcig1.ps  dimensions: width=9cm
Figure 4.

file name: //adcig1.ps  dimensions: width=9cm
Figure 5.

file name: //synth1.ps  dimensions: width=10cm
Figure 6.

file name: //angle_stack.ps    dimensions: width=15.0cm
Figure 7.

file name: //cags.ps  dimensions: width=12cm
Figure 8.

file name: ./envelopes.ps  dimensions: width=12cm
Figure 9.

file name: //radon_comp.ps  dimensions: width=12cm
Figure 10.

file name: //comp_prim1.ps  dimensions: width=12cm
Figure 11.

file name: //comp_mult1.ps  dimensions: width=12cm
Figure 12.

file name: ./comp_mult2.ps   dimensions: width=12cm
Figure 13.

file name: //comp_prim1_stack.ps  dimensions: width=15cm
Figure 14.

file name: ./comp_mult1_stack.ps  dimensions: width=15cm
Figure 15.

file name: ./mul_sketch17.ps    dimensions: width=8cm