Math 128A Spring 2002 | Handout # 3
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Homework 2: Nonlinear Equations: Bisection and Regula Falsi (due on February 7)

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1. Prove that the equation

$$f(x) = x + a\cos x = 0 \tag{1}$$

   has at least one solution (for every real a).

   Hint: Use the intermediate value theorem. You don't have to prove that $f(x)$ is continuous.

2. If the bisection method is applied on the initial interval from $a = 14$ to $b = 16$, how many iterations will be required to guarantee that the root is located to the maximum accuracy of the IEEE double-precision standard?

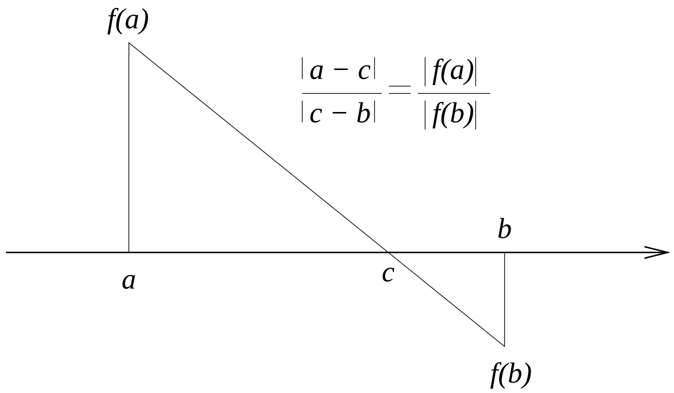
   Hint: A number in the IEEE double-precision standard can have the maximum of 53 significant digits in the binary format.

3. The bisection method can be implemented with the following schematic algorithm

```
BISECTION(f(x), a, b, xtol, ftol)
  fa ← f(a)
  fb ← f(b)
  ASSERT : SIGN(fa) ≠ SIGN(fb)
  for n ← 1, 2, ...
  do
     d ← b − a
     c ← a + d/2
     fc ← f(c)
     if |d| < xtol and |fc| < ftol
        then return c
     if SIGN(fc) = SIGN(fa)
        then
             a ← c
             fa ← fc
        else
             b ← c
```

   In this assignment, you will design an analogous algorithm for a different method. In this method, the input and the general strategy are the same as those in bisection, but instead of dividing the interval in half, we will divide it in proportion $|f(a)|/|f(b)|$ (see the figure.) This method is known as the method of chords or the method of false position or *regula falsi*.

   Modify the algorithm above to transform it to an algorithm for *regula falsi*.



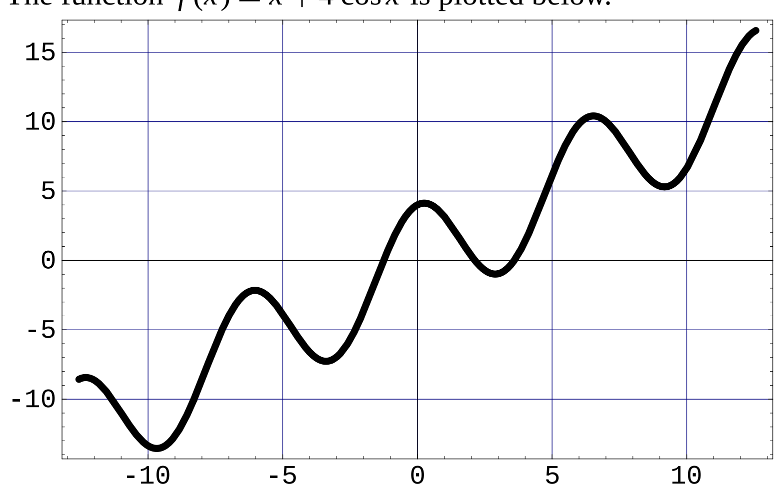
4. (Programming) Implement the bisection algorithm in the programming language of your choice and use it to solve the following three problems. In each case, perform 20 iterations and output your results in a table

n	a_n	b_n	c_n	$\lvert f(c_n)\rvert$

   (a) To test your program, start by computing something familiar. How about π? Solve the equation
   $$\sin x = 0 \tag{2}$$
   using the initial interval $a = 2$ and $b = 4$.
   Determine (experimentally) the number of iterations required to compute the number π with six significant decimal digits.

   (b) The equation
   $$x + e^x = 0 \tag{3}$$
   has the solution around $x \approx -0.6$. Find a better approximation using the initial interval $a = -1$ and $b = 0$.

   (c) The equation
   $$x + 4\cos x = 0 \tag{4}$$
   has three solutions. Locate all of them, selecting an appropriate initial interval for each. The function $f(x) = x + 4\cos x$ is plotted below.



5. (Programming) Implement your *regula falsi* algorithm and repeat the computation for the three problems above. Compare the tables. Which of the two methods appears to converge faster?