

Due Wednesday, May 1, 2013  
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## Lab 3: Depth imaging using Reverse Time Migration

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### ABSTRACT

In this exercise you will familiarize yourself with full wave-equation modeling, reverse time migration (RTM) and sub-surface common image gathers. You will be given a partially completed reverse time migration code to complete. In order to gain first-hand experience, you will then depth-migrate several pre-stack images of the SEG-EAGE salt model. Then you will modify the code to construct the sub-surface offset gathers after imaging.

### INTRODUCTION

Although Kirchhoff-based migration is still popular in the industry for its efficiency, wave-equation-based migration is also becoming a processing routine. The difficulties of seismic imaging below the salt bodies and in areas of high lateral velocity variation have drawn attention to reverse time migration (RTM), which, at least theoretically, is able to meet those challenges. By using a full two-way solution to the wave equation, it is able to construct images using all possible arrivals. Unlike the one-way downward continuation migration, RTM can handle extreme lateral velocity variations without imposing any dip limitations.

RTM has its limitations. One major drawback is the artifact produced by back-scattering at sharp velocity contrasts. In addition, due to the restrictions imposed by finite difference modeling, RTM images are restricted to lower frequency band. The computational cost for RTM is still considered to be high compare to other imaging technique.

You will first review the stability and dispersion condition in modeling using the FD isotropic acoustic wave equation. Second, you will complete a partially written RTM code by coding the imaging condition. Using this code, you will produce a depth image of the SEG-EAGE model. Afterward, you will explore some techniques people use to attenuate RTM artifacts. Finally, you will write a code that calculates sub-surface common image gathers.

## Full-waveform modeling

Read the classic paper by Dablain (1986), you will find it very useful.

In wave equation modeling using the finite difference method, one needs to pay attention to two things: 1) numerical stability and 2) numerical dispersion.

### *Numerical stability*

Numerical stability can be verified with the Courant Friedrichs Lewy condition (CFL condition). It is a necessary condition for convergence while solving partial differential equations numerically using finite differences. It arises when explicit FDTD schemes are used for the numerical solution. If the Courant condition is violated, the simulation will produce wildly incorrect results.

In the class note, you are given the necessary CFL condition for the FD modeling of the isotropic acoustic wave equation. For a second order 2D FDTD modeling, the Courant condition is

$$\frac{V_{max}\Delta t}{\min(\Delta x, \Delta y, \Delta z)} < \frac{\sqrt{2}}{2} \quad (1)$$

$V_{max}$  is the maximum value of your velocity model and  $\Delta t$  is the time sampling.  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$  are the spatial sampling in the  $x$ ,  $y$ , and  $z$  direction, respectively.

### *Numerical dispersion*

In numerical modeling, the phase velocity becomes a function of the discretization interval. When the phase velocity is different from the group velocity, certain frequencies in the propagated signal will travel faster or slower than it should be. In general, the dispersion condition can be expressed as,

$$\frac{V_{min}}{f_{max}\max(\Delta x, \Delta y, \Delta z)} > S \quad (2)$$

where  $f_{max}$  is the maximum frequency that you wish to propagate and  $V_{min}$  is the minimum value of your velocity model.  $S$  represents the minimum number of grid point per wavelength. The value of  $S$  depends on the finite difference approximation you use for your laplacian. You can find some reference value in the paper by Dablain.

1. You are given a velocity model of a land seismic acquisition with values in between 2000-4500m/s. What temporal and spatial spacings do you need to ensure numerical stability in 2D?
2. What would the answer be if the modeling is done with 3D?

3. The velocity model you have in question 1 is now added with a layer of ocean at 500m deep. Suppose you want to image at a frequency up to 50Hz, what should your spatial spacing be to avoid numerical dispersion? State the  $S$  value you use and the kind of FD approximation you are using.
4. Following from question 3, what should the time spacing be to ensure numerical stability?
5. In general, when you use a higher order finite difference stencil, the stability condition becomes less restrictive, that is,  $S$  decreases. Comment on the impact on the computational cost given a survey with fixed frequency bound and imaging aperture.

## Reverse time migration

Reverse time migration is based on the time-reversal property of the two-way wave equation and the cross-correlating imaging condition by Claerbout (1985). Two wavefields are independently propagated in shot-profile migration: 1) the receiver wavefield is propagated starting from the recorded data, and 2) the source wavefield is propagated starting from an assumed source wavelet. Both the source wavefield and the recorded wavefield are continued along the time axis; the source wavefield is propagated forward in time, whereas the recorded wavefield is propagated backward in time (hence the name "reverse-time"). The image is formed by cross-correlating the two wavefields and evaluating the correlation at zero-time.

In this section, you will calculate several pre-stack RTM images using a 2D slice of the EAGE-SEG salt model (see figure 1).

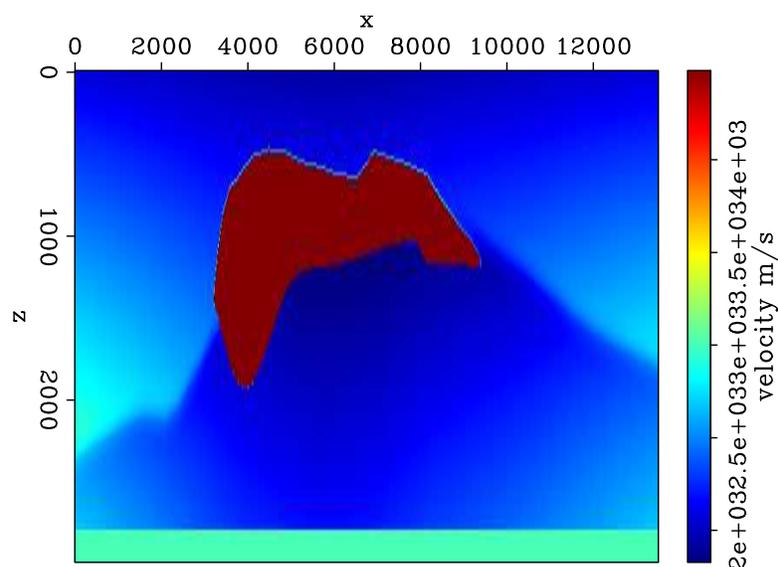


Figure 1: A 2D slice of the EAGE-SEG salt model.

1. Write down the pseudo code for the zero-lag cross-correlating imaging condition in RTM. That is transform equation (4.1) in your text book into a pseudo code. Define all variables and indices.
2. Open the file **src/rtm\_oper.f90** and put in your imaging condition. You might need to look through the code to find the necessary variables. Make sure your code compiles by typing **make bin/ReverseTime\_Mig.x**
3. After you successfully compile your code, calculate a shot-profile pre-stack RTM image by typing **make Data/rtm171.H**. 171 is an index for shot numbers. This survey contains 440 shots spread along the surface at 30m spacing. Include the pre-stack image **Data/rtm171.H** as a figure in this paper.

[Insert Image  
Here]

Figure 2: A pre-stack RTM image **Data/rtm171.H** of the EAGE-SEG salt model.

4. Now, calculate several more pre-stack RTM images with shot number from **172** to **180**. Create a make rule to sum these pre-stack images with the rest of the RTM images (**Data/rtmB.H**) to generate a post-stack RTM image. Include a figure in this paper.
5. Does your final RTM image correctly represent the reflector in the EAGE-SEG velocity model? Name the similarities and differences between the reflectors in your image with that in the velocity model.
6. What kind of migration artifacts do you notice?

## Migration artifacts

Although RTM is capable of illuminating structure in complex areas, it often produces a significant amount of high-amplitude, low-frequency noise above sharp interfaces.

# [Insert Image Here]

Figure 3: A stacked final RTM image of the EAGE-SEG salt model.

Other artifacts arise from cross-correlation of source and receiver wavefields that do not correspond to the same reflection event. The simplest mechanism that serves to attenuate these artifacts is by stacking across all the pre-stack images. However, stacking does not eliminate all noise. Figure 4 explains the formation of the high-amplitude low-frequency artifact in the region above a sharp velocity contrast.

1. Does downward-continuation migration produce the same kind of artifacts found in RTM? Explain.
2. Many techniques have been proposed to attenuate RTM artifacts. One crude way to suppress the low-frequency artifact is by applying a laplacian to the image. Modify the file `src/Laplacian.f90` to apply a laplacian to your image. After you modify it, generate the executable by typing `make bin/Laplacian2d.x`. Include your code in the appendix.
3. Apply a laplacian to your post-stack RTM image by typing `make Data/rtmFilter.H`. Include the figure in this paper.
4. The introduction of the paper by Liu et. al 2011 contains a review of several techniques for suppressing RTM artifacts. Pick three of the methods for suppressing RTM artifacts. Discuss the pros and cons of each technique. Answer this question in bullet form.

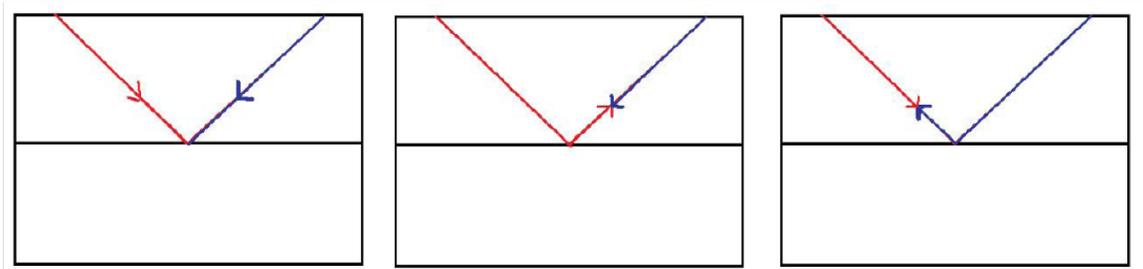


Figure 4: Red ray represents incident source-side wavefield and blue ray is the back-propagated receiver wavefields. Cross-correlation of the source and receiver wavefields yield signal (left) and artifacts (middle and right).

[Insert Image  
Here]

Figure 5: EAGE-SEG salt model RTM image after applying a laplacian filter.

## Common image gather

For velocity analysis and petrophysical studies, it is particularly useful to measure the variations between the partial images at a fixed image point. In class, you have learn that offset-domain common-image gather (ODCIG) and angle-domain common image gather (ADCIG) can be constructed before or after imaging. In this lab, you will construct ADCIG after imaging using the technique proposed by Sava and Fomel 2003.

1. Modify the code `src/rtmOff_oper.f90` to calculate 2D subsurface ODCIG. Include a copy of your code in the appendix.
2. After you modify the code, make sure it compiles by typing `make bin/RTMOff_Mig.x`, calculate one pre-stack ODCIG by typing `make Data/rtmOff175.H`. Include a cubeplot figure in this paper.

[Insert Image  
Here]

Figure 6: A pre-stack offset-domain common image gather at shot number 171.

3. When the migration velocity is correct, what do you expect to see in the ODCIG? Do you observe that in the pre-stack ODCIG you generated in the previous question? Explain your observation.
4. Using the SEP command `OFF2ANG` and a pre-computed ODCIG `Data/offimage.H`, calculate the corresponding ADCIG. Read the manual for `OFF2ANG` carefully to make sure you have all the input parameters. Include a cubeplot figure of your ADCIG.
5. Does the angle illumination range increase or decrease with depth? Explain.

# [Insert Image Here]

Figure 7: A 2D angle-domain common image gather

## ALL DONE

When you are all done, print save a pdf copy of this document (`scons paper.pdf` and upload it to the online dropbox), Clean up your directory (`make clean`).

## REFERENCES

Claerbout, J. F., 1985, Imaging the earth's Interior: Blackwell Science Inc.

Dablain, M.A., 1986, The application of high-order differencing to the scalar wave equation: *Geophysics*, **51**, no.1, 54-66

Liq F., G.Zhang, S.A. Morton, and J.P. Leveille, 2011, An effective imaging condition for reverse-time migration using wavefield decomposition: *Geophysics*, **76**, no.1 , S29-A39

Sava P. and S. Fomel, 2003, Angle-domain common-image gathers by wavefield continuation methods: *Geophysics*, **68**, 1065-1074